Abstract

Many non-scaling FFAG accelerator designs involve magnetic fields that cannot be described in popular accelerator design codes, and complex beam dynamics that require extremely accurate simulations. A recent design of a 1 GeV isochronous non-scaling FFAG is used to compare the codes COSY Infinity and ZGOUBI, both of which are commonly used in FFAG design. Results are presented for the comparison of basic beam dynamics and calculated dynamic aperture.

INTRODUCTION

High power proton accelerators are required for many future applications, including the development of accelerator driven systems (ADS) for the transmutation of nuclear waste or generation of power. The development of very high power proton drivers of around 10 MW or more remains a considerable challenge to the accelerator community. However, the potential success of such facilities may lie in the possibility of accelerators which are small, reliable and relatively low-cost.

A non-scaling FFAG accelerator has recently been designed by Johnstone et al. [1, 2] with isochronous orbits and stable betatron tunes. Such an accelerator could operate in CW mode and has the potential to meet the criteria of an ADS proton driver at 1 GeV.

During the development stages of novel FFAGs, simulation codes are heavily relied upon to provide an understanding of beam dynamics. These codes must typically be able to handle magnetic fields of arbitrary multipole order with flexible geometry, and to incorporate higher order dynamics.

Well-known codes used for the design of accelerators such as cyclotrons and synchrotrons may provide a good first estimate, but can be lacking in their flexibility of magnetic element descriptions or limited in their accuracy, particularly at large particle amplitudes.

There are two codes that are frequently used for detailed characterisation of non-scaling FFAGs; COSY Infinity [3, 4] and ZGOUBI [5]. This paper discusses the basic dynamics of the isochronous non-scaling FFAG design and compares the results of COSY Infinity and ZGOUBI in terms of betatron tunes, isochronicity and dynamic aperture.

ISOCHRONOUS NON-SCALING FFAG

The lattice design is outlined here for clarity, but is described in further detail elsewhere [1, 2]. The design takes advantage the three types of focusing available in circular accelerators; weak focusing, edge focusing and strong focusing. Refinement of the interplay between these three types of focusing means that the betatron tune, orbit location and accelerator footprint can be controlled at the design stage. The lattice is periodic, and the presence of strong focusing allows for relatively long straight sections which are expected to be advantageous when considering RF, injection and extraction regions.

The design consists of four identical cells and is very compact, reaching a 1 GeV proton energy at an average radius of just over 5 metres, while maintaining long straight sections of 2 metres. The main parameters of the lattice design are given in Table 1 and the field map for one quadrant of the machine is shown in Fig. 1.

Table 1: Lattice Parameters of 250-1000 MeV Isochronous Non-scaling FFAG for ADS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>250 MeV</th>
<th>1000 MeV</th>
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<tr>
<td>Avg. Radius [m]</td>
<td>3.419</td>
<td>5.030</td>
</tr>
<tr>
<td>Cell $\nu_x/\nu_y$</td>
<td>0.380/0.237</td>
<td>0.383/0.242</td>
</tr>
<tr>
<td>B Field F/D [T]</td>
<td>1.62/-0.14</td>
<td>2.35/-0.42</td>
</tr>
<tr>
<td>Magnet length F/D [m]</td>
<td>1.17/0.38</td>
<td>1.94/1.14</td>
</tr>
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Figure 1: A field map of one quarter of the lattice showing the vertical magnetic field in Tesla. Magnetic field profiles are measured along the centre of the F and D magnets, shown in Fig. 2.
SIMULATION DETAILS

Original characterisation of the isochronous ns-FFAG design were performed by Johnstone et al. using COSY Infinity, a kinematic code which has been heavily updated in recent years to allow for the simulation of non-scaling FFAGs including fringe field effects, non-linearities and full kinematics with no paraxial approximation. The betatron tunes and time-of-flight results have been verified using the cyclotron code CYCLOPS [6] with a smoothing applied to the magnet fringe fields. The results indicate that the design should maintain isochronicity to within ±3% with total betatron tunes varying by no more than 0.5 throughout acceleration. Initial studies of the machine indicated that it ought to have a 50 to 100 π mm mrad dynamic aperture. All results used herein for comparison have been taken from Refs. [1, 2].

ZGOUBI is a ray-tracing code which can track particles through electric and magnetic fields introduced as field maps or as analytic elements. It has excellent flexibility in the choice of analytic elements, including complex geometry with high-order multipole and combined-function magnets. For this reason it was selected as one of the main codes for the design and characterisation of the first non-scaling FFAG, EMMA [7] at Daresbury Laboratory, and has been used extensively in the design of PAMELA, a non-scaling FFAG accelerator for medical applications [8].

To ensure that no artificial effects were introduced by the existing field maps, an analytic ZGOUBI magnet element was employed to test the dynamics of the lattice design. The geometry of the lattice was inferred from detailed study of the existing field map. Each cell transports the beam through 90° and is followed by a 2 m drift section. The edge of the magnet is taken to be the point where the field falls off to zero, giving the F magnet an opening angle of 25.7° and the D an opening angle of 13.4° as shown in Fig. 1 The multipolar magnetic field is defined from a polynomial fit to the radial magnetic field profile along the centre of each magnet and an Enge type fringe field with a 5 cm extent is employed. The fitted radial field profiles are shown in Fig. 2.

![Figure 2: Radial field profile from field map (blue points) with polynomial fit (black line) for the F (left) and D (right) magnets, taken along the centre line of each magnet.](image)

RESULTS

The closed orbits were established by minimising the phase space area of the oscillations of a single particle launched near the expected closed orbit. The results for a range of energies throughout acceleration are shown in Fig. 3a. To make a basic test of the isochronous condition of the lattice, the variation in the time of flight around the ring for a single particle on the closed orbit was calculated, as shown in Fig. 3b. The time-of-flight result is consistent with the quoted isochronicity of ±3%.

After establishing the closed orbits the betatron cell tunes are calculated by tracking sets of 11 particles through a single cell of the lattice to determine the transfer coefficients of the structure, as described in [5, p.134]. The result is shown in Fig. 3c.

![Figure 3: (a) Variation of the closed orbit position at the centre of the long straight section throughout acceleration, ZGOUBI results are shown as a black line and match well with the original COSY Infinity results (red). (b) Variation in time-of-flight as a percentage of the mean time-of-flight for a single particle on the closed orbit. (c) Variation in the horizontal (blue) and vertical (green) betatron cell tunes throughout acceleration range. The original COSY Infinity results are also shown (red) and the two sets are consistent. The small deviation at low energy is expected to arise from the difference between the field maps and the ZGOUBI analytic model, most notably in the description of the magnet fringe field.](image)
Dynamic Aperture

The 4D dynamic aperture was calculated by tracking single particles with amplitudes in both transverse planes. The coasting beam dynamic aperture at injection was established by tracking these particles over 1000 turns, where the limit of the dynamic aperture is determined to be the largest starting amplitude which does not produce particle loss during the 1000 turn tracking. This does not incorporate any physical beam-pipe aperture but relies simply on loss occurring due to the trajectory becoming unstable or moving outside the region of magnetic field.

The accelerated dynamic aperture was calculated using a similar method, tracking particles throughout acceleration with different acceleration rates, using a simple energy gain per turn model for acceleration. The results of these tracking simulations are presented in Table 2 and coasting beam phase space portraits are shown in Fig. 4. The dynamic aperture appears to be very large, and some interesting high-order island effects between the horizontal and vertical planes are observed at large amplitudes.

Table 2: Dynamic aperture calculated using single particle tracking for coasting beam and various acceleration rates.

<table>
<thead>
<tr>
<th>Accel. rate</th>
<th>turns</th>
<th>DA [$\pi$ mm mrad normalised]</th>
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<tr>
<td>0 (at 250 MeV)</td>
<td>1000</td>
<td>420</td>
</tr>
<tr>
<td>1 MV/turn</td>
<td>750</td>
<td>374</td>
</tr>
<tr>
<td>2 MV/turn</td>
<td>375</td>
<td>411</td>
</tr>
<tr>
<td>4 MV/turn</td>
<td>188</td>
<td>450</td>
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Figure 4: Phase space portraits for single particles tracked for 1000 turns starting with simultaneous horizontal and vertical offsets in steps of 2 mm at injection (upper) and extraction (lower) in the horizontal (left) and vertical (right) planes.

DISCUSSION

Simulations in ZGOUBI confirm the basic dynamics of this novel isochronous non-scaling FFAG lattice design. The predicted dynamic aperture is very large, particularly for rapid acceleration of less than 200 turns. It is envisaged that this rate of acceleration would be required in order to attain good orbit separation at extraction.

This design would most likely require an extraction system similar to that of a high power cyclotron and this may present a significant challenge. More fundamentally, in order for this novel accelerator to meet the criteria of a high power CW proton source, a study of the dynamics in the presence of a high intensity beam is paramount. At present neither COSY Infinity or ZGOUBI incorporate space charge. Developments to include space charge in both codes are in their early stages [9], and the modelling of non-scaling FFAGs in an existing field map tracking code (OPAL [10]) is underway.

ACKNOWLEDGEMENTS

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REFERENCES