SIMULATION OF MICROWAVE INSTABILITY IN LER OF KEKB AND SUPERKEKB

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Abstract

Microwave instability in the LER of KEKB may be one obstacle to achieving high luminosity as expected by beam-beam simulations. To understand the single-bunch beam dynamics of KEKB LER, we constructed a numerical impedance models by calculating ultra-short wake potentials of various vacuum components, resistive wall and coherent synchrotron radiation. The geometrical wakes were calculated by 3D electromagnetic code GdfidL. And CSR impedance were estimated by a dedicated code. Similar work was also done for LER of SuperKEKB. Using these impedance models we simulated the microwave instability at LER of KEKB and SuperKEKB by solving Vlasov-Fokker-Planck (VFP) equation in the longitudinal phase space. The results of impedance calculation and simulations were presented in this paper.

INTRODUCTION

KEKB [1] has been operated for more than 10 years since its first commissioning from Dec. 1, 1998. In June 2009, the peak luminosity reached $2.11 \times 10^{34}\text{cm}^{-2}\text{s}^{-1}$ with stored beam currents of 1.64/1.12A (LER/HER) due to crab crossing and off-momentum optics corrections. One of the merits of KEKB [2] which contributed to such high luminosity is squeezing the vertical beta function at interaction point (IP) to 0.59 cm. Correspondingly, the natural bunch length is around 4.6 mm. And at normal operating bunch current of 1.0 mA at LER, the measured bunch length is around 7 mm.

Since the beam-beam simulations showed that the crab crossing should boost the luminosity by a factor of 2 [3], the present achieved luminosity is still far from expectations. Besides chromatic coupling induced by lattice nonlinearity [4], microwave instability in the LER may be another potential obstacle for KEKB to achieving higher luminosity by way of increasing beam currents.

Recently, Y. Cai et al. studied the microwave instability in the LER of KEKB using a broadband resonator impedance model [5]. In that work, it was demonstrated that the model described the longitudinal beam dynamics very well when comparing with experimental observations. As predicted by Cai's model, the threshold of microwave instability at LER of KEKB is 0.5 mA, which is well lower than the present operating current of 1.0 mA. In this paper, we introduce the studies on microwave instability in the LER of KEKB and SuperKEKB using numerically calculated impedance models.

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QUASI GREEN'S FUNCTION OF WAKE POTENTIAL

To study the longitudinal single-bunch instabilities, we first calculate the ultra-short wake potentials of various vacuum components. GdfidL installed on a cluster with 256 GB memory is available at KEK. As trade-off between the capability of the cluster and the interested frequency range, 0.5 mm bunch length was chosen for most vacuum components of KEKB LER.

Fig. 1 shows the total geometrical wake potentials of LER of KEKB and SuperKEKB. The length of driving gaussian bunch used in GdfidL is 0.5 mm. Due to significant improvements in the vacuum components, the impedance of SuperKEKB rings will be well suppressed. Coherent synchrotron radiation (CSR) is another important impedance source at LER of KEKB and SuperKEKB. The bending radius of normal dipoles at KEKB LER and wigglers are 15.87 m and 16.3 m, respectively. For SuperKEKB LER, only half of the wigglers will remain. Such magnets will produce CSR as bunch length get short to a few minimeter. Thus a dedicated code was developed by K. Oide in 2008 to calculate the CSR impedance in LER of SuperKEKB. In this code, the paraxial approximation was adopted [6]. Electronic fields due to CSR were calculated in the frequency domain and then wake potential was obtained by Fourier transformation. The calculated CSR wake potentials of 0.5 mm bunch are shown in Fig. 2. Interference between adjacent magnets caused modulations at the tail parts of the CSR wake potentials.

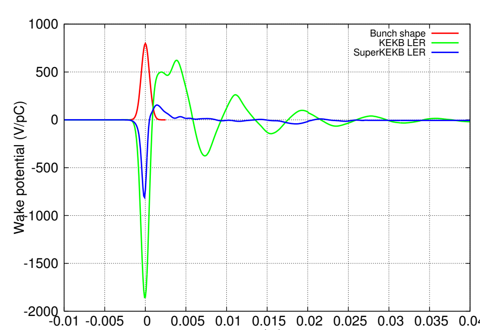

Figure 1: Calculated geometrical wake potentials of 0.5 mm bunch for LER of KEKB and SuperKEKB.

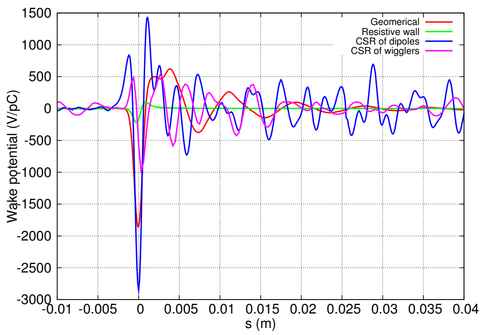

Figure 2: Calculated geometrical, resistive wall and CSR wake potentials of 0.5 mm bunch for LER of KEKB.

SOLVING VFP EQUATION

Basically we follow R.L. Warnock and J.A. Elisson's work [7] to solve Vlasov-Fokker-Planck (VFP) equation numerically. VFP equation including collective wake force is written as

$$\frac{\partial\psi}{\partial s} + \frac{\partial q}{\partial s}\cdot\frac{\partial\psi}{\partial q} + \frac{\partial p}{\partial s}\cdot\frac{\partial\psi}{\partial p} = \frac{2\beta}{c}\frac{\partial}{\partial p}[p\psi + \sigma_p^2\frac{\partial\psi}{\partial p}] \quad (1)$$

$$\psi = \psi(q,p,s) \quad (2)$$

where $\psi(q,p,s)$ is the probability density in the longitudinal phase space and is normalized as $\int\int\psi(q,p,s)dpdq = 1$. $q = z$ is the longitudinal coordinate and $p = \Delta p/p_0$ is the relative momentum deviation. The corresponding longitudinal distribution is calculated from $\psi(q,p,s)$ as

$$\lambda(q,s) = \int\psi(q,p,s)dp \quad (3)$$

and will be used in calculating wake forces. The Hamiltonian's equations are

$$\frac{\partial q}{\partial s} = \frac{\omega_s\sigma_z}{c\sigma_p}\cdot p \quad (4)$$

$$\frac{\partial p}{\partial s} = -\frac{\omega_s\sigma_p}{c\sigma_z}\cdot q - I_n\cdot F(q,s) \quad (5)$$

where $I_n = \dfrac{Ne^2}{E_0C}$ and wake force is

$$F(q,s) = \int_{q'=-\infty}^{\infty} W_0(q'-q)\lambda(q')dq' \quad (6)$$

The unit of W_0 is V/pC. N is bunch population. C is circumference of the ring. E_0 is the design energy of the ring.

Operator splitting

The technique of operator splitting [8, 9], or called time splitting, is widely used in solving partial differential equations (PDEs). To solve the VFP equation, we rewrite the VFP Eq. 1 and split the operators into three parts:

$$\frac{\partial\psi}{\partial s} = \mathcal{L}\psi = (\sum_{i=1}^{3}\mathcal{L}_i)\psi \quad (7)$$

The solution of Eq. 7 is given by

$$\psi^{n+1} = e^{\Delta s\mathcal{L}}\psi^n \quad (8)$$

where

$$\mathcal{L}_1 = -\frac{\omega_s\sigma_z}{c\sigma_p}\cdot p\cdot\frac{\partial}{\partial q} + \frac{\omega_s\sigma_p}{c\sigma_z}\cdot q\cdot\frac{\partial}{\partial p} \quad (9)$$

$$\mathcal{L}_2 = I_n\cdot F(q,s)\cdot\frac{\partial}{\partial p} \quad (10)$$

$$\mathcal{L}_3 = \frac{2\beta}{c}\frac{\partial}{\partial p}[p + \sigma_p^2\frac{\partial}{\partial p}] \quad (11)$$

The $\mathcal{L}_1$ and $\mathcal{L}_2$ represents Liouville operator, and the $\mathcal{L}_3$ is called Fokker-Planck operator. The Liouville operator is reversible and the Fokker-Planck operator is irreversible. A simple first-order splitting is formulated as

$$e^{\Delta s\mathcal{L}} \approx e^{\Delta s\mathcal{L}_1}e^{\Delta s\mathcal{L}_2}e^{\Delta s\mathcal{L}_3} \quad (12)$$

And high-order splitting instead of Eq. 12 cab be applied to achieve better approximation. For example, second-order symmetric splitting scheme can be adopted

$$e^{\Delta s(\mathcal{L}_1+\mathcal{L}_2)} \approx e^{\Delta s2/\mathcal{L}_1}e^{\Delta s\mathcal{L}_2}e^{\Delta s/2\mathcal{L}_1} \quad (13)$$

To get good approximation, usually we split the one-turn map in the ring to k integration steps. This scheme can be written as

$$e^{C\mathcal{L}} \approx [e^{C/k\mathcal{L}_1}e^{C/k\mathcal{L}_2}e^{C/k\mathcal{L}_3}]^k \quad (14)$$

Discrete operator

The discrete version of Liouville operator is Frobenius-Perron operator. Let $\mathcal{F}_1$ and $\mathcal{F}_2$ are the Frobenius-Perron operators corresponding to $\mathcal{L}_1$ and $\mathcal{L}_2$, then the evolution of probability density corresponding to reversible operators can be evaluated as

$$\psi^*(q,p) = \mathcal{F}_1\psi(q,p,n\Delta s) = \psi(R^{-1}(q,p),n\Delta s) \quad (15)$$

$$\psi^{**}(q,p) = \mathcal{F}_2\psi^*(q,p) = \psi^*(K^{-1}(q,p)) \quad (16)$$

where the rotation mapping R is

$$\begin{bmatrix} q' \\ p' \end{bmatrix} = \begin{bmatrix} \cos(\mu_s\Delta s/C) & \beta_z\sin(\mu_s\Delta s/C) \\ -\sin(\mu_s\Delta s/C)/\beta_z & \cos(\mu_s\Delta s/C) \end{bmatrix}\begin{bmatrix} q \\ p \end{bmatrix} \quad (17)$$

and the kick mapping K is

$$\begin{bmatrix} q' \\ p' \end{bmatrix} = \begin{bmatrix} q \\ p - I_nF(q,s)\Delta s/C \end{bmatrix} \quad (18)$$

For Fokker-Planck operator, we propose exponentially fitting scheme (EFS) [10] for discretization

$$\frac{\psi_i^{n+1}-\psi_i^n}{\Delta s} = \frac{2\beta}{c}\psi_i^{n+1} + \frac{\beta p_i(\psi_{i+1}^{n+1}-\psi_{i-1}^{n+1})}{c\Delta p} + \rho_i^{n+1}\frac{2\beta\sigma_p^2}{c}\frac{\psi_{i+1}^{n+1}-2\psi_i^{n+1}+\psi_{i-1}^{n+1}}{\Delta p^2} \quad (19)$$

Table 1: Main parameters of KEKB LER

Parameter	Value	Unit
Circumference	3016.25	m
Beam energy	3.5	GeV
Bunch population	6.6	10^{10}
Natural bunch legnth	4.58	mm
Synchrotron tune	0.024	
Longitudinal damping time	2000	turn
Energy spread	7.27	10^{-4}

where

$$\rho_i^{n+1} = \frac{p_i \Delta p}{2\sigma_p^2} \coth \frac{p_i \Delta p}{2\sigma_p^2} \quad (20)$$

As proved in [10], the EFS has the properties of 1) being uniformly stable for all values of integration step Δs, damping coefficient β and mesh size Δp; 2) being oscillation free.

SIMULATION RESULTS

Based on the algorithms described in the previous section, a code of VFP solver was developed and used in simulations of microwave instability at LER of KEK and SuperKEKB. The main parameters of KEKB LER are listed in Table 1. For SuperKEKB LER, we choose bunch length as 5 mm and other parameters as the same as KEKB LER.

The numerical impedance model predicts much weaker bunch lengthening against measurements [11, 12] as shown in Fig. 3. An pure inductance of around 90 nH should be added in this impedance model to get similar bunch lengthening. But when the pure inductance was added, the threshold of MWI get much higher as shown in Fig. 4. This disagreement indicates that there are unknown impedance sources in the KEKB LER. According to Fig. 4, threshold of MWI with CSR impedance is around 0.7 mA. Without CSR, the threshold is around 1.1 mA. It can be concluded that CSR is important source to drive microwave instability.

For bunch length of 5 mm, no serious bunch lengthening and energy spread growth are seen with bunch current up to 1.6 mA at SuperKEKB LER, as shown in Fig. 5 and Fig. 6.

SUMMARY AND DISCUSSIONS

Accurate impedance model is essential for studying microwave instability. When comparing with beam observations, the numerical impedance model for KEKB LER gave insufficient bunch lengthening and higher threshold for MWI. The discrepancy between numerical model and measurements are around 90nH in case of bunch lengthening.

CSR in storage rings like KEKB LER was not well understood yet. Interference between adjacent bending magnets seems to be strong. Thus more benchmarking on the CSR code is needed.

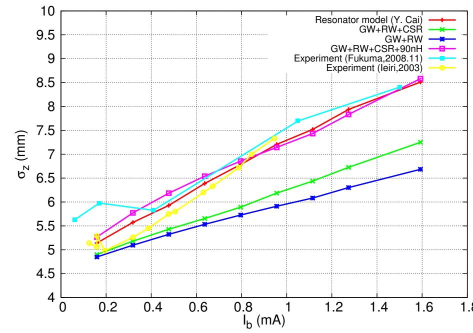

Figure 3: Bunch length as function of bunch current at KEKB LER. The resonator model is given in [5]

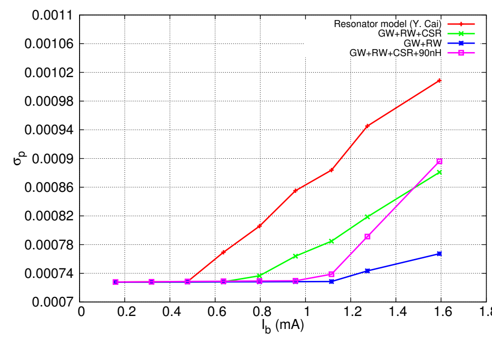

Figure 4: Energy spread as function of bunch crrent at KEKB LER.

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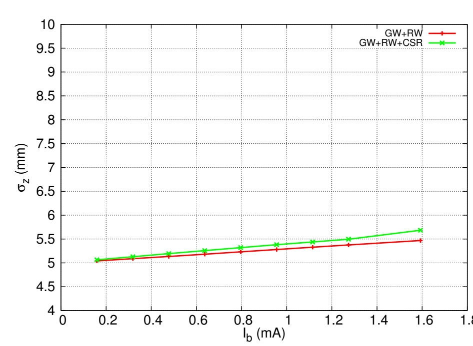

Figure 5: Bunch length as function of bunch current at SuperKEKB LER.

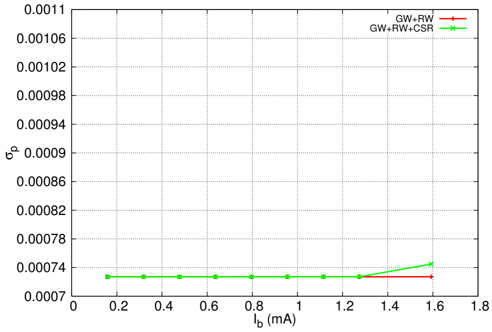

Figure 6: Energy spread as function of bunch crrent at SuperKEKB LER.

REFERENCES

[1] KEKB B-Factory Design Report, KEK-Report-95-7, (1995).

[2] K. Oide, ”KEKB B-Factory, The Luminosity Frontier”, Progress of Theoretical Physics, Vol. 122, No. 1, July 2009.

[3] K. Ohmi, et al., ”Luminosity Limit Due to the Beam-Beam Interactions With or Without Crossing Angle, Phys. Rev. ST Accel. Beams 7, 104401 (2004).

[4] D. Zhou, et al., ”Simulations on Beam-Beam Effect Under the Presence of General Chromaticity”, KEK Preprint 2009-10, May 2009.

[5] Y. Cai, et al., ”Potential-Well Distortation, Microwave Instability, and Their Effects with Colliding Beams at KEKB”, Phys. Rev. ST Accel. Beams 12, 061002 (2009).

[6] T. Agoh and K. Yokoya, ”Calculation of Coherent Synchrotron Radiation Using Mesh”, Phys. Rev. ST Accel. Beams 7, 054403 (2004)

[7] R. Warnock and J. Ellison, ” A General Method for Propagation of Phase Space Distribution, with Application to the Sawtooth Instability”,SLAC-PUB-8404, 2000.

[8] S. I. Tzenov, ”Contemporary Accelerator Physics”, Singapore, World Scientific, p. 217 (2004).

[9] W. Hundsdorfer and J. G. Verwer, ”Numerical Solution of Time-Dependent Advection-Diffusion-Reaction equations”, Springer series in Computational Mathematics, vol. 33, Springer, Berlin, p. 325 (2003).

[10] D.J. Duffy, ”A Critique of the Crank Nicolson Scheme Strengths and Weaknesses for Financial Instrument Pricing”, Wilmott magazine, p. 68-76, July 2004.

[11] T. Ieiri and H. Koiso, ”Measurement of Longitudinal Impedance at KEKB”, Proc. of 14th Symposium on Accelerator Science and Technology, KEK, Tsukuba, p. 443 (2003).

[12] H. Ikeda, et al., ”Negative Momentum Compaction at KEKB”, Proc. of Workshop on e^+e^- in the 1-2 GeV range: Physics and Accelerator Prospects, Alghero (SS), Italy, September 2003.