Abstract

In Coherent Electron Cooling (CEC), it is essential to study the amplification of electron response to a single ion in the FEL process, in order to properly align the electron beam and the ion beam in the kicker to maximize the cooling effect. In this paper, we use Genesis to simulate the amplified electron beam response of single ion in FEL amplification process, which acts as ‘Green’s function’ of the FEL amplifier.

INTRODUCTION

It is essential to reduce both transverse and longitudinal emittance of the hadron beams (cool the beam) in present colliders, such as RHIC, LHC and the future electron ion collider (EIC) eRHIC, which aim on high luminosities. Current cooling techniques, such as stochastic cooling and electron cooling, cannot provide desire cooling speed at high longitudinal intensity and high beam energy scenarios.

The revolutionary coherent electron cooling (CeC) [1] technique promises to revolutionize the performance of high energy hadron collider and EIC by boosting, by about one order of magnitude, the quality of the beams and their attainable luminosity. CeC scheme is based on the electrostatic interaction between the electron and hadron beams and an adjustable FEL process to amplify the imprinted imperfection on the electron beam. This scheme benefits from the enormous bandwidth of the FEL amplifier, which is several orders of magnitude larger than that of RF components of the traditional stochastic cooling.

In this paper, we focus on the simulation study of the FEL amplification process of a initial signal which resides within one resonance wavelength. This reflects the electron beam response of the a single ion, since the response is usually within several Debye radius [2], which is smaller than the chosen resonance wave length. We take the parameters from the CeC proof of principle (PoP) experiment [3], as shown in table 1.

UNIFORM E-BEAM CASE

We use Genesis 2.0 [4], which is a widely used 3-D FEL simulation code, to study the evolution of the initial electron beam response to the ion beam within one resonance wavelength. First, a train of slices is generated with quiet start algorithm and given beam parameters, i.e. the bunching factor \(< e^{i\theta} >\) is zero. Then a slice with small bunch factor and its desire phase is generated and attached to the tail of the slice train. All slices, as illustrated in figure 1, with or without initial bunching factor, have same beam parameters including the beam size, emittance and beam energy, which corresponds to an electron beam with uniform longitudinal distribution. The electron response to the signal ion resides in the bunch tail and will propagate towards only to the head because the radiation carries the information and always travels faster than the electron beam. The number of slices is determined by the smaller value of i) the bunch length and ii) the undulator period, since the interaction between electron beam and its radiation only carry the information up to the slippage distance \(N\lambda_r\). It is important to synchronize the position of maximum bunching-amplified electron beam slice with the ion which creates it initially.

1-D FEL theory anticipates that the group velocity of the

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Table 1: Parameter of the Electron Beam and the FEL Process

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy (MeV)</td>
<td>21.8</td>
</tr>
<tr>
<td>Charge per bunch (nC)</td>
<td>0.5-4</td>
</tr>
<tr>
<td>Norm. emittance (mm mrad)</td>
<td>5</td>
</tr>
<tr>
<td>Peak current in FEL (A)</td>
<td>60-100</td>
</tr>
<tr>
<td>rms energy spread</td>
<td>1 × 10^{-3}</td>
</tr>
<tr>
<td>Undulator period (m)</td>
<td>0.04</td>
</tr>
<tr>
<td>Undulator length (m)</td>
<td>8</td>
</tr>
<tr>
<td>Undulator parameter (a_w)</td>
<td>0.577</td>
</tr>
</tbody>
</table>

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Figure 1: Top: Lay out of CEC PoP experiment. Bottom: Cartoon for layout of electron beam slices. Red slice corresponds to the one with initial non zero bunching factor; blue slices corresponds to the zero bunching factor generated by ‘quiet start algorithm’ in Genesis. Each slices has the length of the resonance radiation wavelength.
wave package in the exponential growth regime gives:

$$\frac{v_g - \bar{v}_{ez}}{c - \bar{v}_{ez}} = \frac{1}{3}$$

(1)

where $v_g$ is the group velocity of the radiation wave package, $\bar{v}_{ez}$ is the average longitudinal velocity of the electron beam in the undulator and is given by:

$$\bar{v}_{ez} = c \left( 1 - \frac{1 + a_w^2}{2\gamma^2} \right)$$

(2)

Although the ion beam travels also in the undulator in economic CEC layout, its trajectory almost is not affected. Therefore the average velocity of ion beam is faster than $\bar{v}_{ez}$, i.e., $v_i = c - c/\left(2\gamma^2\right)$. In this 1D consideration, if we make $v_i = v_g$ as the synchronization condition, the undulator parameter $a_w$ should have value $1/\sqrt{2}$. However, this may not be optimum when reality effects involves.

Figure 2 show the the bunching factors and the radiation power at each slices during and after the FEL process. It can be easily observed that the interaction of the electron beam and the generated radiation carries the initial bunching information forward as the signal is amplified exponentially along the undulator. Figure 3 gives the values and positions of the maximum bunching factor and radiation power. We find that in 3D case, the anticipate group velocity is slower than that of 1D model (Eq. 1). If we fit the linear part of bottom one in figure 3 (after 3 meters), we find that the right hand side of the Eq. 1 should change from $1/3$ to approximately $1/4$. The finite size of the electron beam slows down the group velocity of the wave package and the ‘bunching’ package. Therefore the new approximate value $1/4$ is determined by electron beam size in the undulator, which is 0.28 mm. If we match the ion velocity with this new group velocity, we will get the optimum undulator parameter $a_w = 1/\sqrt{3} = 0.577$, instead of $1/\sqrt{2}$. In addition, the group velocity of the bunching wave package is a constant only at exponential growth stage. In the first 3 meters of this case (figure 3), the beam slice with initial bunching factor coherently radiate without amplification and the velocity of the bunching wave package is close to zero. At the undulator exit, the bunching wave package only travels to the 34$^{rd}$ slice; while the ion, that create the initial signal at first slice, now resides at 51$^{th}$ slice relatively. From figure 2, this results less than factor of 2 reduction of amplification and increment of cooling time, which is acceptable in CEC PoP experiment.

If we require that the ion beam resides exactly on the
location of the maximum bunch factor, the undulator parameter $a_w$ needs to be further decreased to about $1/\sqrt{5}$. However, this cause dramatic decrement in bunching factor at undulator’s exit since its length is fixed. In this example, the bunch factor at the undulator exit drops about 50 times when $a_w$ drops from $1/\sqrt{3}$ to $1/\sqrt{5}$.

## NON-UNIFORM E-BEAM CASE

We may extend similar analysis to general cases. In reality, the electron beam has various parameters along its longitudinal directions, such as peak current and average beam energy. There is recent theoretical calculation[6] on cooling phase due to the peak current and beam energy variation. In CEC PoP experiment, the electron beam after the bunch compressor hardly has an ideal uniform distribution. We may use a Gaussian function to model its peak current. In additional, the beam will be accelerated on crest by a 704 MHz RF cavity to reach 21.8 MeV. The curvature of the accelerating field will produce the beam energy variation, which can be characterized by a cosine function. Both effects cause output bunching and its phase dependence on the initial ion position. We may synchronize and adjust the cooling phase to one value, which lead to the maximum cooling speed. However, this may not be the optimum value for all ions that overlaps the electron beam in the modulator. In the following exercise, we move the position of the initial signal from the single ion along the longitudinal axis of the electron beam. At the end of the undulator, we synchronize the ion at the peak current of the electron beam with the electron slice of largest amplified response. There will be relative phase difference among the longitudinal positions. If the phase difference exceeds $\pi/2$, the tail of the electron beam will heat the ion beam instead of cool it.

Followed by the uniform e-Beam study, we can generate $N$ slices with their own peak current and mean energy, where $N = l_e/\lambda_r$, $l_e = 8\sigma_{ez}$ is the full length of the electron beam, where we assume the longitudinal distribution is Gaussian with $4 - \sigma$ cutoff. The initial signal from ion is changed through the $N$ slices to demonstrate the effects of varying parameters in the longitudinal direction. Figure 4 shows the amplification of initial bunching decrease along with current reduction and the phase difference becomes significant due to both the energy and current variation. However, we confirmed the theoretical result in [6], that the effects from these two sources are in opposite direction. And the simulation gives that at rms bunch length of 2 mm, the phase of the bunching that the ion beam meets at the end of FEL process is flat for both the full synchronized case (ideal) and the economic mode (as in CEC PoP experiment). Although the tails has phase difference larger than $\pi/2$, the bunching amplification is too tiny to affect the outcome.

![Figure 4: The output bunch factor (top) and phase (bottom) that the ion encounters at the exit. The bunch length is rms value.](image)

## CONCLUSION

We demonstrate that the simulation platform of Genesis and home-made pre- and post-processing codes, becomes powerful tool to analyze and optimize the FEL amplifier of CEC. It may includes 3D effects and arbitrary distributions of the electron beam, which yield more accurate prediction of the cooling performance.

## REFERENCES