COUPLING IMPEDANCE OF ROUGH RESISTIVE PIPE*

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Abstract
A new version of modelling of the surface roughness impact by thin dielectric layer in the round resistive beam pipe is suggested. The calculation method of coupled resistive-roughness impedance is developed.

INTRODUCTION
In the XFEL projects due to shortness of bunches (about 8-25 µm [1,2]) the surface roughness destructive effect is expected to be significant. In the simplest case of the homogeneous beam pipe with the finite conducting wall there are two main sources of wakefields: wall’s finite conductivity and wall’s surface roughness. Overall, it is a single integrated effect of the rough resistive walls. Nevertheless, till present the roughness and resistivity influences are described separately probably owing to absence of the acceptable simple solution for the corrugated resistive tube. In this paper the possible way of the coupled consideration of the resistivity and roughness wakefield effects is suggested.

ROUGHNESS EFFECT: PERFECT CONDUCTING AND RESISTIVE WALLS
At present different models have been developed [3-5] to study the clear effect of the surface roughness in the beam pipe. The proposed consideration is based on the model [5], then the roughness on the perfectly conducted wall inner surface is approached by the thin dielectric coating (Fig.1). The parameters of dielectric layer (dielectric constant and layer thickness) are determined by comparison with the wakefield effects in periodically corrugated tube with the rectangular corrugations. The main parameters of corrugations are taken equal to the RMS parameters of roughness: \( \delta \) (depth), \( p \) (period) and \( g \) (gap) by equalization of the fundamental resonance frequency \( k_0 \) for both cases. For the corrugated rectangular tube \( k_0 = \sqrt{2p/bd\delta} \) (b tube inner radius), while for the tube with thin dielectric layer with thickness \( \delta' \) \( k_0 = \sqrt{2\epsilon/bd\delta'(\epsilon - 1)} \). Thus by equalization of two resonance frequencies in the case of \( \delta' = \delta \) one obtains:

\[ \epsilon = (1 - g/p)^{-1} \]  

(1)

By using the same dielectric layer as an inner cover of the real resistive smooth pipe it is possible to obtain the summary resistive-roughness effect of the resistive rough pipe. Thus, the problem comes to the impedance of the two-layer tube with outer finite conducting metallic and inner dielectric layers. This problem is solved by the method of multi-layer tube impedance calculation [6], that gives an exact solution for any infill of layers. In the subsequent sections the impedances and wake potentials dependence on the g/p ratio is investigated. As basic model the small-gap undulator round aluminium vacuum chamber \( (b = 2\text{ mm}) \) with 1mm wall thickness is taken.

IMPEDANCES
The longitudinal resistive impedance for the two-layer round tube at high enough frequencies in the common case of the arbitrary infill of layers may be presented as

\[ Z_l = j\frac{Z_0}{nkb^2} \left( 1 + \frac{2}{kb\epsilon_0\chi_1} \frac{\epsilon_2\chi_2}{\epsilon_1\chi_1} \frac{1}{\chi_2 - \chi_1} \right) \]  

(2)

where \( d_1, \epsilon_1, \chi_1 \) and \( d_2, \epsilon_2, \chi_2 \) are the thickness, permittivity and transverse wave number of the inner and outer layers respectively and \( k = \omega/c \) (\( \omega \) frequency and \( c \) speed of light) is the wave number. For the special case of the outer metallic and inner dielectric loss less layers:

\[ \epsilon_1 = \epsilon'_1\epsilon_0, \quad \chi_1 = jk\sqrt{\epsilon'_1-1} \]
\[ \epsilon_2 = \epsilon_0 + j\sigma_2/\omega, \quad \chi_2 = (1 - j)\sqrt{\omega\mu_0\sigma_2} \]  

(8)

where \( \epsilon_0, \mu_0 \) are the electrical permeability and magnetic permeability of vacuum, \( \epsilon'_1 = \epsilon \) (1) is a relative dielectric permittivity of the inner layer and \( \sigma_2 \) is a conductivity of the outer metallic layer.

Longitudinal Impedance
Ordinary small-gap undulator section aluminum vacuum chamber is considered. The vertical size of vacuum chamber is limited by the undulator pole gap (6mm). The wall thickness is taken equal to 1mm, thus in the round tube approximation the tube radius should be taken equal to 1mm. The roughness on the inner surface is

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approximated by periodical rectangular corrugations. The depth of corrugations is taken equal to 20, 10, 1 and 0.1 \( \mu m \) and \( g/p \) ratio equal to 0.5, 0.25 and 0.125. These corrugations are assumed equivalent to the inner dielectric layer with dielectric constant (1) and corresponding thickness. The coupling resistive-roughness longitudinal monopole impedances for the mentioned parameters are presented below (Fig.2). Dashed lines on the Figure show the fundamental resonance frequencies. As it is seen, there are the following main properties of impedance distribution behaviours.

1) Periodicity, conditioned by dielectric layer thickness and permittivity (\( g/p \) ratio);
2) Common increasing of fundamental resonance frequency with decreasing of depth of roughness and \( g/p \) ratio as well;
3) Increasing shift to the low frequencies between the main resonances of clear roughness and coupling impedance. The shift conditioned by the impact of resistivity and increases with \( g/p \) ratio or \( \delta \) reduction as well. In both cases, due to finite conductivity, resonance level is decreasing and resonance curves become smoother and tend to the ordinary resonance of the impedance of resistive tube with smooth inner surface.

Transverse impedance

The same regularities may be observed during the transverse impedance investigations. Dipole transverse impedance distributions for 1 and 0.5 \( \mu m \) and for \( g/p = 0.5, 0.25 \) and 0.125 are presented in Fig.3.

![Figure 2: Real part of the longitudinal coupling impedance distributions; g/p=0.5 (top), 0.25 (middle), 0.125 (bottom). Also shown (Smooth Surface) corresponds to the non-corrugated (ideal) case.](image)

![Figure 3: Imaginary part of the transverse coupling impedance distributions; g/p=0.5 (top), 0.25 (middle), 0.125 (bottom). Also shown (Smooth Surface) corresponds to the non-corrugated (ideal) case.](image)

WAKES

Longitudinal Wake

Longitudinal monopole wake potentials for the coupling resistive and roughness wakefield for the
Gaussian bunch RMS length $\sigma_z = 9 \mu m$ are plotted below.

The longitudinal wakefield in the case of the large depth of roughness (for 50, 10 and even 1 $\mu m$) leads to the bunch deformation and due to large long range wake acts on the sequential bunches as well. For the given RMS bunch length ($\sigma_z \approx 9 \mu m$) the RMS depth of roughness $\leq 0.1 \mu m$ is acceptable.

**INTEGRAL PARAMETERS**

The integral parameters, loss and kick factors characterize the quality of pipe. The developed method permits one to estimate directly the destructive influence of roughness by comparison with the same parameters value for the smooth resistive pipe.

The roughness influence for the several parameters of roughness are illustrated in Table 1 and Table 2, where the loss and kick factors for the different depths and different values of the parameter $g/p$ are given.

| Table 1: Loss factor for small-gap vacuum chamber (V/pC/m) |
|---|---|---|
| Depth | $0.05\mu m$ | $0.1\mu m$ | $1\mu m$ |
| $g/p=1/8$ | 636 | 756 | 2646 |
| $g/p=1/4$ | 756 | 1036 | 3369 |
| $g/p=1/2$ | 1036 | 1592 | 3868 |

| Table 2: Kick factor for small-gap undulator vacuum chamber (kV/pC/m²) |
|---|---|---|
| Depth | $0.05\mu m$ | $0.1\mu m$ | $1\mu m$ |
| $g/p=1/8$ | 6 | 7 | 16 |
| $g/p=1/4$ | 7 | 8.8 | 2.1 |
| $g/p=1/2$ | 8.8 | 11 | 28 |

For the resistive walls without roughness the longitudinal loss factor for the discussed small-gap undulator is equal to 540 V/pC/m. The kick factor for the same case is equal to 5 kV/pC/m².

**CONCLUSION**

The suggested method permits one to calculate coupling resistive-roughness impedances and wake potentials in the rough resistive beam pipe instead of additive summation. The common properties of the coupling impedance are developed from the presented examples. The coupling resistive-roughness effect influence on the beam longitudinal and transverse stability may be estimated by the help of integral parameters calculation. The results can be used for the thin dielectric coating influence estimation as well.

**REFERENCES**