

## MULTIPASS MUON RLA RETURN ARCS BASED ON LINEAR COMBINED-FUNCTION MAGNETS\*

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### Abstract

Recirculating Linear Accelerators (RLA) are an efficient way of accelerating short-lived muons to the multi-GeV energies required for Neutrino Factories and TeV energies required for Muon Colliders. In this paper we present a design of a two-pass RLA return arc based on linear combined function magnets, in which both charge muons with momenta different by a factor of two are transported through the same string of magnets. The arc is composed of  $60^\circ$ -bending symmetric super cells allowing for a simple arc geometry closing. By adjusting the dipole and quadrupole components of the combined-function magnets, each super cell is designed to be achromatic and to have zero initial and final periodic orbit offsets for both muon momenta. Such a design provides a greater compactness than, for instance, an FFAG lattice with its regular alternating bends and is expected to possess a large dynamic aperture characteristic of linear-field lattices.

### INTRODUCTION

Figure 1 shows a schematic layout of a dog-bone-shaped muon RLA [1] consisting of a single linac with droplet return arcs. Reusing the same linac for multiple beam passes provides for a more compact accelerator design and leads to significant cost savings. In the conventional scheme with separate return arcs, different energy beams coming out of the linac are separated and directed into appropriate arcs for recirculation. Each pass through the linac requires a separate fixed-energy arc, increasing the complexity of the RLA. We present a novel return-arc optics design based on linear combined functions magnets with variable dipole and quadrupole field components, which allows two consecutive passes

with very different energies to be transported through the same string of magnets.

In the scheme illustrated in Fig. 1, a 0.9 GeV/c muon beam is injected in the middle of a 0.6 GeV/pass linac. The linac is then traversed by the beam four times. Therefore, one of the return arcs accommodates 1.2 and 2.4 GeV/c muon momenta, while the other arc accommodates 1.8 and 3.0 GeV/c momenta. The two arcs can be designed using the same approach. Below we will focus our discussion on the 1.2/2.4 GeV/c arc whose design is somewhat more challenging due to the greater fractional momentum difference of the two passes.

### OPTICS DESIGN

#### Arc Design Concept

Each droplet arc consists of a  $60^\circ$  outward bend, a  $300^\circ$  inward bend and another  $60^\circ$  outward bend so that the net bend is  $180^\circ$ . This arc geometry has the advantage that if the outward and inward bends are composed of similar cells, the geometry automatically closes without the need for any additional straight sections, making it simpler and more compact.

The super cells constituting the arc are designed to satisfy the following basic conditions:

- Each super cell possesses periodic solutions for the orbit and the Twiss functions.
- At the beginning and at the end of each super cell, the periodic orbit offset, dispersion and their slopes are all zero.

These conditions are met at both momenta. The former condition ensures that the super cells bending in the same direction are optically matched while the latter one

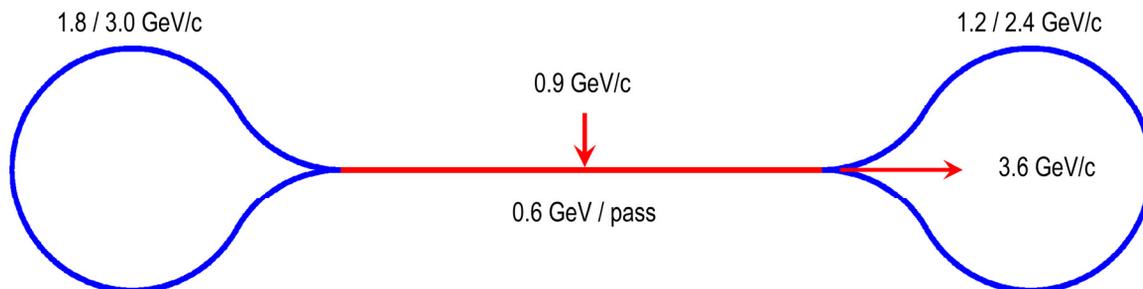


Figure 1: Schematic layout of a 4.5-pass 3.6 GeV/c muon RLA.

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ensures optical matching of the cells bending in the opposite directions. The latter condition also implies that the beam is centered in the linac and that the linac is dispersion free.

### Linear Optics

We earlier developed a linear optics solution for a multi-pass arc [2-4] based on the conventional linear NS-FFAG lattice [5]. The disadvantage of such a conventional FFAG approach is an inefficient usage of the channel's length due to the alternating outward-inward-outward bends in the underlying triplet structure. In other words, bending the beam by a certain net angle requires a total bend of three times that angle. That made the multi-pass arc very long and hard to compete with the separate arc solution. In the design presented in this paper, we deviate from the conventional FFAG scheme by not requiring regular alternating bends.

A solution satisfying the requirements discussed in the section above can be obtained using only same-direction bends, which can shorten the arc by almost a factor of 3. However, in our parameter range of relatively low energies and large momentum ratio such a solution would still not be optimal in terms of the channel length and magnet parameters. The number of magnets required to meet all of the requirements would make the channel unnecessarily long. Therefore, we introduce another innovation to increase the number of available parameters without increasing the number of magnets. We make the bending angle of each combined function magnet variable with a constraint that the bending angles of all magnets in a super cell must add up to the required fixed total bend. Such a solution combines compactness of the design with all the advantages of our earlier linear NS-FFAG solution [4], namely, large dynamic aperture and momentum acceptance essential for large-emittance muon beams, no need for a complicated compensation of non-linear effects, simpler combined-function magnet design with only dipole and quadrupole field components, etc.

To study the optics for large momentum offsets, we used the Polymorphic Tracking Code (PTC) module of MAD-X [6]. While perturbative method codes are not suitable for such a study, PTC allows symplectic integration through all elements with user control over the precision with full or expanded Hamiltonian.

We assume that the arc is composed of identical super cells. In our energy range, we use the maximum possible bend of 60° per super cell to have the largest possible number of magnets in the super cell and therefore the largest number of free parameters for optics tuning. The super cell consists of 24 combined function magnets with dipole and quadrupole field components. The magnets are 0.5 m long and are separated by 0.2 m gaps. The total arc length is 117.6 m.

Due to the arc's geometric closing, the first and the last few magnets of the arc overlap. These magnets cannot contain quadrupole field components because the symmetry would otherwise lead to the quadrupole fields having opposite slopes in the overlapping magnets. Therefore, we keep the first two magnets of each super cell as pure dipoles. Their bending angles are fixed and are chosen to provide a sufficient separation of the incoming and outgoing higher-momentum beam while

keeping the separation of the lower and higher-momentum beams within acceptable limits. Also based on these considerations, we choose the higher 2.4 GeV/c momentum as the reference momentum going through the magnet centers. The beam trajectories at the beginning of the arc are shown in Fig. 2.

The super cell is symmetric with respect to its center. Therefore, out of the 24 magnets constituting the super cell, 12 are independent. As discussed above, 2 of these magnets are pure dipoles with a fixed bending angle of 6° each. The remaining 10 magnets each have variable dipole and quadrupole field components with a constraint that the bending angles of all super cell's magnets add up to a net bend of 60°. This gives a total of 19 independent parameters.

When solving for the periodic orbit and the periodic Twiss functions of the super cell, the initial values of the orbit offset, dispersion, their slopes and the alpha functions were all set to zero at both momenta. The initial values of the horizontal and vertical beta functions were set to 2 m at both momenta to provide easy matching to the linac and to keep the peak values of the beta functions inside the super cell at an acceptable level. The 19 independent parameters discussed above were then adjusted to give zero slopes of the orbit offset, dispersion and beta functions at the center of the super cell at the two momenta. The super cell's symmetry then ensures the appropriate properties at the super cell's exit. Since the 2.4 GeV/c beam goes through the magnet centers, its periodic orbit by definition has zero offset everywhere. This results in a total of 7 constraints. The extra free parameters were used to control the maximum values of the orbit deviation, beta functions and dispersion. In terms of magnetic field requirements, the maximum needed dipole field is about 1.7 T while the maximum quadrupole gradient is about 28 T/m. Figures 3 and 4 show solutions for the periodic orbit, dispersion, and beta functions of the outward-bending super cell at 1.2 and 2.4 GeV/c, respectively. An inward-bending super cell is identical to the outward-bending cell except that its bends are reversed.

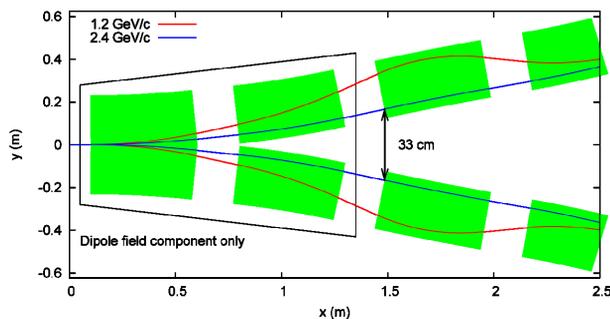


Figure 2: Beginning of the arc with the pure dipoles acting as a spreader/ recombiner of the different momenta trajectories.

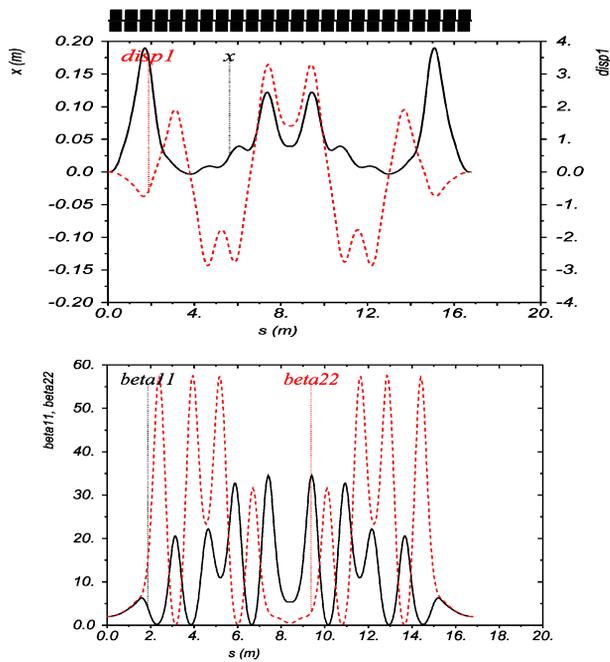


Figure 3: 1.2 GeV/c periodic orbit, dispersion (top) and beta functions (bottom) of the outward bending super cell.

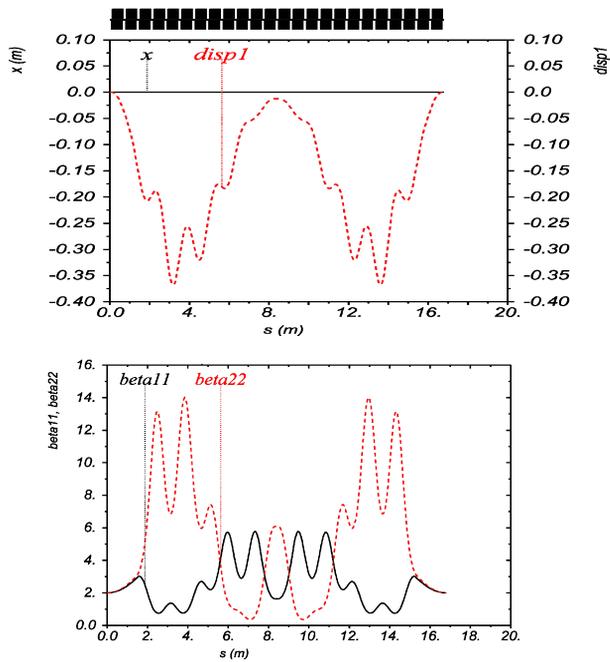


Figure 4: 2.4 GeV/c periodic orbit, dispersion (top) and beta functions (bottom) of the outward bending super cell.

Figure 5 shows geometric layouts of the 1.2 and 2.4 GeV/c closed periodic orbits. The displacement of the 1.2 GeV/c orbit was enhanced by a factor of 10. Note that because of the varying bending angles, the arc is not perfectly circular. The largest orbit separation occurs only in a small number of magnets and is caused by the necessity to spread/recombine the different momenta orbits at the beginning of the arc. The maximum orbit

deviation is reduced for smaller momentum ratios such as that of the 1.8/3.0 GeV/c arc.

At few-GeV energies muons are not ultra-relativistic; there is a non-negligible difference in the speeds of 1.2 and 2.4 GeV/c muons. Because of the short arc circumference it was not possible to compensate the time of flight difference by adjusting the path lengths. One can attain an appropriate synchronization with the linac by placing a path-length chicane in front of the arc. Such a chicane would have a cumulative effect on the opposite-direction passes.

We are currently studying the dynamic aperture and momentum acceptance of the arc. Earlier studies with a similar linear lattice yielded promising results [4]. We will investigate chromatic effects and if necessary, implement their control and compensation. Another important aspect that requires investigation is the design sensitivity to magnet misalignments and magnetic field errors. Establishing tolerance levels on these errors is crucial for the costing of large aperture magnets.

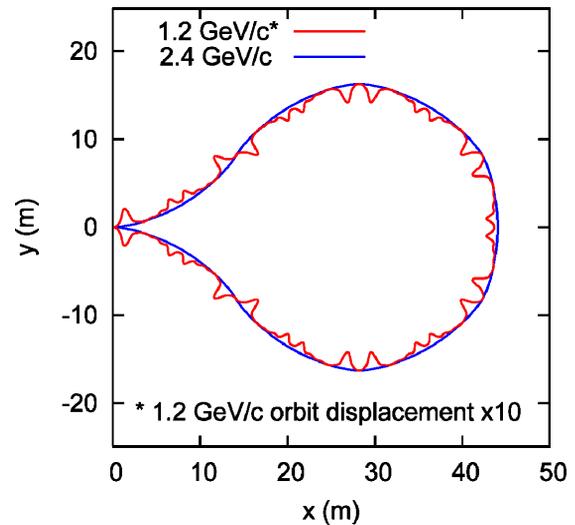


Figure 5: Horizontal (top) and vertical (bottom) maximum-amplitude stable phase-space trajectories for the linear and non-linear NS-FFAG designs.

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