ULTRA-SHORT LOW CHARGE OPERATION AT FLASH AND THE EUROPEAN XFEL

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Abstract

The Free Electron Laser in Hamburg (FLASH) is a SASE FEL user facility and in addition serves as a prototype for the European XFEL. The recent upgrade of FLASH with a higher harmonic RF module opens a new possibility for ultra-short low charge operation. The advantage of small transverse emittance at low charges can be used only with strong, linearized bunch compression. At this report we consider simulations of the beam dynamics at low charges and estimate the expected properties of the radiation at FLASH and the European XFEL. We present first experimental results at FLASH.

INTRODUCTION

Several X-ray Free Electron Laser (FEL) projects are developed worldwide [1-3]. Recent successful lasing of LCLS with low charges and with ultra-small emittance have lead to the interest to use such scenarios at Free Electron Laser in Hamburg (FLASH) and at the European XFEL [4].

FLASH is a SASE FEL user facility and in addition serves as a prototype for the European XFEL. The recent upgrade of FLASH with a higher harmonic RF module opens a new possibility for ultra-short low charge operation. The advantage of small transverse emittance at low charges can be used only with strong, linearized bunch compression. At this paper we consider simulations of the beam dynamics at low charges and estimate the expected properties of the radiation at FLASH and the European XFEL.

With the help of self-consistent beam dynamics simulations for FLASH and European XFEL it will be shown that we are able to provide a well conditioned electron beam for different charges. However, the RF tolerances for low charges are tough. From FEL simulations for FLASH we can conclude that the charge tuning (20-1000 pC) in SASE mode allows to tune the radiation pulse energy (30-1400 J) and the radiation pulse width (FWHM 2-70 fs).

MULTISTAGE BUNCH COMPRESSION WITH COLLECTIVE EFFECTS

The nonlinearity of the radio frequency (RF) fields and of the bunch compressors (BC’s) can be corrected with a higher harmonic RF system. An analytical solution for cancellation of RF and BC’s non-linearities for a multistage stage bunch compressor system is presented in [5]. A more general solution for a system with collective effects (space charge forces, wakefields, a coherent synchrotron radiation (CSR) within the chicane magnets) can be found by an iterative tracking procedure based on this analytical result.

Collective Effects and Tracking Codes

The analytical solution introduced in [5] neglects the collective effects in the main beam line. In order to take them into account we do tracking simulations taking into account the collective effects through analytical estimations (space charge forces, wakefields), or through direct numerical solution with tracking codes.

To take into account coherent synchrotron radiation (CSR) in bunch compressors we use code CSRtrack [6]. This code tracks particle ensembles through beam lines with arbitrary geometry. It offers different algorithms for the field calculation: from the fast “projected” 1-D method [7] to the most rigorous one, the three-dimensional integration over 3D Gaussian sub-bunch distributions [8].

For high peak currents the compression is affected by wakefields from the vacuum chamber and by space charge forces. The free space longitudinal space charge impedance and the corresponding wake function for bunch with Gaussian transverse profile are given by [9]

\[
\frac{dZ(\omega)}{dz} = \frac{1}{2\gamma^2 c} \frac{1}{2\pi} e^{\omega t} \Gamma(0, \alpha^2) \]

\[
\frac{dw(s)}{dz} = \theta(s) \frac{Z_0 c}{8\pi\sigma_{\perp}^2} \left[ \frac{\xi(s)}{\xi(0)} \right] - \frac{\sqrt{\pi}}{2} \frac{\xi(s)}{\xi(0)} e^{s^2/4\sigma_{\perp}^2} \text{Erfc} \left[ \frac{\xi(s)}{2\sigma_{\perp}} \right],
\]

\[
\alpha = \frac{\omega \sigma_{\perp}}{c\gamma}, \quad \Gamma(0, \alpha) = \int_0^\infty e^{-t^2} dt, \quad \xi(s) = \frac{\sigma_{\perp}^2}{\sigma_{\perp}^2},
\]

where \( \sigma_{\perp} \) is the transverse RMS size of the beam, \( \theta(s) \) is the Heaviside step function, \( Z_0 \) is the free space impedance, \( c \) is the vacuum light velocity.

Let us consider the bunch accelerated from energy \( \gamma_0 \) to the energy \( \gamma_{\perp} \) along distance \( L \). Then we use an adiabatic approximation which takes into account the slow change of the RMS size of the bunch during the acceleration:

\[
Z(\omega) = \int_0^L \frac{dZ(\omega, r_s, \gamma, \gamma)}{dz} dz = \frac{\omega Z_0}{4\pi c} \int_0^{\gamma_0} e^{\alpha(s)} \Gamma(0, \alpha(z)^2) dz,
\]

\[
\alpha(z) = \frac{\omega \sigma_{\perp}(z)}{c\gamma(z)}, \quad \sigma_{\perp}(z) = \left[ \frac{\varepsilon_0}{\gamma(z)} \right]^{1/2},
\]

where \( \langle \beta \rangle \) is the averaged optical beta function along distance \( L \), \( \varepsilon_0 \) is the normalized transverse emittance.

Along with the above analytical estimations we use an alternative approach based on the straightforward tracking
with code ASTRA [10]. This program tracks particles through user defined external fields taking into account the space charge field of the particle cloud.

The both codes, CSRtrack and ASTRA, do tracking in free space neglecting the impact of the vacuum chamber on the self fields. We use coupling impedances (or wake functions) to take into account interactions of the bunch with the boundary. The wakefield code ECHO [11] was used to estimate the wake functions of different beam line elements.

The FLASH facility contains 56 TESLA accelerating cavities. Their wake function is given by [12]

\[ w(s) = 10^{-\theta(s)} 43 e^{-34 \delta} \]

The wake function of the harmonic module with 4 cavities reads [13]

\[ w(s) = 10^{-\theta(s)} \left( 318 e^{-34 s \delta} + 0.9 \cos(5830 s + 83) \sqrt{s + 195} + 0.036 \delta(s) \right), \]

where the last term with the Dirac delta function describes the reduction of the pipe radius from 39 mm to 20 mm at the position of the third harmonic module.

\[ \alpha = \frac{\bar{\delta}_1}{E_1^0}, \quad \beta = \frac{Z^* - \bar{Z}^*}{X_2}, \]

\[ \hat{X}_2 = \frac{\bar{r}_{562} E_2^0}{E_2} \hat{Z}_2 - 6 \bar{u}_{562} \left( \delta_1^3 \right)^3 - 6 \bar{r}_{562} \delta_2 \delta_x^*, \]

\[ \delta_x^* = \frac{\alpha E_1^0 \bar{Y}_2 + \bar{\delta}_2}{E_2^0}, \quad \hat{\delta}_2 = k' Z_1^* Y_2 - 3 k^2 Z_1^* X_2 \bar{k}_Y Z_1^* \bar{k}_Y \hat{Z}_1, \]

\[ \hat{\delta}_1 = -6 \bar{u}_{561} \alpha_1 - 6 \bar{r}_{561} \alpha_1 \alpha_2, \quad Z_1 = \bar{r}_{561} \alpha_2 - 2 \bar{r}_{561} \alpha_1^2. \]

The RF parameters in ACC1, X1, Y1, and in the third harmonic module ACC39, X1, Y1, can be found through relations (5) from paper [5]

\[ \begin{align*}
X_{1,1} &= \frac{F_1 + F_2 (3 k)^2}{8 k^2}, \quad Y_{1,1} = -\frac{F_1 + F_2 (3 k)^2}{8 k^3}, \\
X_{1,3} &= -\frac{F_1 + F_2 k^2}{8 k^2}, \quad Y_{1,3} = \frac{F_1 + F_2 k^2}{24 k^3}, \\
F_1 &= E_1^0 - E_0^0, \quad F_2 = E_1^0 \alpha_{i,i} - E_0^0 \frac{\partial^i \delta_{i,j}}{\partial x_{i,j}} (0), \quad i = 2, 3, 4.
\end{align*} \]

An Iterative Tracking Procedure with Collective Effects

The analytical solution for RF parameters given above will not produce the required compression in reality. The strong self fields can severely deteriorate the properties of the compressed bunch. In order to take the collective effects into account we have to carry out tracking simulations. For the adjustment of the RF parameters we use an iterative procedure, which starts from the values of the RF parameters obtained through the analytical solution introduced in the previous section.

The problem without self fields can be written in operator form

\[ \mathbf{A}_0 (\mathbf{x}) = \mathbf{f}_0, \]

where non-linear operator \( \mathbf{A}_0 (.) \) is defined in [5] and the right-hand side \( \mathbf{f}_0 \) and the unknown vector of the RF parameters \( \mathbf{x} \) are given by relations

\[ \begin{align*}
\mathbf{f}_0 &= (E_1^0, E_2^0, Z_1^0, Z_2^0, Z_3^0)^	op, \\
\mathbf{x} &= (X_{1,1}, Y_{1,1}, X_{1,3}, Y_{1,3}, X_2, Y_2)^	op.
\end{align*} \]

The previous section describes the inversion of this operator for a given vector of the macroparameters \( \mathbf{f}_0 \). We write the solution of the problem formally in the operator form

\[ \mathbf{x}_0 = \mathbf{A}_0^{-1}(\mathbf{f}_0), \]

where \( \mathbf{A}_0^{-1} \) is the inverse operator.

The general problem with self fields included reads

\[ \mathbf{A}_i (\mathbf{x}) = \mathbf{f}_i, \]

where non-linear operator \( \mathbf{A}_i (.) \) is realized by a tracking procedure for the given RF parameters vector \( \mathbf{x} \). Let us note that the tracking operator depends on this vector.

We would like to use the analytical solution as a “preconditioner” at each iteration. Our experience shows that such approach results in fast convergence (~ 5
iterations). In order to derive the iteration scheme let us rewrite Eq. (5) in an equivalent form
\[ x = A_0^{-1} \left( A_0(x) + f_0 - A_0(x) \right). \]
From the last equation the iterative scheme
\[ x_{n+1} = A_0^{-1} \left( A_0(x_n) + f_0 - A_0(x_n) \right), \]
(6)
can be suggested. It can be rewritten in a more convenient form, where one iteration includes the following steps:
\[ f_{n+1} = A_0(x_n) - \text{doing the numerical tracking,} \]
\[ \Delta f_{n+1} = f_0 - f_{n+1} - \text{calculation of the residual in the macroscopic parameters,} \]
\[ g_{n+1} = g_n + \Delta f_{n+1} \cdot x_n = A_0^{-1}(g_n), \]
- doing the analytical correction of the RF parameters.

The iterative scheme is robust and converges fast to the solution. We apply this iterative algorithm in the next section in order to find the working points for two-stage bunch compression system in FLASH.

**MODELLING OF TWO STAGE BUNCH COMPRESSION IN FLASH FACILITY**

The Free-Electron Laser FLASH at DESY is the first user facility for VUV and soft X-ray laser like radiation using the SASE scheme. Since summer 2005, it provides coherent femtosecond light pulses to user experiments with impressive brilliance [14]. It includes two bunch compressors, a C-chicane and an S-chicane. These two chicanes have to compress the electron bunches to achieve the peak current of 2.5 kA. The initial peak current after the gun is about 52 A.

The initial conditions \( E_0^0, \delta_0(0), \delta_0(0), \delta_0(0) \) are obtained from numerical simulations of the gun with code Astra [10]. The initial energy from the gun \( E_0^0 \) is about 5 MeV. The current profile and the longitudinal phase space after the RF gun, before the booster ACC1, are shown in Fig.2.

The initial peak current after the gun is about 52 A. Hence, in order to reach the peak current of 2.5 kA we need the total compression given by
\[ C \equiv Z_1^{-1} = 48. \]
(7)
After the recent upgrade the FLASH facility has the following technical constraints on the achieved voltages:
\[ V_{1,3} \leq 150 \text{ MV}, \quad V_{1,3} \leq 26 \text{ MV}, \quad V_2 \leq 350 \text{ MV}. \]
The deflecting radii in the bunch compressors have to fulfill the restrictions
\[ 1.4 \leq r_1 \leq 1.93, \quad 5.3 \leq r_2 \leq 16.8. \]

In order to correct the nonlinearity induced by the fundamental harmonic module ACC1 before compressor BC2 we need to use a deceleration in the third harmonic module ACC39. And for the voltages the relation \( V_{1,3} = V_{1,1}/9 \) approximately holds. Hence, the nominal energies in BC2 and BC3 are fixed with safety margin of 5% as follow
\[ E_1^0 = 0.95 \left[ E_0^0 + \frac{8}{9} \epsilon \max V_{1,1} \right] = 130 \text{MeV}, \]
\[ E_2^0 = 0.95 \cdot (E_1^0 + \epsilon \max V_2) = 450 \text{ MeV}. \]

Figure 2: The initial particle distribution after the gun.

Now we are going to choose the deflecting radius \( r_1 \) in compressor BC2. In order to reduce the space charge forces between the bunch compressors we aim to use only a weak compression in BC2. Hence the deflecting radius of the first bunch compressor is fixed at the maximum
\[ r_1 = 1.93 \text{ m}. \]
(9)
This solution has two additional benefits: small CSR fields in compressor BC2 itself and a possibility of a larger energy chirp after it. The last feature reduces the voltage requirement on RF modules ACC2 and ACC3.

Let us now choose the compression factor \( C_1 \equiv (Z_1^0)^{-1} \) in the first bunch compressor. We would like to take it as small as possible. For the time being we will fix the free parameters of the global compression at zero: \( Z_2^0 = 0 \), \( Z_2^0 = 0 \). From the analytical solution we build the plot shown in Fig. 3. It has three areas. In region I we need a very high voltage for the third harmonic module: \( V_{1,3} > 26 \text{ MV} \). In region II we need a very high voltage for the second accelerating module: \( V_2 > 360 \text{ MV} \). Hence our solution has to belong to region III. It can be seen from Fig. 3 that, due to the restriction on voltage \( V_{1,3} \), the compression in the first BC can not be less than 2. In order to have a reserve in \( V_{1,3} \) for adjustment of global compression parameter \( Z_2^0 \) and for the self-fields effects compensation we choose
\[ C_1 = 2.84. \]
(10)
Now we are going to choose the deflecting radius $r_2$ in S-chicane BC3. At the first step we will fix temporarily the phase $\phi_2$ between the bunch compressors near to the maximum $\phi_2 = 0.9 \cos^{-1} \left( \frac{E_2^0 - E_1^0}{\text{max}(V_z)} \right) = 22^\circ$. It means that we aim to produce the largest possible chirp with the RF system $(V_2, \phi_2)$. Hence, for the fixed compression factor $C_1$ the energy chirp at entrance of BC3 will be as large as only possible. Such solution uses a larger deflecting radius $r_2$ and it results in weaker CSR fields in the last chicane.

Bunch compressor BC3 is of S-type and the deflecting radius is given by relation

$$r_2 = \frac{L_0}{\sin \sqrt{\frac{r_{562}}{L_0} (3L_0 + 4L_0)}} = 6 \text{ m}, \quad (11)$$

where $L_0 = 0.5$ is the magnet length and $L_0 = 0.5$ is the drift length between the magnets.

Figure 5: Impact of $Z_2^{10}$ parameter on the bunch shape (left plot). Longitudinal phase space after the second bunch compressor (right plot).

Equations (7)-(11) give 6 macroparameters from eight required to define system (3). We need now to choose values of $Z_2^{10}$ and $Z_2^{10}$. It follows from the definition of function $Z(s)$ that in order to have a local maximum of the compression at $s = 0$ we need $Z_2^{10} = 0$, $Z_2^{10} > 0$. Let us first to consider meaning of the parameter $Z_2^{10}$. The left plot in Fig. 4 compares two compression curves for different values of this parameter. We see that for $Z_2^{10} = 0$ a very strong compression in the head of the bunch exists. We can avoid it by choosing $Z_2^{10} > 0$.

<table>
<thead>
<tr>
<th>$V_{1,1}$, $\phi_{1,1}$, $V_{1,3}$, $\phi_{1,3}$, $V_2$, $\phi_2$, MV</th>
<th>deg</th>
<th>MV</th>
<th>deg</th>
<th>MV</th>
<th>deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without self</td>
<td>148.5</td>
<td>10.5</td>
<td>21.0</td>
<td>180.8</td>
<td>345</td>
</tr>
<tr>
<td>With self</td>
<td>144.1</td>
<td>-4.7</td>
<td>22.6</td>
<td>144.7</td>
<td>350</td>
</tr>
</tbody>
</table>

In order to fix the positive value of the parameter $Z_2^{10}$ we consider the right plot shown in Fig. 4. It presents the required voltages vs. parameter $Z_2^{10}$. In order to minimize the requirement on the voltage we choose

$$Z_2^{10} = 2000 \text{ m}^2. \quad (12)$$

Finally, we would like to fix the last parameter $Z_2^{10}$. With the help of this parameter we can shift the maximum of the compression to the right or to the left as shown in Fig. 5. In order to symmetrize the current we use

$$Z_2^{10} = 1 \text{ m}^2. \quad (13)$$

Equations (7)-(13) completely define system (1) and from the analytical solution we can find the RF parameters given in Table 1 (the first row).
Figure 6: The RF tolerances in accelerating module $M_1$ vs. global compression parameter $Z_2^2$.

![Figure 6](image)

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\Delta V$</th>
<th>$\Delta I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.00111$</td>
<td>$0.0022$</td>
<td>$0.0026$</td>
</tr>
<tr>
<td>$0.00096$</td>
<td>$0.0075$</td>
<td>$0.0042$</td>
</tr>
<tr>
<td>$0.00072$</td>
<td>$0.0021$</td>
<td>$0.0022$</td>
</tr>
</tbody>
</table>

Table 2: RF Tolerances in the Working Point

Figure 7: The RF tolerance in accelerating module $M_{1,1}$ vs. compression in the first BC.

Let us estimate tolerances for relative change of compression $\Theta = \Delta C_2/C_2 = 0.1$. We use the analytical estimations of [5]. The left plot in Fig. 6 presents the estimation of the relative voltage and phase deviations admissible in module ACC1

$$\frac{\Delta V_{1,1}}{V_{1,1}} = \frac{Z_2^0}{V_{1,1}} \left[ \sqrt{\rho_{1,1} Z_2^2} \right], \quad |\Delta \phi_{1,1}| \leq \frac{Z_2^0}{|\rho_{1,1} Z_2^2|} \Theta.$$

These tolerances are obtained from equations (13) of paper [5]. By the solid line we show the strongest tolerance in two dimensional space $(X_{1,1}, Y_{1,1})$. It is given through the gradient as follows

$$\frac{\Delta v_{1,1}}{V_{1,1}} = \frac{Z_2^0}{V_{1,1}} \Theta.$$ 

The same tolerances are shown for the third harmonic module at the right plot in Fig. 6. Table 2 presents all RF tolerances for the working point defined in this section.

Finally, we show in Fig. 7 the dependence of the strongest tolerance in the booster ACC1 on the choice of the compression factor $C_1$ for the fixed global compression factor $C = 48$, and other parameters chosen as described above. It is easy to see that the chosen value $C_1 = 2.84$ is near to the optimum. Let us note that the approximate solution given by Eq.(15) in [5] results in the value $C_1 = 2.67$.

### Working Points for Low Charges

In our simulations we have used the same laser pulse length at the cathode of the gun for all charges. Hence, in order to reach the same peak current with low charges we need to increase the compression factors in the bunch compressors. In order to scale the bunch compression factor in the first bunch compressor we can consider a trajectory equation in FODO cell [15]

$$x' + k_x x = \frac{1}{I_0} \frac{\beta_0' y'}{\sigma_x (\sigma_x + \sigma_y)} x.$$

In order to have the same defocusing due to space charge forces we have to keep the relation

$$\frac{I}{\sigma_x (\sigma_x + \sigma_y)} = \text{const}.$$

If we suggest that the emittance scales as a square root from the charge then our scaling law for compression factor in the first bunch compressor reads

$$C_1(q) = O\left(\frac{\sqrt{q}}{I_0(q)}\right) = O\left(\frac{1}{\sqrt{q}}\right),$$

where $I_0(q)$ is the peak current after the gun for charge $q$. In our simulations we have used a more aggressive scaling of the compression in the first bunch compressor. It is given in Table 3.

<table>
<thead>
<tr>
<th>$q$, nC</th>
<th>$E_1$, [MeV]</th>
<th>$E_2$, [MeV]</th>
<th>$r_1$, [m]</th>
<th>$r_2$, [m]</th>
<th>$C_1$</th>
<th>$Z_2^1$, [m]</th>
<th>$Z_2^2$, [m$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>48</td>
<td>1</td>
<td>2e3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>130</td>
<td>450</td>
<td>1.93</td>
<td>6.93</td>
<td>4.63</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>0.25</td>
<td>7.8</td>
<td>6.57</td>
<td>150</td>
<td>0.7</td>
<td>4e3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>9.3</td>
<td>10.3</td>
<td>240</td>
<td>0</td>
<td>4e3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>15.17</td>
<td>31.8</td>
<td>1000</td>
<td>-0.5</td>
<td>5e3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Tracking Simulations with Collective Effects

In this section we present results for simulations with all collective effects included. We have implemented two different tracking procedures. The first procedure uses the analytical model of accelerating modules and tracks the transverse phase space by linear optics transform matrices. The longitudinal space charge forces are taken into account analytically as described at the first section. The second procedure uses code ASTRA to track the particles through the accelerating sections of the beam line. The bunch compressors in both procedures are tracked with the help of code CSRtrack. The first procedure is fast. It takes only about 10-20 minutes on one processor. The second procedure is very time-consuming.
consuming and takes hours of heavy parallelized calculations. We use the first model to implement the iterative procedure described by Eq. (6). It takes about 5-10 iterations to solve the problem. After it we check the results with the full three dimensional calculations implemented in the second procedure.

Table 4: RF Tolerances for Low Charges.

<table>
<thead>
<tr>
<th>q, nC</th>
<th>0.5</th>
<th>0.25</th>
<th>0.1</th>
<th>0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACC1</td>
<td>$</td>
<td>\Delta V/V</td>
<td>$</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\Delta \Phi</td>
<td>$, deg</td>
<td>0.025</td>
</tr>
<tr>
<td>ACC39</td>
<td>$</td>
<td>\Delta V/V</td>
<td>$</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\Delta \Phi</td>
<td>$, deg</td>
<td>0.061</td>
</tr>
<tr>
<td>ACC 2/3</td>
<td>$</td>
<td>\Delta V/V</td>
<td>$</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\Delta \Phi</td>
<td>$, deg</td>
<td>0.15</td>
</tr>
</tbody>
</table>

We have checked with the tracking that the tolerances are left approximately the same as described in Table II for the situation without self fields.

Figure 9 presents properties of the bunch with charge 100 pC at the undulator entrance. Properties of the bunches for other charges are summarized in Table V of the next section.

The RF tolerances for different charges are summarized in Table 4.

**FEL Simulations**

The results of beam dynamic simulations are used as an input for FEL code ALICE [16] to estimate properties of radiation for different charges.

We have followed the standard way of preparing input data for an FEL code: a macro-particle distribution at the undulator entrance was cut into longitudinal slices. A mean energy, RMS energy spread, current, RMS emittance and other parameters were calculated for each slice. Then the centre of the bunch was perfectly matched to the undulator entrance.

Figure 10: The averaged radiation energy in the pulse along the undulator for different charges.

Figure 11: Temporal structure of the radiation pulse at $z = 20\, \text{m}$ for charges 1 nC (the left plot) and 20 pC (the right plot). Solid line and the dashed one correspond to an averaged and a single pulse profile, respectively. Dotted line shows profile of the electron bunch.

Figures 10, 11 present some properties of the radiation for different charges. It can be seen from the last plot that a single x-ray spike with full longitudinal coherence may be expected for the bunch charge of 20 pC.

The main properties of the electron bunch and the radiation produced in the FLASH undulators are summarized in Table 5.

The most right column presents the parameters estimated for the “spike mode” without third harmonic
cavity [17]. It can be seen that due to the better emittance and higher peak current the saturation length is drastically reduced.

Table 5: Radiation and Beam Properties for Different Charges.

<table>
<thead>
<tr>
<th></th>
<th>with harmonic module</th>
<th>without harmonic module</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch charge, nC</td>
<td>1 0.5 0.25 0.1 0.02 0.5-1</td>
<td></td>
</tr>
<tr>
<td>Wavelength, nm</td>
<td>6.5 6</td>
<td></td>
</tr>
<tr>
<td>Beam energy, MeV</td>
<td>1000 1000</td>
<td></td>
</tr>
<tr>
<td>Peak current, kA</td>
<td>2.5 2.1 1-1.5 1.3-2.2</td>
<td></td>
</tr>
<tr>
<td>Slice emittance, mm</td>
<td>1-1.3 0.7-0.9 0.5-0.7 0.4-0.3 1.5-3.5</td>
<td></td>
</tr>
<tr>
<td>Slice energy spread, MeV</td>
<td>0.1-0.2 0.25 0.4 0.25 0.3</td>
<td></td>
</tr>
<tr>
<td>Saturation length, m</td>
<td>13 12 11 10 11 22-32</td>
<td></td>
</tr>
<tr>
<td>Energy in the rad. pulse, mJ</td>
<td>1000-1400 700 500 200 30 50-150</td>
<td></td>
</tr>
<tr>
<td>Radiation pulse duration FWHM, fs</td>
<td>70 30 17 7 2 15-50</td>
<td></td>
</tr>
<tr>
<td>Averaged peak power, GW</td>
<td>5.7 2.4</td>
<td></td>
</tr>
<tr>
<td>Spectrum width, %</td>
<td>0.4-0.6 0.8-1 0.4-0.6</td>
<td></td>
</tr>
<tr>
<td>Coherence time, fs</td>
<td>4-5 - - -</td>
<td></td>
</tr>
</tbody>
</table>

**FIRST EXPERIMENTAL DATA AT FLASH**

Figure 12: The first experimental data of compression with third harmonic module in FLASH for the bunch charge of 200pC.

After upgrade of the FLASH facility and installation of the third harmonic module the first experiments on linearized bunch compression for low charges started and the first lasing was obtained. Figure 12 presents the measured longitudinal phase space and the estimated current profile for bunch charge of 200 pC. The measurements are done with transverse deflecting cavity by Ch. Behrens. We see that for the used RF parameters we have non-symmetric bunch shape with stronger compression at the head of the bunch (The bunch head is at the left side here). For a better compression we need to change the RF parameters in order to reach the conditions $Z'' > 0$, $Z' > 0$.

**MODELLING OF THREE STAGE COMPRESSION IN EUROPEAN XFEL**

In this section we consider beam dynamics simulations for the European XFEL. We will follow the same formalism used already for FLASH. The main difference to the previous study is the three stage bunch compression scenario of the European XFEL.

**Explicit Form of the Solution for Three Stage Bunch Compression System**

The European X-ray Free Electron Laser (XFEL) will use a three stage bunch compression scheme with third harmonic module for the longitudinal phase space linearization. In this case we have to define 8 RF parameters ($X_1, Y_1, X_2, Y_2, X_3, Y_3$). In order to define 8 equations [5] we have to fix 15 independent parameters:

$E_0, \delta_0, \delta_0'$ - initial conditions (as obtained from the gun simulations);

$r_i, r_i', r_i^0, E_i, E_1^0, E_2^0$ - deflecting radii and nominal energies in the bunch compressors;

$Z_1, Z_2' - inverse$ compression factors after compressor $BC_1$ and after compressor $BC_2$;

$Z_3, Z_3', Z_3'' - parameters$ of compression after $BC_3$.

The solution for this configuration can be written explicitly:

$$X_3 = E^0_3 - E_3^0, \quad Y_3 = \delta_3 E^0_3 - \delta_3' E_3^0, \quad \delta_3 = \frac{Z_3 - Z_3'}{r_{563}},$$

$$\alpha_3 = \frac{y_3}{E_3}, \quad \gamma_3 = \frac{Z_3 - \hat{x}_3}{x_3}, \quad \bar{x}_3 = \bar{x}_3 - \frac{r_{563}}{E_3} \bar{y}_3 - 2t_{563} (\delta'_3)^2,$$

$$\bar{y}_3 = \bar{y}_3 - k^2 Z_3^2 X_3 - kY_3 \bar{x}_3, \quad \bar{x}_3 = \bar{x}_3 - \frac{r_{563}}{E_3} \bar{y}_3,$$

$$\bar{y}_3 = \bar{y}_3 - kY_3 \bar{x}_3, \quad \alpha_3 = \frac{\hat{y}_3}{E_3}, \quad \hat{y}_3 = \frac{Z_3'' - \hat{x}_3}{\bar{x}_3},$$

$$\hat{x}_3 = \hat{x}_3 - \frac{r_{563}}{E_3} \hat{y}_3 - 6u_{563} (\delta'_3)^3 - 6t_{563} \delta_3' \delta_3''$$

$$\delta_3 = \alpha_3 E_3^0 \bar{y}_3 + \bar{x}_3, \quad \hat{x}_3 = \hat{x}_3 + kY_3 \bar{x}_3, \quad 3k^2 Z_3^2 X_3 - kY_3 \hat{x}_3,$$

$$Z'_3 = Z_3' - r_{563} \delta_3 - 2t_{563} (\delta'_3)^2.$$  

Other RF parameters can be found by the same relation as for two bunch compression system (see Eq. (1)).
Choosing of the Working Points and Beam Properties from Beam Dynamics Simulations

From technical constrains of the current layout and from the required bunch properties we can fix most of the parameters. But 5 parameters \( r_1, r_2, r_3, Z_1, Z_2 \) require an additional consideration.

\[
\delta E_i = \frac{\Delta E_i}{V_{i1}} \times 10^4
\]

Figure 13: The dependence of RF tolerance in the first module from the bunch compression factors for the bunch charge of 20 pC.

\[
r_i = \max(r^0_{i3}, \min r^0_{i3}), \quad r^0_{i3} = \left( \frac{1}{C_1 C_2} - \frac{1}{C} \right) \Delta E_i / \Delta W_i \quad (14)
\]

where \( \Delta W_i = qW_N C \), \( W_N = 43 \text{ V/pC} \) (see the first section) and \( N_C = 512 \) is the number of the TESLA cavities in the main linac.

Let us introduce the energy variation after the bunch compressor number \( i \) as

\[
\delta E_i = \left( \max E_i(s) - \min E_i(s) \right) / E_i^0.
\]

If we have restriction on the energy variation in the first bunch compressors then the deflecting radii can be found from the conditions

\[
r^0_{i3} = \max(r^0_{i3}, \min r^0_{i3}), \quad r^0_{i3} = \frac{L_i}{\delta E_i C_1} \left( 1 - \frac{1}{C_2} \right), \quad (15)
\]

\[
r^0_{i3} = \max(r^0_{i3}, \min r^0_{i3}), \quad r^0_{i3} = L_i \delta E_i \left( 1 - \frac{1}{C_1} \right). \quad (16)
\]

Taking into account relations (14)-(16) we are doing two dimensional scan of the RF tolerance dependence on the bunch compression factors in the first and the second bunch compressors. It is presented in Fig. 13 for the bunch charge of 20 pC and \( \delta E_i = 4\% \), \( \delta E_i = 2.5\% \). The white circle presents the choice of the working point.

Finally, Fig. 14 shows the main properties of the bunch at the undulator entrance as obtained from the beam dynamic simulations for bunch charge of 20 pC.

The main beam properties at the undulator entrance for different charges are summarized in Table 6.

Table 6: Bunch Properties at the European XFEL

<table>
<thead>
<tr>
<th>Bunch charge, nC</th>
<th>1</th>
<th>0.5</th>
<th>0.25</th>
<th>0.1</th>
<th>0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak current, kA</td>
<td>~ 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slice emittance, mm-mrad</td>
<td>0.7</td>
<td>0.4-0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Slice energy spread, MeV</td>
<td>0.1-0.2</td>
<td>0.1-0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Bunch length FWHM, fs</td>
<td>160</td>
<td>80</td>
<td>40</td>
<td>10</td>
<td>1-2</td>
</tr>
</tbody>
</table>

I would like to thank all members of DESY beam dynamics group for useful discussions and suggestions.

REFERENCES