COUPLING IMPEDANCE CONTRIBUTION OF FERRITE DEVICES:  
THEORY AND SIMULATION∗

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Abstract
Beam coupling impedances have been identified as an appropriate quantity to describe collective instabilities caused through beam-induced fields in heavy ion synchrotron accelerators such as the SIS-18 and the planned SIS-100 at the GSI facility [1]. The impedance contributions caused by the multiple types of beamline components need to be determined to serve as input condition for later stability studies. This paper will discuss different approaches to calculate the coupling impedance contribution of ferrite devices, exploiting the abilities of both commercial codes such as CST STUDIO SUITE® and specific extensions of this code to address kicker related problems in particular.

INTRODUCTION
Ferrite kickers are commonly used for synchrotron injection and extraction. The beam-induced electromagnetic fields inside the kicker may lead to instabilities and have to be determined. The dominating impedance contributions originate from the ferrite yoke (continuous part) and from the adjacent pulse forming network (PFN, resonant spikes). Further contributions may originate from conducting and dielectric parts included in the support structures. It has been shown that it is possible to treat these contributions individually and superimpose the results.

Z(ω) = Z_{yoke} + Z_{PFN} + Z_{else}  \hspace{1cm} (1)

The scope of this work will be focused to the ferrite contributions in frequency range of 1MHz up to 100MHz to serve as a first step for a complete impedance investigation of the SIS-100 kicker system.

The longitudinal and transverse beam coupling impedances are defined as follows [2]:

\begin{align*}
Z_{||}(\omega) &= \frac{1}{q^2} \int d^3 x E_z J_{z,ext}, \\
Z_{x,y}(\omega) &= \frac{i}{q^2 \Delta} \int d^3 x \rho_{x,y}(E_{x,y} \mp v B_{y,x}),
\end{align*}

ELECTRODYNAMICS

E_z is the longitudinal electric field and E_{x,y} and B_{y,x} are the transverse electric and magnetic fields excited by the beam. We assume a rigid particle beam in form of a circular lamina of radius a that moves along the z-axis with a constant longitudinal velocity \( v = \beta c \) as the source of electromagnetic field excitation, where \( \beta \) is the relativistic factor and \( c \) is the vacuum speed of light. For the charge density and current density we have:

\begin{align*}
\varrho_V (\vec{r}, t) &= \sigma_A (\varrho, \varphi) \delta (z - vt) \\
J_{z,ext} (\vec{r}, t) &= \varrho_V (\vec{r}, t) \vec{v} = \sigma_A (\varrho, \varphi) \delta (z - vt) \vec{v}
\end{align*}

\hspace{1cm} (4)
with homogenous transverse charge distribution:

$$\sigma_A (\varrho) = \left\{ \begin{array}{ll} \frac{Q}{\pi a^2}, \varrho \leq a \\ 0, \varrho > a \end{array} \right. \quad (5)$$

Applying the Fourier Transform, we gain an expression for a point charge circulating with $\omega$:

$$\varrho V (\varrho, z, \omega) = \sigma_A \frac{1}{\pi} e^{i \frac{\varrho}{a} z}$$

$$J_{z,\text{ext}} (\varrho, z, \omega) = \sigma_A e^{i \frac{\varrho}{a} z} \quad (6)$$

The expressions above appear on the right hand side of the wave equation, which is derived from Maxwells equations.

$$\nabla \times \frac{1}{\mu} \nabla \times \vec{E} (t) + \frac{\partial^2}{\partial t^2} \varepsilon \vec{E} (t) + \frac{\partial}{\partial t} \kappa \vec{E} (t) = -\frac{\partial}{\partial t} J_{\text{ext}} (t) \quad (7)$$

and respectively in frequency domain:

$$\nabla \times \frac{1}{\mu} \nabla \times \vec{E} (\omega) - \omega^2 \varepsilon \vec{E} (\omega) = -i \omega J_{\text{ext}} (\omega) \quad (8)$$

If a time domain approach is chosen the impedance is obtained through a Fourier transformation of the wake function [3]. Ferrites exhibit magnetization losses which are proportional to the area enclosed by the $B$-$H$- magnetization loop. This energy is commonly taken into account using a frequency dependant complex permeability $\mu = \mu' + j \mu''$. The resulting $B$-$H$- characteristics is an ellipse fitted inside the magnetization loop having a different shape at every frequency point.

**COMPUTATIONAL MODELS**

For most practical applications the electromagnetic fields can not be determined analytically. Thus coupling impedance calculation can either be adressed in the time domain using FDTD wake field solvers or in frequency domain, making use of the time harmonic description of a circulating point charge. For testing purposes and to gain better understanding these methods will be compared. For the SIS-100 heavy ion and proton beams the frequency spectra of interest lies in the range of 1 to 100 MHz. When applying conventional wakefield codes very large wavelengths need to be calculated. Operating in frequency domain seems to be more elegant in this case and allows to include lumped impedances such as PFN. The drawing of the preliminary SIS-100 kicker design has been loaded into CST STUDIO SUITE® modeler using the import routine for CATIA® software. The model has been reduced to the ferrite yoke and a vacuum surrounding has been added. For simple structures CAD import is not mandatory but it makes sure that the dimensions are consistent with the design and allows to switch to the full model easily. A further simplification used here is the assumption of relativistic beam $v = c_0$. Identical models for time domain calculation and frequency domain approach are used. The yoke is about 300 mm wide, 200 mm high and 750 mm long. The discretization uses approx 72000 meshcells, which appears sufficient for the frequency range of interest. The ferrite material data is provided as frequency dependant complex permeability $\mu = \mu' + j \mu''$ according manufacturer datasheet [7] for 8C11 ferrite type.

![Figure 2: Simplified model consisting of the ferrite yoke only.](image)

![Figure 3: Complex ferrite permeability: manufacturer data and fitted data from second order dispersion fit.](image)

For the time domain wake field calculation CST PARTICLE STUDIO® software is used. A built-in second order dispersive model [6] is used for the ferrite, based on the permeability data. It can be seen that the second order dispersion fit does not entirely capture the permeability characteristics. The choice of bunch length determines the spectra of the exciting bunch and usable frequency range [6]. To obtain sampling points in the low frequency region around 1 MHz bunch lengths above 1500 mm must be used and multiples for the Wavelength to be calculated. For even lower frequencies the whole spectra of interest has to be split using multiple calculations with different bunchlengths. The frequency domain approach is based on Finite Integration Technique using the CST input file, the Trilinos package [8] and python script language [9] and has been introduced in [4].
NUMERICAL RESULTS

The results for the longitudinal coupling impedance are presented below. The location of the global Maximum of the real part strongly depends on the dimensions of the cross-section of the yoke and thus serves as design parameter in case of beam stability issues.

![Figure 4: Real part of the longitudinal coupling impedance.](image)

![Figure 5: Imaginary part of the longitudinal coupling impedance.](image)

The results for the transverse coupling impedance are presented below.

![Figure 6: Real part of the transverse coupling impedance.](image)

![Figure 7: Imaginary part of the transverse coupling impedance.](image)

The results for the transverse coupling impedance are presented below.

SUMMARY AND OUTLOOK

The results of both simulation approaches appear reasonable and agree well. The observed deviation is most likely related to the ferrite dispersion model that has been used for the time domain calculation and requires further investigation. Even though the desired frequency range requires extreme settings for time domain calculation, this is still an option. Including the complex coil geometry and adjacent parts in the model requires higher computational capacities. Currently the limitation for the frequency domain approach is set by the addressable memory of a 32 bit computer system. Testing on a 64 bit computer system is in progress. Further headroom could be gained by the use of parallel computing. After these improvements have been made the full kicker geometry can be taken into account.

REFERENCES