PARAMETRIC-RESONANCE IONIZATION COOLING IN TWIN-HELIX*

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Abstract

Parametric-resonance Ionization Cooling (PIC) is proposed as the final 6D cooling stage of a high-luminosity muon collider. For the implementation of PIC, we developed an epicyclic twin-helix channel with correlated optics. Wedge-shaped absorbers immediately followed by short rf cavities are placed into the twin-helix channel. Parametric resonances are induced in both planes using helical quadrupole harmonics. We demonstrate resonant dynamics and cooling with stochastic effects off using GEANT4/ G4beamline. We illustrate compensation of spherical aberrations and benchmark COSY Infinity, a powerful tool for aberration analysis and compensation.

INTRODUCTION

Combining muon ionization cooling with parametric resonant dynamics should allow much smaller final transverse muon beam sizes than conventional ionization cooling alone [1-2]. Thus, high luminosity would be achieved in a collider with fewer muons. Parametric-resonance Ionization Cooling (PIC) is accomplished by inducing a half-integer parametric resonance in a muon cooling channel. The beam is then naturally focused with a period of the channel's free oscillations. The channel is designed [3-5] with correlated values of the horizontal and vertical betatron periods so that focusing occurs in both planes simultaneously. Absorber plates for ionization cooling followed by energy-restoring RF cavities are placed at the beam focal points. At the absorbers, ionization cooling limits the angular spread while the parametric resonance causes a strong reduction of the beam spot size. The normalized equilibrium transverse emittance achievable in this resonant cooling scheme is given by [2]

$$\varepsilon_{\perp}^{n} = \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w\,, \qquad (1)$$

where $\beta = \upsilon / c$ is the relativistic velocity factor, Z is the absorber material's atomic number, m_e and m_μ are the electron and muon masses, respectively, and w is the average absorber thickness in the beam direction. Compared to the conventional ionization cooling, the emittance of Eq. (1) is reduced by at least an order of magnitude by a factor [2]

$$\frac{\pi}{\sqrt{3}}\frac{w}{\lambda_x} = \frac{\pi}{2\sqrt{3}}\frac{\gamma'_{acc}}{\gamma'_{abs}}\,, \qquad (2)$$

where λ_x is the period of the horizontal betatron oscillations and γ'_{acc} and γ'_{abs} are the RF acceleration and intrinsic absorber energy loss rates, respectively. The equilibrium beam size σ_a and angular spread θ_a at the absorber and the equilibrium momentum spread $\Delta p/p$ are given by [2]

$$\sigma_a^2 = \frac{1}{8}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu}w^2\,, \qquad (3)$$

$$\theta_a^2 = \frac{3}{2}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu}\,, \qquad (4)$$

$$(\Delta p/p)^2 = \frac{3}{8}\frac{(\gamma^2+1)}{\gamma\beta^2}\frac{m_e}{m_\mu}\frac{1}{\log}\,, \qquad (5)$$

where γ is the muon relativistic energy factor and log is the Coulomb logarithm of ionization energy loss for fast particles. For instance, for 250 MeV/c muons in a cooling channel with 5 mm thick Be absorbers, Eqs. (1) and (3)-(5) give $\varepsilon_{\perp}{}^{n} \approx 60$ μm·rad, $\sigma_a \approx 0.2$ mm, $\theta_a \approx 130$ mrad, and $\Delta p/p \approx 0.02$.

PARAMETRIC RESONANCE

To induce a half-integer parametric resonance, we earlier [5] used short lumped quadrupole magnets. That scheme, however, was primarily used for demonstration purposes. Its practical implementation would be difficult and the quadrupole fringe fields could potentially drive beam resonances and produce aberrations. Moreover, inducing a resonance in one plane was causing a dynamic instability in the other plane [5]. This simple model helped to gain a qualitative understanding of the parametric resonance driving scheme. We then used a pair of opposite-helicity but otherwise identical helical quadrupoles [6, 7] to excite a parametric resonance in each plane. The periodicity of the helical quadrupole pair was twice shorter than the respective betatron period [5]. Locations of the focal points were set with respect to the main twin helix by adjusting the parametric quadrupole phases using the fact that the focal point is located 1/8 of the respective betatron period from the maximum of the

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quadrupole field [5]. Using this approach, we were able to independently induce a parametric resonance in each plane without causing instability in the other plane.

The horizontal and vertical phase-space trajectories of a 250 MeV/c μ^- at the desired absorber location with half-integer parametric resonances excited in both planes are shown by the red dots in the top and bottom parts of Fig. 1, respectively. The hyperbolic shape of the horizontal phase-space trajectory near the graph's center is indicative of a half-integer parametric resonance. The vertical phase-space motion is a little more complicated because it is strongly coupled to the horizontal motion. Regions of hyperbolic motion in the vertical phase space suggest that the particle goes in and out of the vertical parametric resonance depending on the particle's horizontal position. Tracking in this and later studies presented in this paper was done using the GEANT-based G4beamline program [8].

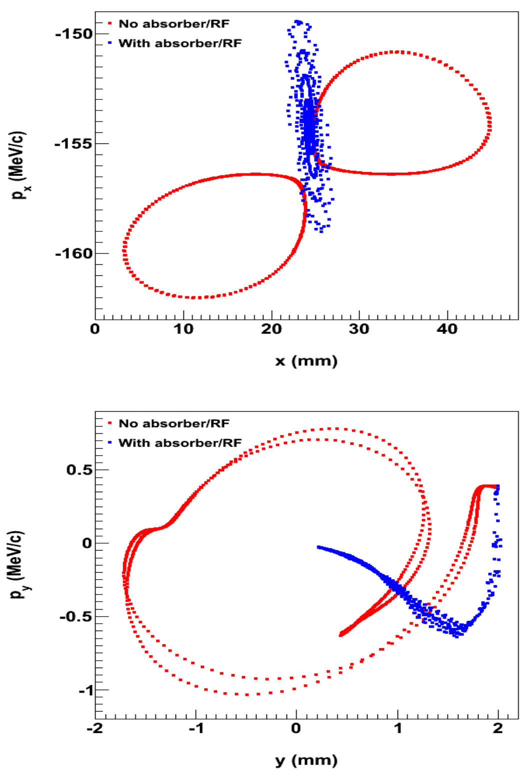

Figure 1: Horizontal (top) and vertical (bottom) phase-space trajectories with half-integer parametric resonances excited in both planes without (red) and with (blue) absorbers/RF. Stochastic effects are off.

COOLING SIMULATION

We next placed 2 cm thick Be wedge absorbers with a thickness gradient of 0.3 at the resonance focal points. Locations of the focal points were chosen at the places in the twin helix with 3 cm dispersion [5]. This dispersion value combined with the absorber thickness gradient should provide sufficient longitudinal cooling to maintain the momentum spread at a constant level [5]. The wedge absorbers were immediately followed by short 201 MHz RF cavities, which completely restored the absorber energy loss. At this level of study, the cavities were made unrealistically short to avoid large energy perturbations and to decouple from the transit time effects.

A 250 MeV/c μ^- was tracked through the system with stochastic effects turned off. The resulting horizontal and vertical phase-space trajectories at the absorber location are shown by the blue dots in Fig. 1 and compared to those without the absorbers/RF. Note an overall reduction of the size at an increase of the angular spread characteristic to PIC. Figure 2 shows evolution of the horizontal phase space along the cooling channel. Note the two distinct branches in the phase-space trajectory due to the half-integer parametric resonance.

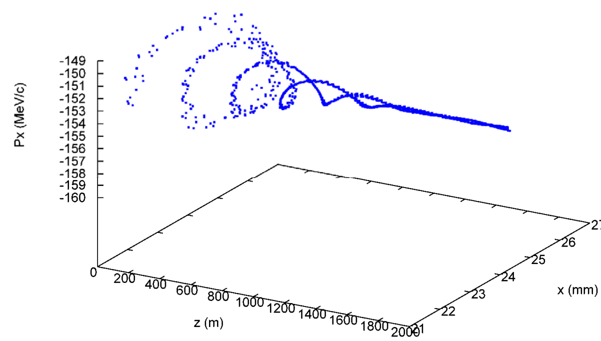

Figure 2: Evolution of the horizontal phase space along the cooling channel with absorbers/RF under the parametric resonance condition. Stochastic effects are off.

ABERRATION COMPENSATION

Horizontal Spherical Aberration

When the stochastic effects are turned on in the cooling simulation described above, the particle motion becomes unstable. This is due to aberration impact between consecutive focal points. Both chromatic and spherical aberrations must be compensated to a degree where they are small compared to the beam size at the absorber. Since, according to Eq. (4), the equilibrium angular spread in on the order of a hundred milliradians, the spherical aberrations must be precisely compensated over an angular range of a few hundred milliradians. This is a challenging task. However, some of the intrinsic symmetries of the correlated optics reduce the number of the aberration compensation conditions. The number of conditions can be further reduced by using a coupling resonance [2].

The conventional approach to aberration compensation is to correct aberrations sequentially from lower to higher order by successively introducing appropriate higher-order multipoles. However, it cannot be done in such a straightforward way in a twin helix since the reference orbit does not pass through the center of the magnetic system. Introducing a higher-order multipole in a twin helix feeds-down lower-order multipole components along the orbit. Aberration compensation has to be done by adjusting different-order multipole components simultaneously. Here we demonstrate compensation of the horizontal spherical aberration using a straight octupole and a pair of helical decapoles. Their strengths were tuned using a generic fitting algorithm to minimize the smear of a monochromatic point-source beam with ±200 mrad horizontal angular spread after 2 helix periods. The μ^- tracks over 2 helix periods from one absorber location to another are shown before and after the compensation in Fig. 3. Figure 4 illustrates impact of the compensation on the channel's dynamic acceptance shown in terms of the initial horizontal and vertical momentum components, for which the particle later remains dynamically stable.

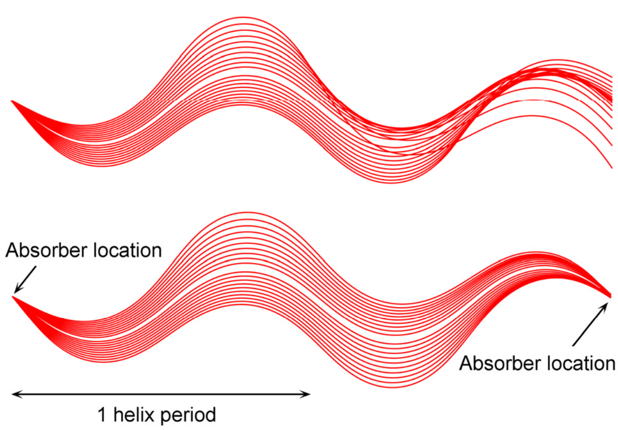

Figure 3: μ^- tracks distributed over ±200 mrad in the horizontal plane before (top) and after (bottom) compensation of the horizontal spherical aberration.

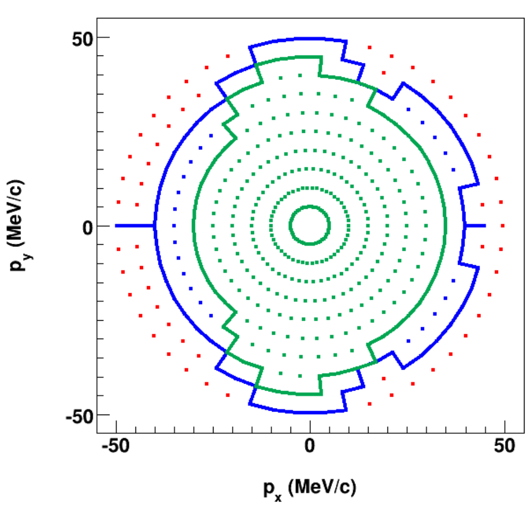

Figure 4: Dynamic acceptance in terms of the stable initial horizontal and vertical momentum components of a monochromatic point-source beam. The green and blue lines show the boundaries of the dynamic acceptance before and after the compensation, respectively.

COSY Infinity Benchmark

To systematically approach aberration correction, we plan on using COSY Infinity [9,10], a well-known code for aberration analysis and compensation. COSY works by expanding a particle's trajectory around a reference orbit to an arbitrary order in the 6D phase-space coordinates. Therefore, COSY is well-suited for our purpose. It has been extended to include helical harmonics and to calculate the reference orbit in a twin helix. The results of applying various orders of COSY to calculate the smear of a ±160 mrad monochromatic point-source beam after 2 helix periods are compared to a G4beamline simulation in Fig. 5 with a good agreement.

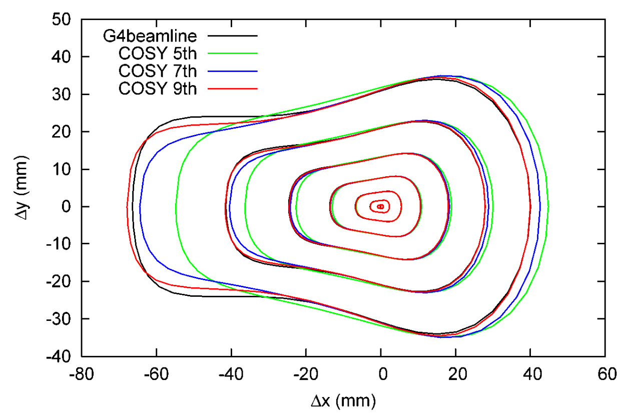

Figure 5: Beam smear due to spherical aberrations after 2 helix periods. G4beamline simulation is compared to various-order COSY Infinity calculations.

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