Abstract

In this article, we will review the techniques to measure the off-momentum β-beat and the correction algorithms with the chromatic sextupoles in RHIC. We will focus on the measurement and correction of the off-momentum β*-beat at the interaction points. The off-momentum β* is measured with the quadrupole strength change and a high resolution phase lock loop tune meter. The results of off-momentum β* correction performed in a dedicated beam experiment in the 2012 RHIC 250 GeV polarized proton run are presented.

INTRODUCTION

Based on the perturbation approach, the relative first order off-momentum β-beat is given by

\[
\frac{1}{\beta_{x,y}(s)} \frac{\partial \beta_{x,y}(s)}{\partial s} = \pm \frac{1}{2 \sin(2 \pi s \phi_{x,y})} \int \beta_{x,y}(s') \left[ K_1(s') \pm K_2(s') D_x(s') \right] \cos(2(\phi_{x,y}(s) - \phi_{x,y}(s')) - 2\pi Q_{x,y}) ds'.
\] (1)

Here β_{x,y} are the betatron amplitude functions, φ_{x,y} are the betatron phase advances. δ = dp/p₀ is the relative momentum deviation. K₁ and K₂ are the strengths of the quadrupoles and sextupoles. D_x is the horizontal dispersion.

And the second order chromaticities are given by [1, 2]

\[
\xi_{x,y}^{(2)} = -\frac{1}{2} \xi_{x,y}^{(1)} + \frac{1}{2 \pi} \int \left[ K_1(s') \pm K_2(s') D_x(s') \right] \frac{\partial \beta_{x,y}}{\partial s} ds + \frac{1}{2 \pi} \int K_2(s') D_x(s') ds,
\] (2)

For RHIC, the second term in Eq. (2) is the dominant which is related to the first order off-momentum β-beat.

According to Eq. (1) and (2), both the first off-momentum β-beat and the second order chromaticity are determined by the half integer resonance driving terms which are defined as

\[
h_{2001} = \sum_i N \left[ -(K_1 L)_i + (K_2 D_x L)_i \right] \beta_{x,i} e^{-i2\phi_{x,i}},
\] (3)

\[
h_{0021} = \sum_i N \left[ -(K_1 L)_i + (K_2 D_x L)_i \right] \beta_{y,i} e^{-i2\phi_{y,i}}.
\] (4)

Therefore, we can minimize the half integer resonance driving terms to correct both the first order off-momentum β-beat and the second order chromaticity.

In this article we focus on first order off-momentum β*-beat correction with the chromatic sextupoles in the arcs. In RHIC, the number of independent chromatic sextupole power supplies were doubled from 12 to 24 in the summer of 2007. In each sextant of RHIC, there are 4 independent sextupole power supplies. Considering the phase advances of each FODO cell is close to 90°, we split the focusing and defocusing sextupoles into 2 sub-families in each sextant. The sextupoles in each sextant are sorted as SF1, SF2, SF3, SF4. There are 24 sub-families in total in each RHIC ring.

In the following, we first present the correction algorithm to the off-momentum β-beat. Then we carry out a numerical simulation to verify the correction method with the Blue ring lattice in the 2011 RHIC Au-Au ion run. In the end we present the experimental results of off-momentum β*-beat measurement and correction in the Blue ring in the 2012 RHIC 250 GeV polarized proton run.

ALGORITHM AND SIMULATION

The off-momentum β-beat can be corrected based on the optics model and its online measurement. If the linear optics model reflects the accelerator precisely, the off-momentum β-beat correction will be straight-forward by simply dialing in the offline calculated correction strengths into the machine. However, if there is a large discrepancy in the linear optics between the offline model and the real machine, we have to perform online off-momentum β-beat correction based on its actual measurement. Ref. [3] shows that 15% on-momentum β-beat could cause 30% change in the second order chromaticity. The on-momentum β-beat comes from the quadrupole gradient errors.

No matter of offline or online correction, the correction algorithms to the off-momentum β-beat are the same. For example, with the popular singular value decomposition method, we could calculate the correction strengths with the linear response matrix between the off-momentum β-beat and the sextupole families,

\[
\begin{pmatrix}
\Delta \frac{d\beta}{ds}^1 \\
\Delta \frac{d\beta}{ds}^2 \\
\Delta \frac{d\beta}{ds}^3 \\
\vdots \\
\Delta \frac{d\beta}{ds}^M
\end{pmatrix} = A \cdot 
\begin{pmatrix}
\Delta (k_2 L)_1 \\
\Delta (k_2 L)_2 \\
\Delta (k_2 L)_3 \\
\vdots \\
\Delta (k_2 L)_{N-1} \\
\Delta (k_2 L)_N
\end{pmatrix}.
\] (5)

Here the left sides are the off-momentum β-beat changes together with the linear chromaticity change. The right side are the sextupole families. We can use the off-momentum
for the 2011 Au-Au run. The working point of this lattice is (31.23, 32.22). The $\beta^*$ at IP6 and IP8 are 0.7 m. At other non-colliding IPs, the $\beta^*$s are 5 m. The second order chromaticities without correction are (-7500,2800).

Fig. 1 shows the off-momentum horizontal $\beta^*$ at IP6 without and with correction. Fig. 2 shows the off-momentum $\beta_x$-beat along the ring without and with correction. Fig. 3 shows the off-momentum horizontal tunes without and with correction. The correction strengths are: $\Delta K2L$ for SFPI is 0.12 m$^{-2}$, $\Delta K2L$ for SFMI is -0.12 m$^{-2}$. From Fig. 2 and Fig. 3, after the off-momentum $\beta^*$-beat correction, the global off-momentum $\beta$-beats and the second order chromaticity are also reduced.

**EXPERIMENTAL RESULTS**

The key to the online off-momentum $\beta$-beat correction is to measure the off-momentum $\beta$ functions. There are several techniques available in RHIC to measure $\beta$ functions. We can use an AC dipole to excite the beam to generate a coherent beam motion [4] or use a tune meter to kick the beam to generate a de-coherent beam motion. Based on the 1024 turns of turn-by-turn BPM data along the ring, we can determine $\beta$s at valid BPMs. The data processing algorithms for the two methods are very similar.

To change the off-momentum $\beta$s, we shift the horizontal beam position inside or outside of the beam pipe by adjusting the RF frequency through a radial loop. For the following experiment in the 2012 RHIC 250 GeV polarized proton run, a 1.0 mm radial shift corresponds to $|dp/p_0| = 0.93 \times 10^{-3}$. We also measure the second order chromaticity with the radial shifts.

We normally set the AC dipole driving tune 0.01 away from the betatron tune. The driving tune of the AC dipole should be out of the beam tune spectrum in order to keep the beam losses low during the measurement. Therefore, it is important to reduce the beam tune spread before excitation by setting the linear chromaticity close to zero.
Using the tune meter kicker also requires low chromaticities to obtain a longer coherent beam motion. However, in the beam experiments we observed the linear chromaticity change after radial shifts. To measure $\beta$ functions, we had to adjust linear chromaticities again which may affect the off-momentum $\beta$-beat’s measurement and correction.

When looking only at the off-momentum $\beta^*$-beat at the IPs, we can measure the $\beta^*$s with the strength changes of the first quadrupoles (Q1) next to the IPs. This method does not need to adjust chromaticity at different radial shifts. The tune changes are measured with a high resolution phase lock loop based tune meter whose absolute resolution is normally better than $10^{-4}$. Fig. 3 shows the tune data during one measurement. At each IP, we change the strengths of the two Q1 quadrupoles on both sides of the IP. Each measurement gives both horizontal and vertical $\beta^*$ at the IP and takes about 2 minutes.

Figure 4: Tune readings during $\beta^*$ measurement with Q1 quadrupole strength change method.

Figure 5: Measured $\beta^*_x$ at IP6 versus $dp/p_0$ without and with correction.

Limited by the tight beam time in the beam experiment, we were only able to measure the horizontal off-momentum $\beta^*$ at IP6 with the Q1 quadrupole strength change method in the Blue ring in the 2012 RHIC 250 GeV polarized proton run. The measured horizontal second order chromaticity is 4600 without correction. With a moderate correction, the horizontal second order chromaticity is reduced to 3000. The correction strengths are: $\Delta(K2L)$ for SFPI family is $-0.09 m^{-2}$, $\Delta(K2L)$ for SFMI family is $0.09 m^{-2}$.

Fig. 5 shows the measured $\beta^*_x$ at IP6 versus the relative off-momentum deviation. Fig. 6 shows the measured off-momentum tunes versus the relative off-momentum deviation. The maximum radial shifts are 0.5 mm and 1.0 mm during the off-momentum $\beta^*$ and tune measurements respectively. From Figs. 4 and 5, both the horizontal off-momentum $\beta^*$ at IP6 and the horizontal second order chromaticity are reduced with the above correction strengths.

SUMMARY

In this article we reviewed the techniques to measure and correct the off-momentum $\beta$-beat in RHIC. We focused on the measurement and correction of $\beta^*$-beat at the IPs, especially at the colliding IPs. With the Blue ring lattice in the 2012 RHIC 250 GeV polarized proton run, we obtained some preliminary experimental results of the off-momentum $\beta^*$-beat measurement and correction at IP6.

REFERENCES