ON THE PROBLEM OF THRESHOLD CHARACTERISTICS FOR FELWI

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Abstract

For a free-electron laser without inversion (FELWI), estimates of the threshold laser power are found. The large-amplification regime should be used to bring an FELWI above the threshold laser power.

THRESHOLD FOR FELWI

In a free-electron laser (FEL), coherent stimulated radiation is produced by the accelerated motion of electrons in the ponderomotive potential formed by the combined field of the wiggler and the electromagnetic wave. The so-called free-electron laser without inversion (FELWI) proposed recently [1]-[6] relies on the non-collinear arrangement of the electron and laser beams. An FELWI aims to improve performance of FELs and optical klystrons and to extend operation domain to shorter wavelengths. This is to be achieved by a two-wiggler design employing an advanced laser-induced electron phasing in the first wiggler. It reveals itself in the proportionality of the laser-induced changes of the electron energy and the laser-induced angular deviation of electron velocity, which is specific to the non-collinear interaction geometry of an electron beam a laser beam. Engineering of the angular-dependent electron path length in the drift region between the two wigglers makes it possible to tune most of the electrons exiting the first wiggler closer to the phase, which is favorable for efficient amplification in the second wiggler. As a result, an FELWI gain G becomes positive for almost every detuning $\Omega \equiv \omega(v_0 - v_{res})/c$, which characterizes deviation of the electron velocity or the laser frequency from the resonance condition, $\Omega = 0$: $\int G(\Omega) d\Omega > 0$ [1]-[6]. This amplification mechanism can only work if the laser-induced angular spread α of electrons in the first wiggler is essentially larger than the natural electron beam spread α_{beam}.

The interaction of electron beam with laser field can be described by laws of conservation for momentum $\mathbf{p}_e + \mathbf{p}_L = \mathbf{p}'_e + \mathbf{p}'_L$ and energy $\mathcal{E}_e + \mathcal{E}_L = \mathcal{E}'_e + \mathcal{E}'_L$. Here $\mathbf{p}_e$ and $\mathbf{p}'_e$ are initial and final momentums of electrons, $\mathbf{p}_L$ and $\mathbf{p}'_L$ are initial and final momentums of laser field; $\mathcal{E}'_L$ and $\mathcal{E}_L$ are initial and final energies of light beam and $\mathcal{E}'_e$ and $\mathcal{E}_e$ are initial and final energies of electrons. The density of electromagnetic wave momentum is $\mathbf{P}_L = 1/(4\pi c)[\mathbf{EB}] = \mathbf{k}\omega/(4\pi c)\mathbf{A}_L^2$, where $\mathbf{A}_L$ is an amplitude of a vector-potential of laser field. We can write for $\mathbf{A}'_L = \mathbf{A}_L \exp(k'' L)$, where k'' is a spatial growth rate of laser field in a medium of an electron beam; L is a length of interaction. From law of conservation we can expect that $|\triangle \mathbf{p}| = |\mathbf{p}'_e - \mathbf{p}_e| = |\mathbf{p}'_L - \mathbf{p}_L| = \mathbf{A}_L^2[\exp(2k''L) - 1]$. We can see that the change of electron momentum $|\triangle \mathbf{p}|$ depends on the spatial growth rate k'': with the growth rate k'' rising, the change of electron momentum rises too. This means that for noncolinear interaction the deviation of electron from its original direction depends on both the spatial growth rate k'' and the amplitude A_L of laser field. The growth rate k'' is a function on electron beam current; and the amplitude depends on laser power. Therefore, the condition $\alpha > \alpha_{beam}$ leads to the threshold of either the laser power or the electron beam density.

We consider the induced radiation by a mono-energetic beam of electrons propagating in a wiggler. We assume that the static magnetic field of a plane undulator $\mathbf{A}_\mathrm{w}$ is independent on the transverse coordinates x and y. Also we approximate the static magnetic field by a harmonic function $\mathbf{A}_\mathrm{w} = A_\mathrm{w}\mathbf{e}_y = (A_0 e^{-i\mathbf{k}_\mathrm{w}\mathbf{r}} + \mathrm{c.c.})\mathbf{e}_\mathrm{y}$, where $\mathbf{k}_\mathrm{w} = (0, 0, k_\mathrm{w})$ is the wiggler wave vector; "c.c." denotes the complex conjugation, $\mathbf{e}_y$ is the unit vector along y axis. The wiggler field causes an electron to oscillate along the y-axis. For this reason, the electron interacts most efficiently with a light wave if the latter is linearly polarized. We assume that the vector potential of the laser wave has a linear polarization $\mathbf{A}_L = A_L(t, x, z)\mathbf{e}_y = \mathbf{a}_+ e^{i(\mathbf{k}-\mathbf{k}_\mathrm{w})\mathbf{r} - i\omega t}$.

The early theoretical constructions assume infinite electron and laser beams. In reality both electron and laser beams are restricted in the transverse directions. This means that the non-collinear arrangement of electron and laser beams leads to the finite area of them interaction (fig. 1). The length of laser amplification in the medium of electron beam is $L_L = 2r_b/\sin(\alpha + \theta)$, where $2r_b$ is a width of the electron beam in the xz-plane (fig. 1). The length, at which the electrons move acting by force of laser field, is equal to $L_e = 2r_L/\sin(\alpha + \theta)$, where $2r_L$ is a width of the laser beam in the xz-plane.

The solution of the linearized equations for slow motion

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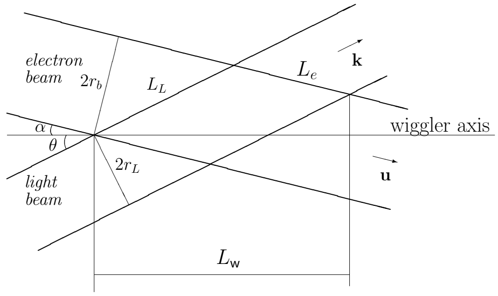

Figure 1: The scheme in xz-plane of undulator with non-collinear arrangement.

of the electron in the xz-plane is [7]:

$$\delta \mathbf{v}_{\|} = K^2 \frac{c^2}{\gamma_0^3} \sum_{j=1}^{4} \frac{\beta_1 \mathbf{k}_j - \frac{\omega}{c^2}\beta_2 \mathbf{u}}{D_{b(j)}} a_j e^{i\xi_0 - i\Delta_{\omega(j)} t} + \text{c.c.} \tag{1}$$

Here K is the undulator strength parameter, defined as normalized dimensionless vector-potential of the undulator magnetic field $K = e/(mc^2)|A_0|$. The total relativistic factor of electrons γ_0 is defined as $\gamma_0 = \sqrt{1 + 2K^2}(1 - u^2/c^2)^{-1/2}$, where the initial velocity of electrons is $\mathbf{u} = (-u \sin\alpha; 0; u\cos\alpha)$. The dispersion relation for the wiggler with electron beam determines 4 branches $k_\nu = k_\nu(\omega)$ of oscillations, namely, two beams and two laser waves [7]. Here $a_j = a_{+(j)}/A_0$ is the dimensionless amplitude of wave j at the entrance of the first undulator and $\Delta_{\omega(j)} = \omega - (\mathbf{k}_j \mathbf{u})$ is the detuning, $D_b = (\omega - \mathbf{k}\mathbf{u})^2 - \Omega_b^2$ is the dispersion function of electron beam wave associated with the beam frequency Ω_b, where $\Omega_b^2 = \omega_b^2 \gamma_0^{-1}\left[1 - (\mathbf{k}\mathbf{u})^2/k^2c^2\right]$. Here $\omega_b^2 = 4\pi e^2 n_b/m$ is square of the Langmuir frequency of the electron beam. $\xi_0 = \mathbf{k}_0 \mathbf{r}_{\|0}$, where $\mathbf{r}_{\|0}$ is the initial coordinate in the XZ plane. The coefficients are $\beta_1 = \gamma_0(\omega - (\mathbf{k}_0\mathbf{u})) - \omega_b^2(\mathbf{k}_0\mathbf{u})/k_0^2c^2$ and $\beta_2 = \gamma_0(\omega - (\mathbf{k}_0\mathbf{u})) - \omega_b^2/\omega$.

Under the condition of appreciable amplification $k''L \geq 1$, where k'' is the growth rate of instability in undulator, we can omit all waves in Eq.(1) except the amplified one. Assuming that the electron, incoming in the laser field at $t = 0$ and interacting during the time $t = L_e/u$, deviates from the initial direction by the angle $\Delta\alpha$, which depends on the initial phase as $\cos\xi_0$, we obtain the maximal value of this angle deviation:

$$\Delta\alpha_{max} \simeq K^2 c^2 \frac{\beta_1}{D_b \gamma_0^3} \frac{k_0}{u} a \sin(\alpha + \theta)[e^{k'' L_e} - 1]. \tag{2}$$

Here a is initial amplitude of a laser wave at the entrance of the first wiggler, because our approach implies that there is an exterior laser, wave of which is amplified with electron beam in the wiggler.

For single-electron approximation (Thompson regime), Eq. 2 reduces to

$$\Delta\alpha_{max} \simeq K^2 \left(\frac{c}{u}\right)^2 \frac{k_0}{\gamma_0^2} \frac{\Omega_b}{k'' u} a \sin(\alpha + \theta) \frac{e^{k'' L_e} - 1}{k''}. \tag{3}$$

Here k'' is given by formula [9]:

$$k'' = \frac{\sqrt{3}}{2} \left(\frac{K^2}{2}\right)^{1/3} k_0 \left[\frac{\Omega_b \omega}{\gamma_0 (\mathbf{k}_0 \mathbf{u})^2} \left(1 + \frac{\omega_b^2}{\omega \Omega_b \gamma_0}\right)\right]^{2/3}. \tag{4}$$

For the Thompson regime, $\Omega_b/k''u \ll 1$, and $\Omega_b/(k''u) \sim \omega_b^{1/3} \sim \sqrt{k''}$. Hence, the angle deviation of electrons depends on the growth rate as $\Delta\alpha_{max} \sim \sqrt{k''}(e^{k''L_e} - 1)/k''$. As the growth rate diminishes $k'' \to 0$, the angle of deviation $\Delta\alpha$ goes to zero as $\Delta\alpha_{max} \sim \sqrt{k''} \to 0$.

The excess of $\Delta\alpha_{max}$ over the natural dispersion of the beam $\Delta\alpha_{beam}$ gives the threshold value of initial laser amplitude of the exterior laser. We rewrite formula (2) using the overall exterior laser power $P = \frac{c}{4}(k_0 r_L)^2 |a_+|^2$, which needs to use in experiment, namely

$$P > \frac{c}{8} \left(\frac{mc^2}{e}\right)^2 \frac{(\Delta\alpha_{beam})^2 \gamma_0^4}{2K^2 f(k'' L_e)} \left(\frac{k'' u}{\Omega_b}\right)^2. \tag{5}$$

Here $f(x) = \left(\frac{e^x - 1}{x}\right)^2$. For estimation we consider the case of small amplification $k''L_e \sim 1$, when $f(k''L_e) \sim 1$. We assume that $\Omega_b/k''u = 0.3$. Using the following values of parameters [6]: $\gamma_0 = 15$, $K = 0.635$ and $\Delta\alpha_{beam} = 5 \cdot 10^{-4}\ rad$, we obtain value of threshold: $P > P_{th} \simeq 10^8$ W. This power exceed the saturation power of the laser field for which the nonlinear regime occurs.

This means, that the regime with $k''L_e \gg 1$ or $f(k''L_e) \gg 1$ should be used of to realize the FELWI application. It can be anomalous Thompson or Raman regime of amplification. For collective (Raman) regime the maximal angle of deviation is

$$\Delta\alpha_{max} \simeq \frac{1}{2} K^2 \left(\frac{c}{u}\right)^2 \frac{k_0}{\gamma_0^2} X a \sin(\alpha + \theta) \frac{e^{k'' L_e} - 1}{k''}, \tag{6}$$

where $X = 1 + \omega_b^2(\mathbf{k}_0\mathbf{u})/(k_0 c \Omega_b \gamma_0)$. Here k'' is given by formula [9]:

$$k'' = \frac{K}{2} \frac{k_0}{\gamma_0} \frac{\omega \sqrt{\Omega_b}}{(\mathbf{k}_0 \mathbf{u})^{3/2}} \left(1 + \frac{\omega_b^2}{\omega \Omega_b \gamma_0}\right). \tag{7}$$

and, therefore, k'' takes the large value. The threshold power of laser is

$$P_{th} = \frac{c}{4} \left(\frac{mc^2}{e}\right)^2 \frac{(\Delta\alpha_{beam})^2 \gamma_0^4}{K^2 f(k'' L_e)}. \tag{8}$$

In paper [6] the tolerance of the FELWI gain to the electron beam energy spread has been demonstrated. For this spread $\delta\gamma$ has been taken to be extremely large, namely, $\delta\gamma = 2.0$ while the emittance was $\epsilon = 2\pi \times 10^{-6}$m rad. Simulations have been performed to obtain the dependence of the FELWI gain on the electron beam current. The results show that the gain is about 2 orders of magnitude larger than that for ordinary FEL. The simulation have been carried out with the following set of realistic electron beam and wiggler parameters that are sufficiently close to experimental situations [8]: electron energy $E = 29.35\ MeV$

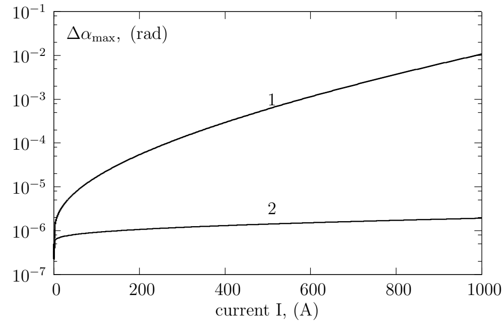

Figure 2: The deviated angle $\Delta\alpha_{max}$ as a function of a beam current I for two values of the laser beam width: line 1 corresponds $r_L = 1.0\ cm$ and line 2 corresponds $r_L = 0.1\ cm$. Other parameters are: electron energy $\gamma = 15$, rms electron beam radius $r_b = 70\ \mu m$, laser wavelength $\lambda_L = 359\ \mu m$, period of the wiggler magnets $\lambda_W = 2.73\ cm$, normalized wiggler field $K = 0.635$, angle between laser and electron beam $\alpha + \theta = 0.13\ rad$.

($\gamma = 15$), emittance up to $\epsilon = 2\pi \times 10^{-6}$ m rad, rms beam radius $r_b = 70\ \mu m$, laser wavelength $\lambda_L = 359\ \mu m$, period of the wiggler magnets $\lambda_W = 2.73\ cm$, number of magnets per section $N = 32$, normalized wiggler field $K = 0.635$, angle between laser and electron beam $\alpha + \theta = 0.13\ rad$.

For our calculations we choose the same parameters and assume that the power of an exterior laser is $P = 100\ W$. The results are presented in Fig. 2, which shows the angle deviation $\Delta\alpha_{max}$ as a function of a beam current I for two values of the laser beam width: line 1 corresponds $r_L = 1.0\ cm$ and line 2 corresponds $r_L = 0.1\ cm$.

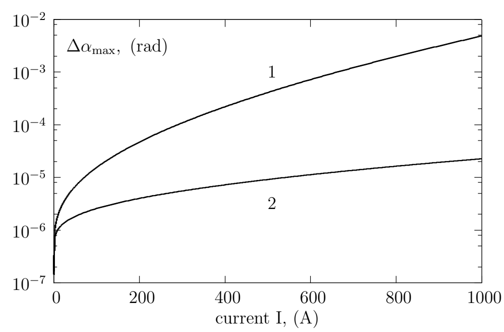

Figure 3: The deviated angle $\Delta\alpha_{max}$ as a function of a beam current I for two values of the angle between laser and electron beams: line 1 corresponds $\alpha + \theta = 0.05\ rad$ and line 2 corresponds $\alpha+\theta = 0.13\ rad$. Other parameters are: electron energy $\gamma = 15$, rms electron beam radius $r_b = 0.02\ cm$, laser wavelength $\lambda_L = 359\ \mu m$, period of the wiggler magnets $\lambda_W = 2.73\ cm$, normalized wiggler field $K = 0.635$, $r_L = 1.0\ cm$

Under the condition $I > 10\ A$, Raman amplification takes place. The dependence of the angle deviation $\Delta\alpha_{max}$ on the laser beam width has simple explanation. On the one hand, with increasing width r_L the laser amplitude drops, under the condition $P = const$. But on the other hand, the length of interaction L_e increases proportional to the width r_L. Hence the exponential term $\exp(k'' L_e)$ grows.

The length of interaction can be changed with angle $\alpha + \theta$ between the electron and the laser beams. Fig. 3 presents results for different values of angle $\alpha + \theta$: line 1 corresponds $\alpha + \theta = 0.05\ rad$ and line 1 corresponds $\alpha + \theta = 0.13\ rad$. The widths of the electron and the laser beams are $r_b = 0.02\ cm$ and $r_L = 1.0\ cm$, respectively. One can see that geometrical parameters, such as the widths of the electron r_b and the laser r_L beams, the angle between the directions of propagation of the electron and the laser beams, allow us to choose an optimal scheme for FELWI operation.

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