ENERGY LOSS AND LONGITUDINAL WAKE FIELD OF RELATIVISTIC SHORT ION BUNCHES IN ELECTRON CLOUDS *

O. Boine-Frankenheim, F. Yaman †, E. Gjonaj, T. Weiland, TU Darmstadt, Germany
G. Rumolo, CERN, Geneva, Switzerland

Abstract

The aim of our study is the numerical computation of the wake field and energy loss per unit length for a relativistic, short (< 10 ns) ion bunch penetrating an electron cloud residing in the beam pipe. We use two different self-consistent PIC codes: A standard 2-D electrostatic PIC code and a higher order 3-D PIC code which is based on the full-wave approach for the Maxwell equations in the time domain. The parameter scope of the codes refers to the CERN LHC and SPS accelerators.

INTRODUCTION

The interaction between an ion beam and an electron cloud may lead to coherent instabilities and beam loss. In this context, wake fields induced by a short, relativistic ion bunch in an electron cloud have already been obtained numerically in Ref. [1, 2]. Furthermore, analytic expressions for wake fields and impedances in the framework of the dielectric response theory were reported in Ref. [3]. In the present work we focus on the energy loss of bunches, as a result of the longitudinal wake field induced in the electron cloud in field free and dipole sections.

ENERGY LOSS AND RF PHASE SHIFT

This study has been motivated by observations in SPS and LHC. In both machines an increase in the shift of the synchronous phase $\Delta \phi_s$ with decreasing bunch spacing has been observed [4]. During beam storage the phase shift in a rf bucket is

$$\sin(\Delta \phi_s) = \frac{\Delta W_p}{q V_{rf}}$$  (1)

where $q$ is the ion charge, $V_{rf}$ is the rf amplitude and $\Delta W_p$ is the energy loss per particle and per turn. The measurement of $\Delta \phi_s$ can be used to gain information on the longitudinal impedance spectrum [5]. In LHC the observed dependence of the phase shift on the bunch spacing indicates that electron clouds can be the source of the energy loss. In general the stopping power $S$ (energy loss of the total bunch per length unit) can be written as

$$\frac{dW}{ds} = - \int \rho_i(\vec{r}) E_z(\vec{r}) d^3r \approx -q \int \lambda(z) E_z(z) dz$$  (2)

where $\rho_i$ is the bunch charge density, $E_z(z)$ is the longitudinal electric field induced by the bunch, $\lambda(z)$ is the line density of the bunch and $q = Ze$ is the ion charge. The energy loss per ion and per turn is

$$\Delta W_p = \frac{L}{N_i} \frac{dW}{ds}$$  (3)

where $L$ is the ring circumference and $N_i$ the number of ions in the bunch.

ELECTRON EQUATION OF MOTION

The rigid bunch of velocity $v_0 \approx c$ interacts with the electrons via its transverse electric field $E^i_{\perp}(r,z)$. Here we ignore the beam’s magnetic field as well as any magnetic field induced by the electrons. For a transverse Gaussian beam profile one obtains for the electric field

$$E^i_{\perp}(r,z) = \frac{q \lambda(z,t)}{2\pi e\sigma_i^2} \left[ 1 - \exp \left( -\frac{r^2}{2\sigma_z^2} \right) \right]$$  (4)

For the sake of simplicity we assume a round beam of radius $a = 2\sigma_z$. The line density of the bunch is assumed to be Gaussian with

$$\lambda(z) = \frac{N_i}{\sqrt{2\pi\sigma_z}} \exp \left( -\frac{z^2}{2\sigma_z^2} \right)$$  (5)

where $z = z_0 - ct$, $N_i$ is the number of ions in the bunch, $\sigma_z$ is the rms bunch length. The resulting electron equation of motion is

$$r'' + \kappa^2(r,z) r = \frac{e E^i_{\perp}(r,z)}{m_e c^2}$$  (6)

where $\kappa(r,z)$ represents the focusing force due to the beam’s transverse electric field and $E^i_{\perp}$ is the electric field of the electron cloud. For $r < a$ the focusing gradient is

$$\kappa(z) = \frac{\sqrt{2\lambda(z)r_e}}{a}$$  (7)

The electron space charge field $E^e_{\perp}(r,z)$ induced by the bunch in the cloud has to be obtained from Gauss law with the electron charge density $\rho_e(r,z)$.

STOPPING POWER FOR SHORT BUNCHES

First we will ignore the effect of electron space charge. Furthermore we will assume that the bunch length $\sigma_z$ is short relative to the electron oscillation length in the bunch center $\kappa^{-1}(0) = \kappa_0^{-1}$.

$$\kappa_0 \sigma_z \ll 1$$  (8)

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† yaman@temf.tu-darmstadt.de

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and that the electrons are homogeneously distributed inside
the beam pipe of radius \( R_p \gg a \). In this case the majority
of the electrons will simply receive a transverse impulse kick \( \Delta \rho_\perp (b) \) from the passing bunch (see also Ref. [6]). \( b \) is the impact parameter or transverse distance between the
electron and the beam axis. The total energy gain of the
electrons per unit length is

\[
\frac{dW_e}{ds} = \frac{1}{2} m_e n_e \int_0^{R_p} 2\pi \Delta \rho_\perp^2 (b) b db
\]  

(9)

and the stopping power

\[
S = -\frac{dW_e}{ds} \approx 4\pi Q_i^2 n_e e_\perp \ln \left( \frac{R_p}{a} \right)
\]  

(10)

The corresponding rf phase shift per unit length is (assuming \( \Delta \phi_s \ll 1 \))

\[
\frac{d\Delta \phi_s}{ds} \approx \frac{4\pi Q_i^2 n_e e_\perp}{V_{rf}} \ln \left( \frac{R_p}{a} \right)
\]  

(11)

In a strong dipole field the electrons can only respond along the
magnetic field lines and the stopping power given by
Eq. 10 is reduced by the factor 1/2. With electron space we
use Mulser’s ‘oscillator model’ [7] in order to calculate the
stopping power. The energy loss of the bunch is obtained
from the energy transferred into plasma waves

\[
S = \frac{Q_{i2}^2 \kappa_e^2}{4\pi \varepsilon_0} \ln \left( \frac{R_p}{a} \right) \exp(-\kappa_e^2 \sigma_z^2)
\]  

(12)

which is exactly Eq. 10 multiplied by an exponential factor. \( \kappa_e = \omega_{pe}/c \) is the inverse ‘dynamical Debye length’
and \( \omega_{pe} \) the electron plasma frequency. For electron cloud
densities exceeding \( \kappa_e \sigma_z \approx 1 \) the stopping power is re-
duced by the plasma shielding effect of the cloud.

SIMULATION MODEL

In order to go beyond the impulse kick approximation
and to study the effect of the self-consistent electron
space charge field a two-dimensional (2D), electrostatic
Particle-In-Cell (PIC) code was employed. For selected parameters the results were compared to a full-wave, three-
dimensional (3D) PIC code [8]. In the 2D model the elec-
trons evolve in a \((x, y)\) plane perpendicular to the bunch
direction of motion. Poisson’s equation is solved in 2D
for the electrostatic potential each time step using the elec-
tron charge density \( \rho_e (x, y, t) \) and the known bunch density
\( \rho_i (x, y, z = z_0 - ct) \). At the beam pipe electrons are ei-
erally reflected, absorbed or multiplied according to the
SEY given in Ref. [9]. In our single bunch simula-
tions we observe that the detailed choice of the SEY does
not affect the stopping power. Only the electron density
further behind the bunch can be affected. Finally, the stop-
ping power in the 2D simulation model is obtained from
Eq. 2 with the longitudinal electric field

\[
E_z(z) = -\frac{1}{\pi a^2} \int_0^a \frac{\partial \phi}{\partial z} dx dy
\]  

(13)

Figure 1: Stopping power as a function of the ion number
in the bunch. The analytic results obtained from Eq. 10 is
represented by the solid curve. The symbols represent the
results obtained from the simulation. The red dashed line

where the potential \( \phi (x, y, z) \) is obtained numerically from
the 3D Poisson equation at the end of a simulation run using the
stored 2D densities \( \rho_e (x, y, t) \) for each time step.

SIMULATION RESULTS

For a bunch length \( \sigma_z = 0.1 \) m, beam radius \( a = 0.002 \)
m and pipe radius \( R_p = 0.01 \) m, which are close to the
LHC parameters at injection energy, the stopping power di-
vided by the number of bunch particles \( N_i \) is shown in Fig.
1 as a function of \( N_i \). The electron density is chosen as
\( n_e = 10^{12} \) m\(^{-3} \). The analytic results from Eq. 10 is com-
pared to the stopping power obtained by solving Eq. 6 nu-
merically (see previous Section). Electron space charge is
included in the simulations, but it does not affect the stop-
ping power and the wake field for \( n_e = 10^{12} \) m\(^{-3} \). The
comparison of the analytical and numerical results shown in
Fig. 1 indicates that the kick approximation is valid for
\( \kappa_0 \sigma_z \lesssim 10 \). Using Eq. 10 for an LHC bunch with
\( N_i = 10^{11} \) we arrive at \( S \approx 6 \) eV/m for the energy loss
per ion and unit length, which corresponds to \( \Delta \phi_s \approx 10^{-4} \)
deg/m, for \( V_{rf} = 3.5 \) MV. For \( N_i = 10^{11} \), bunch length
\( \sigma_z = 0.25 \) m, beam radius \( a = 0.004 \) m and pipe radius
\( R_p = 0.02 \) m, which are close to the SPS parameters at
extractions energy, the stopping power obtained from Eq.
12 as a function of the electron cloud density is shown in
Fig. 2. The symbols represent the simulation results and
the vertical line indicates that the dynamical Debye length
\( \kappa_e^{-1} \) is equal to the rms bunch length \( \sigma_z \). One can observes
that the stopping power from the simulations starts to drops
at \( \kappa_e \sigma_z \approx 1 \). However, Eq. 12 underestimates the simu-
lation results for electron densities above this value. It is
worth noting that Eq. 12 with the substitution \( \sigma_z \approx \sigma_z /2 \)
reproduces the simulation results very well, also in the case of a shorter LHC bunch. For electron cloud densities above \( \kappa_e \sigma_z \approx 1 \) the bunch excites undamped plasma waves in the electron cloud. This can clearly be seen in Fig. 4. In Fig. 3 we compare the longitudinal electric field obtained from the 2D model with the result from 3D EM simulations for \( n_e = 10^{12} \, \text{m}^{-3} \) and SPS bunch parameters. For \( n_e = 10^{12} \, \text{m}^{-3} \) and \( \kappa_e \sigma_z \ll 1 \) electron space charge plays only a very minor role. The shape of the two wake fields shown in Fig. 3 is very similar, we only observe some slight discrepancies in the wake field amplitudes. For \( n_e = 10^{15} \, \text{m}^{-3} \) \( (\kappa_e \sigma_z \approx 5) \) the wake fields are shown in Fig. 4. Both simulations show the excitation of a long range plasma wave behind the bunch. In the 3D simulation the wave decays faster, which can be a result of longitudinal effects, that are absent in the 2D model.

CONCLUSIONS

The energy loss and wake field of a short, relativistic ion bunch in an initially homogeneous electron cloud has been studied within a 2D electrostatic simulation model. The results were compared to a 3D full EM simulation. We found that for sufficiently short bunches or \( \kappa_0 \sigma_z \lesssim 10 \) the energy loss can be described very well by an analytic formula. For electron densities well above the typical \( 10^{11} - 10^{12} \, \text{m}^{-3} \) or \( \kappa_e \sigma_z \gtrsim 1 \) the space charge field of the electrons reduces the energy loss. This 'plasma-shielding' effect is more important for longer bunches. For \( n_e = 10^{12} \, \text{m}^{-3} \) and LHC bunch parameters we estimate a rf phase shift per unit length of \( \Delta \phi_s \approx 10^{-4} \, \text{deg/m} \).