BEAM TILT DUE TO TRANSVERSE WAKEFIELDS FOR DAΦNE, SUPERB, KEKB and SUPERKEKB

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Abstract

When a beam bunch traverses a transverse impedance, the bunch head generates a transverse wakefield that kicks the bunch tail, generating a betatron motion of the tail relative to the head. In a storage ring, in a steady state, this kick to the bunch tail produces a transverse closed orbit (e.g. in the y-direction) of the bunch tail relative to the bunch head, which means the beam now has a y-z tilt. Such beam tilt due to transverse wakefields may cause a loss of luminosity in storage ring colliders or loss of brightness in light sources. In this paper, we present a preliminary study of the beam tilt effect for the colliders DAΦNE, SuperB, KEKB and SuperKEKB.

INTRODUCTION

SuperB [1] and SuperKEKB [2] are next generation B-factories aiming at luminosities more than one order of magnitude higher than present B-factories [3]. In order to achieve the extremely high luminosity of up to 1×10^{36} cm^{-2}s^{-1}, the most promising scheme is to minimize the vertical emittance and squeeze the vertical beam size at IP by more than one order smaller than that of present colliders. In Table 1, we collected some basic parameters considered in our studies for the low energy rings (LERs) of these two projects [2]. The relevant parameters of DAΦNE [3] and KEKB [4] are also shown for purpose of comparison.

The transverse beam tilt effect was first studied for PEP-I [5] and later for SSC [6]. It is a transverse analogy of the longitudinal potential-well distortion effect [7]. Consider a bunch traversing a transverse impedance localized at one point in a ring, the bunch head generates a transverse wakefied that kicks the bunch tail. Due to this nonuniform kick, the bunch head and tail experience different closed orbits, i.e. the bunch now has a tilt in the transverse direction. In a collider, if there is such a tilt at the IP, and the tilt angle is larger than $\sigma_y^*/\sigma_{z,\mathrm{eff}}$, then there is a loss of luminosity, where $\sigma_{z,\mathrm{eff}}$ is the effective bunch length. For most storage ring colliders, $\sigma_{z,\mathrm{eff}}$ is just the nominal bunch length σ_z. For DAΦNE [8], SuperB and SuperKEKB, it is given by σ_x^*/ϕ, where $\pm\phi$ is crossing angle.

Synchrotron oscillation will exchange the head and tail particles. However, as long as we stay away from synchro-betatron resonances, the synchrotron motion can be considered slow and this head-tail exchange is adiabatic. The analysis through closed-orbit will be sufficient for the purpose.

Table 1: Machine parameters of DAΦNE, SuperB, KEKB and SuperKEKB. Only selected parameters for LERs are shown due to limited space. The superscript of asterisks indicate parameters at IP. S-B and S-KEKB indicates SuperB and SuperKEKB, respectively.

Param.	**DAΦNE**	**S-B**	**KEKB**	**S-KEKB**
E (GeV)	0.51	4.18	3.5	4
σ_z (mm)	17	5	7.5	6
σ_x^* (μm)	250	8.8	120	10.2
σ_y^* (μm)	2.5	0.035	1	0.059
β_y^* (cm)	0.93	0.0206	0.59	0.027
ϕ (mrad)	25	33	11	41.5
N ($\times 10^{10}$)	2.5	6	8.66	9

ANALYTIC MODEL

The bunch tail receives a transverse kick due to a localized transverse impedance. The wake kick for a particle at longitudinal position z is calculated by [5],

$$\Delta y'(z) = \frac{Ne^2}{E}\int_0^\infty dz'\ \rho(z'+z)W_\perp(y_b,z') \tag{1}$$

where E is beam energy, $\rho(z)$ is the normalized longitudinal bunch density at position z, and $W_\perp(y_b,z)$ is transverse wake function at distance z with $z>0$ and transverse offset y_b. The quantity y_b indicates the nominal unperturbed transverse displacement of the bunch when it has zero beam current. For a Gaussian bunch,

$$\rho(z) = \frac{1}{\sqrt{2\pi}\sigma_z}e^{-\frac{z^2}{2\sigma_z^2}}. \tag{2}$$

The transverse wake function can be approximated by its Taylor series in terms of y_b, i.e.

$$W_\perp(y_b,z) \approx W_\perp^0(z) + W_\perp^1(z)y_b \tag{3}$$

where $W_\perp^0(z) = W_\perp(0,z)$ and $W_\perp^1(z) = \frac{\partial W_\perp(y_b,z)}{\partial y_b}\Big|_{y_b=0}$ denotes the uniform and dipolar components, respectively.

If the object considered is cylindrically symmetric, there is $W_\perp^0(z) = 0$ and the dipolar component is dominant in Eq. (3). Then the transverse kick is proportional to the transverse displacement of the bunch head. For a broad-band resonator impedance, for example, we have

$$W_\perp^1(z>0) = \frac{k_r c R_s^1}{Q\bar{k}_r}e^{-\alpha_r z}\sin\left(\bar{k}_r z\right), \tag{4}$$

where c is the speed of light in vacuum, k_r is the resonant frequency, R_s^1 is the transverse shunt impedance, and Q is the quality factor. The remaining parameters are defined by $\alpha_r = \frac{k_r}{2Q}$ and $\bar{k}_r = \sqrt{k_r^2 - \alpha_r^2}$. Substituting Eqs. (2) and (4) into Eq. (1), one can find the explicit form of $\Delta y'(z)$ as follows [9]

$$\Delta y'(z) = \frac{Ne^2}{E}\frac{k_r c R_s^1 y_b}{2Q\bar{k}_r} e^{-\frac{\sigma_z^2}{2}(\bar{k}_r^2-\alpha_r^2)+\alpha_r z}\times$$

$$\mathrm{Im}\left\{e^{-i\bar{k}_r(z+\alpha_r\sigma_z^2)}\left[1+\mathrm{Erf}\left[\frac{(i\bar{k}_r-\alpha_r)\sigma_z^2-z}{\sqrt{2}\sigma_z}\right]\right]\right\}, \quad (5)$$

where the symbol Im$\{\}$ denotes the imaginary part of the quantity concerned, and Erf$[z]$ is the error function.

The effect of the transverse kick is translated to the displacement of closed orbit relative to the bunch head at IP. The rms estimate of the displacement Δy due to a single transverse impedance is [5]

$$\Delta y/\sigma_y^* = \sqrt{0.5\beta_y/\epsilon_y}\Delta y', \quad (6)$$

where σ_y^* , ϵ_y and β_y are the vertical beam size at the IP, vertical emittance and vertical beta function at the location of the impedance, respectively. Considering many localized impedances distributed around a ring, the net effect can be estimated via [6]

$$\Delta y = \sum_i \sqrt{0.5\beta_y^*\beta_{yi}}\Delta y_i', \quad (7)$$

where i denotes the i-th component generating transverse impedance, and β_y^* is the vertical beta function at the IP.

NUMERICAL RESULTS

DAΦNE

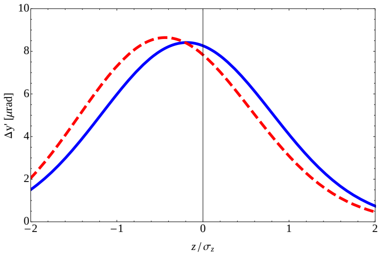
For DAΦNE [10], we take $R_s^1/k_r = 150$ kΩ/m, $y_b = 1$ mm, and $Q = 1$. The kick angle across the bunch with $k_r = 334\ \mathrm{m}^{-1}$ and $k_r = 167\ \mathrm{m}^{-1}$ is shown in Fig. 1(a). Here k_r is set to be the order of $2\pi/\sigma_z$. It is seen that the maximum kick angle is around $8\ \mu$rad.

If we pessimistically assume that the impedance is 100% localized, with $\beta_y = 10$ m, then, at the IP, the beam tail that receives the maximum wakefield kick of $8\ \mu$rad will have a displacement relative to the bunch head by around $1.7\ \mu$m, according to Eq. (6). This displacement is smaller than the vertical beam size at IP. Thus the beam tilt effect is not so serious for DAΦNE.

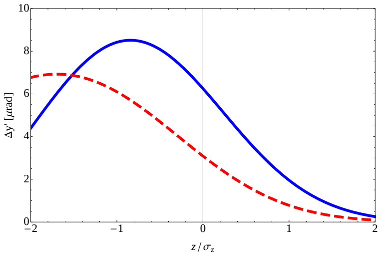
SuperB

For SuperB, we take the same parameters as DAΦNE for the impedance model and settle on $y_b = 1$ mm. Using beam parameters listed in Table 1, the kick angle across the bunch is calculated as shown in Fig. 1(b).

Again we assume that the impedance is 100% localized at one location with $\beta_y = 10$ m. Then, at the IP where $\beta_y^* = 0.206$ mm, the beam tail that receives the maximum



(a) DAΦNE



(b) SuperB

Figure 1: The kick angle as a function of z/σ_z across the bunch. $z > 0$ is bunch head. Blue solid lines: $k_r = 334\ \mathrm{m}^{-1}$; red dashed lines:$k_r = 167\ \mathrm{m}^{-1}$.

wakefield kick of 8.5 μrad according to the above figure will have a wake-induced transverse displacement relative to the bunch head by around $0.27\ \mu$m, which is much larger than the vertical beam size at the IP, i.e. $\sigma_y^* = 35$ nm. However, various factors will help:

- We should compare the orbit differential in a distance of σ_x^*/ϕ, not across the entire bunch length. Take $\sigma_x^* = 8.8\ \mu$m and $\phi = 33$ mrad, the kick angle difference between $z = -\sigma_x^*/\phi$ to σ_x^*/ϕ is around $0.5\ \mu$rad when $k_r = 334\ \mathrm{m}^{-1}$, yielding an orbit differential of 22 nm, to be compared with $\sigma_y^* = 35$ nm.

- Orbit can be controlled to better than 1 mm. In fact, careful orbit corrections down to sub-millimeter level should put this effect under control, at least for cylindrically symmetric objects.

The main uncertainty of the present calculation lies in two factors: (a) we have used the same DAΦNE impedance for SuperB, and (b) we have assumed the impedance is localized at one particular point in the ring. To obtain an accurate estimate, we will need to know the SuperB impedances, and to know the distribution of these impedances. The use of DAΦNE impedance for SuperB is optimistic because SuperB has more vacuum chamber components. On the other hand, the assumption of lumped impedance is pessimistic because if, for instance, the impedances are distributed into 10 lumps, instead of a single lump, then the closed-orbit distortion will reduce by a factor of $\sqrt{10}$ [6]. This uncertainty will require more careful and systematic study.

KEKB and SuperKEKB

For KEKB, we choose $k_r = 400\,\mathrm{m}^{-1}$, $R_s^1/k_r = 10^6$ Ω/m, $Q = 1$ and $y_b = 1$ mm. Here the parameter k_r is set to be $3/\sigma_z$ by referring to [4]; the shunt impedance is extracted from measured vertical betatron tune shift as function of bunch current, i.e. $d\nu_y/dI = -3.7$ A^{-1} [11]. For SuperKEKB, we take the same parameters as KEKB for the impedance model and settle on $y_b = 1$ mm.

The kick angle across the bunch for KEKB and SuperKEKB is shown in Fig. 2. It is seen that the maximum kick angles are around 65 μrad and 75 μrad, respectively. If we pessimistically assume that the impedance is 100% localized, with $\beta_y = 10$ m, the consequent displacements at the bunch tail translated to the IP are around 11 μm and 2.8 μm, respectively. In both cases, the displacements are much larger than the vertical beam sizes at the IP. As we stated, this is a rather pessimistic estimate without considering distribution of impedances. For SuperKEKB, efforts in reducing impedance of vacuum chamber components [12] will help to suppress the beam tilt effect.

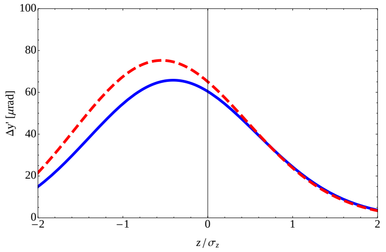


Figure 2: The kick angle as a function of z/σ_z across the bunch for KEKB and SuperKEKB. $z > 0$ is bunch head. Blue solid line: KEKB; red dashed line:SuperKEKB.

3-D OBJECTS

When the impedance has cylindrical symmetry, the wakefield comes from y_b; it vanishes if the nominal closed orbit is zero. A good closed orbit distortion at low beam currents will minimize this beam tilt effect.

When the impedance is a 3-D object, however, the wakefield is nonzero even with good closed orbit control. As an order of magnitude estimate, we can apply the same formulas above, except that the wake function should include only the 3-D objects, but y_b should be replaced by the transverse dimension of the 3-D object.

For example, movable masks (or collimators) are one kind of 3-D object needed to be particularly concerned. In colliders, collimators are usually used for reducing background noise in the detector. For KEKB, asymmetric movable masks were used [13]. For SuperKEKB, symmetric collimators were proposed [12]. The impedance of such components can be calculated using standard numerical codes or by simplified analytical formulas [14].

This estimate for 3-D objects like collimators needs to be studied further. Since transverse dimension of a typical 3-D object is ~ 1 cm, it is suggested that no more than approximately 3-5% of the vacuum chamber discontinuities can be 3-D. A great majority of discontinuities should be cylindrically symmetric.

TENTATIVE CONCLUSIONS

Provided the transverse impedance model and level of closed orbit are the same, the kick angle is in the same order for DAΦNE and SuperB according to our preliminary calculations. This is also true for KEKB and SuperKEKB. Since the vertical emittance is more than one order smaller, the beam tilt effect might play a role in the two next-generation colliders.

Beam tilt due to wakefields is not significant for DAΦNE. It is generally to be controlled in SuperB and SuperKEKB by (a) controlling closed orbit distortion to sub-millimeter, (b) controlling the total impedance not to be substantially more than present machines, and (c) keeping impedance of 3-D objects to below 3-5% of the total impedance.

For SuperB and SuperKEKB, it is suggested that transverse impedances should be carefully budgeted and cataloged with records on their locations around the ring, as well as whether they are cylindrically symmetric or 3-D. This will be needed to make sure the effect is under proper control. Eventually its effect on luminosity can be evaluated via beam-beam simulations with transverse impedance included.

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