INVESTIGATION OF THE $R_{56}$ OF A PERMANENT MAGNET PHASE SHIFTER

Y. Li, J. Pflueger
European X-Ray Free Electron Laser (XFEL). Notkestrasse 85, 22603 Hamburg, Germany

Abstract

In the European XFEL permanent magnet phase shifters will be used routinely between two undulator segments. Their main purpose is to control the ponderomotive phase of the electrons with respect to the emitted radiation. In addition the path length is dependent on the electron energy, which corresponds to a small $R_{56}$. In this paper we investigate the $R_{56}$ of a permanent magnet phase shifter and propose to use it to fine tune $R_{56}$ by adjusting the phase shifter gap.

INTRODUCTION

X-ray free electron laser facilities, especially the ones based on self-amplified spontaneous emission (SASE) configuration [1, 2], require tens of meters or even over 100 m long undulators to achieve saturation. Such a long undulator system can not be built as one continuous device: They must be segmented due to mechanical reasons as well as efficient beam control. As a result, additional instruments such as quadrupoles, beam position monitors, synchrotron radiation absorbers, vacuum pumps, etc. are needed in regular intervals. For example, the European x-ray free electron laser facility [3] will be constructed with 5 m long undulator segments and the intersection between two segments is 1.1 m.

Once the undulator gap is changed the resonance wavelength changes accordingly, therefore it is a convenient way to tune the radiation wavelength. However, the phase advance between electrons and photon field through the intersection in between two adjacent undulator segments varies too. Consequently a device called phase shifter is needed, which allows an additional and adjustable phase delay to properly match the phase between individual undulator segments.

On the other hand, phase shifters are also needed for some specialized purposes: For instance, it can be used for polarization control in a crossed undulator system [4, 5, 6, 7].

A phase shifter could either use a chicane consisting of three electron magnets (EMs) [8, 9] or permanent magnets [10]. Due to some specific concerns such as occupied space, field quality and dissipated heat, et.al., The European XFEL adopts the latter choice. Nevertheless, both types are in principle magnet chicanes, therefore they unavoidably induce some extent of $R_{56}$ to the beam. The $R_{56}$ of a phase shifter is, however, very small. Therefore not much attention has been paid to this aspect. Whereas in x-ray FELs the beam is micro bunched on the 1 Angstrom scale, therefore even a very small $R_{56}$ by the phase shifters maybe enormously affect the the properties of the radiation.

The present paper addresses the $R_{56}$ studies for a permanent magnetic phase shifter. The study uses the configuration used in the European XFEL. The expression of $R_{56}$ is deduced and its impact to the phase space of electron beam is discussed. Further more, the potential use of a phase shifter as a $R_{56}$ fine tuner is also investigated.

$R_{56}$ OF A PHASE SHIFTER

The field of a permanent magnetic phase shifter can be described as a double sinus like curve, as shown in Fig. 1 [10]:

\[
\begin{align*}
\lambda_p & = (1 + \frac{1}{2\gamma^2}) L_D \\
+ \frac{1}{2} \left( \frac{e}{m\gamma} \right)^2 \int_{-L_D/2}^{L_D/2} dz \left( \int_{-L_D/2}^{z} B_y(z')dz' \right)^2,
\end{align*}
\]

(1)

where $L_D$ is the length of magnet field, $c$ the velocity of light and $B_y(z)$ the magnet field distribution: The field extends from $z = - \frac{L_D}{2}$ to $z = \frac{L_D}{2}$. Outside it is zero. The magnet field direction is supposed along $y$ direction and $z$ is the coordinate along the beam path. $e$ is the electron charge, $m$ its mass and $\gamma$ the electron’s energy in unit of its rest mass (0.511 MeV).

Figure 1: Description of the field of a permanent phase shifter: The black curve of double sinus shows the model, $\lambda_p$ is its period.

The flying time $T$ of electrons though a magnet field is given by:

\[
cT = \left(1 + \frac{1}{2\gamma^2}\right) L_D
+ \frac{1}{2} \left( \frac{e}{m\gamma} \right)^2 \int_{-L_D/2}^{L_D/2} dz \left( \int_{-L_D/2}^{z} B_y(z')dz' \right)^2
\]


Inducing the magnet vector potential: \( \nabla \times \vec{A} = \vec{B} \), so
\[ B_y = \frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z. \quad (2) \]
Assume \( B_y \) independent to \( x \), which is the normal case, the second term in Eq. (2) equals to zero, and the field integration is just the potential \( A_x \):
\[ A_x = \int_{-L_D/2}^{L_D/2} B_y(z') dz'. \quad (3) \]
In addition, the first term of \( L_D \) in Eq. (1) can be dropped to calculate the time difference \( \Delta T \) between electrons and photons, because in the same time \( T \) photon passes the length of \( L_D \). Consequently, the phase advance can be calculated:
\[ \phi = \frac{2\pi}{\lambda_R} \int_{-L_D/2}^{L_D/2} A_x^2(z) dz, \quad (4) \]
where \( \lambda_R \) is the radiation wavelength. To be noticed, in Ref. [10], the term of \( \int_{-L_D/2}^{L_D/2} A_x^2(z) dz \) is called phase integral \( (PI) \). To simplify our analysis, \( PI \) is calculated by half part of the phase shifter, named \( PI_h \). The whole phase shifter’s \( PI \) is simply double of \( PI_h \):
\[ PI_h = \int_{-L_D/2}^{L_D/2} A_x^2(z) dz, \quad (5) \]
where \( \lambda_p \) is one sinus period as shown in Fig. 1.

Further more, according to Fig. 1 the field distribution \( B_y \) follows sinus function. Similar to the undulator parameter \( K \), a phase shifter parameter \( K_p \) can be defined:
\[ K_p = \sqrt{\frac{2e}{mc}} \int_{-\lambda_p}^{\lambda_p} \frac{1}{\lambda_p} \int_{-L_D/2}^{L_D/2} A_x^2(z) dz, \quad \lambda_p = \frac{2\pi}{\lambda_R}. \quad (6) \]
And the relationship between \( K_p \) and \( PI_h \) is:
\[ PI_h = \frac{1}{2} \left( \frac{mc}{e} \right)^2 \lambda_p K_p^2. \quad (7) \]
\( PI_h \) can further be expressed by the peak field \( B_{y0} \):
\[ PI_h = \frac{a}{2\pi} B_{y0}^2 \lambda_p^3, \quad (8) \]
and the \( K_p \) accordingly be expressed by \( B_{y0} \) too:
\[ K_p = 0.934 \sqrt{3} B_{y0} T [\lambda_p [cm]], \quad (9) \]
where it is \( \sqrt{3} \) times of the normal undulator parameter \( K \) [10].

Substitute Eq. (8) and Eq. (9) into Eq. (4) the phase shift \( \phi \) by the first half part of phase shifter is:
\[ \phi = \frac{\lambda_R}{\lambda_p} \left[ \lambda_p + \left( \frac{mc}{e} \right)^2 PI_h \right] = \frac{\lambda_R}{\lambda_p} \left( 1 + \frac{R_{56}}{2} \right). \quad (10) \]
According to Eq. (10) \( \phi \) depends on particle’s energy \( \gamma \). Its differential is:
\[ \frac{1}{d\gamma} \frac{d\phi}{d\gamma} = -2 \frac{\lambda_p^2 \pi}{\lambda_R \gamma^3} \left( 1 + \frac{K_p^2}{2} \right). \quad (11) \]

Assume a reference electron whose energy equals to the average energy, each electron’s relative longitudinal position \( s \) is accordingly defined:
\[ \phi = \frac{2\pi s}{\lambda_R}. \quad (12) \]
Substitute Eq. (12) into Eq. (11):
\[ \delta s = -2 \frac{\lambda_p^2}{\gamma^2} \left( 1 + \frac{K_p^2}{2} \right). \quad (13) \]

The definition of \( R_{56} \) is \( \delta s = R_{56} \frac{\delta \gamma}{\gamma} \), so the \( R_{56} \) of the whole phase shifter is:
\[ R_{56} = -2 \frac{\lambda_p^2}{\gamma^2} \left( 1 + \frac{K_p^2}{2} \right). \quad (14) \]
Eq. (14) is quite similar to the resonance condition of undulator. More over, normally \( K_p^2 >> 1 \), therefore according to Eq. (14), the \( R_{56} \) of a phase shifter is proportional to its peak field squared and third power of period \( \lambda_p \).

Further more, because the iron yoke of a permanent phase shifter sets Neumann boundary conditions, the magnetic situation is fully equivalent to that found in Halbach-type hybrid undulators. The gap dependence can therefore be described with an exponential of the from
\[ B_{y0} = a \exp(b/g/\lambda_p) + c(g/\lambda_p)^2. \quad (15) \]
Where \( g \) is the gap, \( a, b, c \) the dimensionless coefficients. In order to get estimates preliminary values they were taken from Ref. [11]: \( a = 3.694, b = -5.068, c = 1.520. \)

Combine Eq. (9), Eq. (14) and Eq. (15) the dependence of \( R_{56} \) on \( \lambda_p \) can be estimated. The results are illustrated in Fig. 2: It is seen that a phase shifter with \( \lambda_p \) of 10 cm could provide \( R_{56} \) of -100 nm. Such a phase shifter could be used as a \( R_{56} \) fine tuner, as discussed in the next section.

In addition, the maximum \( R_{56} \) of the phase shifter used in the European XFEL could be evaluated accordingly: \( \lambda_p = 0.05m, B_{y0} = 1.5T, \gamma = 34246(17.5 GeV), \) consequently \( K_p = 12 \) and
\[ R_{56} = -6.14 nm. \]

**POTENTIAL USE OF THE \( R_{56} \) OF A PHASE SHIFTER**

*Impact of the Bunch Factor \( b_1 \) to SASE1 of the European XFEL*

As seen by the discussion in the last section, the induced \( R_{56} \) of a phase shifter is as small as several nanometers,
Figure 2: $R_{56}$ versus $\lambda_p$ of phase shifters. The gap is assumed 10 mm and the beam energy is assumed 17.5 GeV.

therefore it could be neglected in most cases. However, the shortest wavelength of the European XFEL is 0.1 nm, so the radiation quality sensitively corresponds to the properties of micro bunched beams. Therefore, a weak $R_{56}$ could also have an impact on the micro bunched beam.

In order to investigate their impact a brief simulation was done for the SASE1 beam line in the European XFEL. Table 1 lists the basic parameters of SASE1. The maximum value of $R_{56}$ is -6.14 nm.

Table 1: Parameters used for simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undulator period $\lambda_U$</td>
<td>35.6</td>
<td>mm</td>
</tr>
<tr>
<td>$K_{Peak}$ at 10 mm gap</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td>$K = K_{RMS}$ at 10 mm gap</td>
<td>2.33</td>
<td></td>
</tr>
<tr>
<td>Undulator segment length</td>
<td>5</td>
<td>m</td>
</tr>
<tr>
<td>Electron beam energy</td>
<td>17.5</td>
<td>GeV</td>
</tr>
<tr>
<td>Energy spread (RMS)</td>
<td>1.5</td>
<td>MeV</td>
</tr>
<tr>
<td>Radiation wavelength</td>
<td>0.1</td>
<td>nm</td>
</tr>
<tr>
<td>Bunch peak current</td>
<td>5</td>
<td>kA</td>
</tr>
<tr>
<td>Normalized emittance (RMS)</td>
<td>1.4</td>
<td>mm mrad</td>
</tr>
<tr>
<td>Average $\beta$-function</td>
<td>32</td>
<td>m</td>
</tr>
<tr>
<td>Maximum $R_{56}$ of phase shifters</td>
<td>-6.14</td>
<td>nm</td>
</tr>
</tbody>
</table>

Due to the interaction between the electron beam and the photon field, the energy spread increases along undulator. It is shown in Fig. 3. In addition the beam transverse size is also illustrated.

Seven reference positions are selected to investigate the variations of bunch factor $b_1$ by phase shifters at different energy spread conditions. They are marked by the circles in Fig.3. The bunch factor $b_1$ is defined by:

$$b_1 = |\langle e^{-i\phi_n} \rangle|,$$

where $\phi_n$ is the $n$th electron’s ponderomotive phase, $\langle \rangle$ means averaging over all electrons of the bunch.

Fig. 4 illustrates the bunch factor at each position: $b_{1,b}$ the number before passing the phase shifter and $b_{1,a}$ the number after the phase shifter. The blue dotted line illustrates their difference, $b_{1,a} - b_{1,b}$.

According to Fig. 4 at the positions before 120 m, phase shifters slightly enhance the bunch factor $b_1$, on the contrary after 120 m phase shifter decreases it. This phenomenon is because $b_1$ saturates at the position around 120 m. The shape of phase space clock wise rotates during the whole FEL process. Before saturation is reached, $b_1$ is increased by clock wise rotation and after saturation this leads to debunching. A negative $R_{56}$ provides an additional clock wise rotation, quite like the performance of dispersion chicane used in HGHG FEL [12]. Therefore it helps to increase $b_1$ before saturation is reached and accelerates debunching afterwards.

Nevertheless the variation of the bunching factor $b_1$ by a phase shifter is smaller than 0.05. Therefore it is beneficial to the FEL process, but its effect is limited.
Phase Shifter as $R_{56}$ Fine Tuner

In some specific cases fine tuning of $R_{56}$ down to several nanometers is needed. For example the Fig. 9 in Ref. [13] shows that a change of $R_{56}$ by some tens of nanometers heavily influences the power and polarization of a crossed undulator placed behind the bunching undulator. $R_{56}$ may be optimized dynamically for a given radiation wavelength. This accuracy is hard to obtain with other means.

The permanent phase shifter studied in this paper gives a possibility of tuning $R_{56}$ with nanometer precision. Fig. 2 shows that a phase shifter with $\lambda_p$ of 10 cm, which is realizable with standard technology, gives $R_{56}$ of -100 nm. More over, $R_{56}$ could be dynamically tuned by adjusting the gap, $g$. Fig. 5 illustrates $R_{56}$ and its differential $dR_{56}/dg$ versus gap. The period $\lambda_p$ of the phase shifter is chosen to be 10 cm. In the range of 10 to 50 mm gap, $R_{56}$ changes from -100 nm to -5 nm and the maximum differential is 10 nm/mm. Because the gap can be reliably adjusted with an accuracy of 10 $\mu$m, in principle $R_{56}$ could be precisely tuned with a resolution of 0.1 nm, which is an extremely small number.

Figure 5: $R_{56}$ (black solid curve) and its differential $dR_{56}/dg$ (red dash curve) versus gap. The period $\lambda_p$ is 10 cm.

SUMMARY

The $R_{56}$ induced by phase shifters based on permanent magnets is investigated. A parameter $K_p$ is defined for easily calculation of $R_{56}$: $R_{56}$ is proportional to $K_p$ squared and $K_p$ is proportional to the product of period $\lambda_p$ and peak field $B_{y0}$. More over, permanent magnet phase shifters satisfy the magnetic situation found in Halbach-type hybrid undulator, which allows easy analytic estimation of the $R_{56}$ as a function of $\lambda_p$ and gap.

The impact on bunch factor $b_1$ by phase shifters are accordingly studied for the European XFEL. Simulations show that it could slightly fasten the bunching process before the saturation, therefore no harmful effects are expected.

In addition, a potential use of permanent phase shifter as a $R_{56}$ fine tuner is proposed. A phase shifter with a period length of 10 cm could provide a $R_{56}$ from 0 to -100 nm as well as dynamically tuning of $R_{56}$ with a resolution of 0.1 nm. The studies in Ref. [13] show that a minor variation of $R_{56}$ could greatly change the power and polarization driven by a system comprised of a planar buncher and a pair crossed undulators, so a phase shifter could provide accurate control in that case. However, further detailed studies involving investigation of ISR and CSR effects should be done in the future work.

REFERENCES