

# HIGHER ORDER MODE PROPERTIES OF SUPERCONDUCTING PARALLEL-BAR CAVITIES\*

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## Abstract

The superconducting parallel-bar cavity [1] has properties that make it attractive as a deflecting or crabbing rf structure. For example it is under consideration as an rf separator for the Jefferson Lab 12 GeV upgrade and as a crabbing structure for a possible LHC luminosity upgrade. Initial cavity shape optimization has been performed to obtain a high transverse deflecting voltage with low surface fields. We present here a study of the Higher Order Modes (HOM) properties of this structure. Frequencies,  $R/Q$  and field profiles of HOMs have been evaluated and are reported.

## INTRODUCTION

The superconducting parallel-bar cavity has many advantages over conventional superconducting deflecting and crabbing structures. The optimized design [2] is proven to have low surface fields that give higher transverse gradient and higher shunt impedance. As the length and height are in the order of  $\lambda/2$ , the compactness of the design supports very low frequencies. Unlike other deflecting and crabbing cavities the main contribution of the deflection is from the transverse electric field generated between the two parallel bars. Currently the superconducting parallel-bar cavity shown in Fig. 1 (a) is being considered for the Jefferson Lab 12 GeV upgrade and the design (b) is considered for the proposed LHC luminosity upgrade. Other deflecting/crabbing cavity concepts are presented in [3]. The cavity properties of the two designs are shown in Table 1.

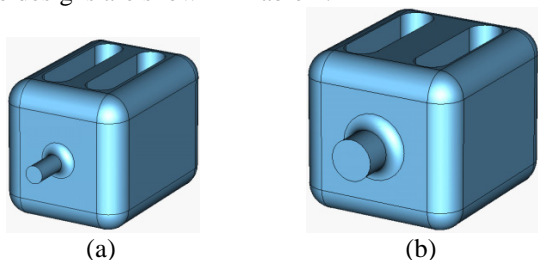


Figure 1: Optimized (a) 499 MHz (b) 400 MHz parallel-bar structures.

The excitation of HOMs can lead to beam instabilities further leading to beam losses. Accurate analysis of HOM properties is critical in identifying these harmful modes

present in the cavity and to damp them appropriately by means of HOM couplers. The HOMs are identified by various cavity parameters such as frequency and  $[R/Q]$ . A detail analysis of HOM properties for the 499 MHz and 400 MHz cavity designs are presented.

Table 1: Properties of Parallel-Bar Structures

Parameter	499 MHz	400 MHz	KEK Cavity <sup>[4]</sup>	Units
Freq. of $\pi$ mode	499.2	400.7	501.7	MHz
$\lambda/2$ of $\pi$ mode	300.4	374.7	299.8	mm
Freq. of 0 mode	517.8	413.05	~ 700.0	MHz
Cavity length	394.4	456.7	299.8	mm
Cavity width	290.0	400.0	866.0	mm
Cavity height	304.8	384.4	483.0	mm
Bars width	67.0	85.0	-	mm
Bars length	284.0	332.0	-	mm
Aperture diameter	40.0	100.0	130.0	mm
Deflecting voltage ( $V_T^*$ )	0.3	0.375	0.3	MV
Peak electric field ( $E_P^*$ )	1.85	2.18	4.32	MV/m
Peak magnetic field ( $B_P^*$ )	6.69	7.5	12.45	mT
Energy content ( $U^*$ )	0.031	0.176	-	J
Geometrical factor	67.96	83.9	220	$\Omega$
$[R/Q]_T$	933.98	317.92	46.7	$\Omega$
$R_T R_S$	$6.3 \times 10^4$	$2.67 \times 10^4$	$1.03 \times 10^4$	$\Omega^2$

At  $E_T^* = 1$  MV/m

## HOM PROPERTIES

There are many parasitic modes other than the fundamental operating mode present in any resonant cavity varying over a wide frequency range, with different field orientations and can be grouped as Lower Order Modes (LOMs), Similar Order Modes (SOMs) or HOMs. For a beam passing through the cavity, one or more of these modes gets activated due to the interaction with the charged particles, generating wake fields that act upon the beam in return. The beam-induced power depends on the intensity and the natural decay time of each mode when any other coupler is not present. The intensity of each mode is determined by the longitudinal or transverse  $[R/Q]$  and the natural decay time is given by  $\tau = Q_{0,n}/\omega_n$  where  $\omega_n$  is frequency of each mode and  $Q_{0,n}$  is its intrinsic quality factor.

The modes excited by the particles may affect other particles in the same bunch or particles in bunches trailing behind, depending on the decay time of the wake fields. The resultant shorter wake fields mostly act on the same bunch leading to single bunch effects, which depend more on the  $[R/Q]$  as the decay times are relatively small. The multiple bunch effects have larger decay times and act on the trailing bunches as well.

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The longitudinal  $[R/Q]$  for modes with an electric field along the beam axis is given by

$$\left[ \frac{R}{Q} \right] = \frac{\left| \int_{-\infty}^{+\infty} \vec{E}_z(z, x=0) e^{\frac{j\omega z}{c}} dz \right|^2}{\omega U} \quad (1)$$

The  $[R/Q]$  for modes with transverse electric field is calculated using two methods. Firstly using the direct integration method given by,

$$\left[ \frac{R}{Q} \right]_T = \frac{\left| \int_{-\infty}^{+\infty} \left[ \vec{E}_T(z, x=0) + j(\vec{v} \times \vec{B}(z, x=0))_T \right] e^{\frac{j\omega z}{c}} dz \right|^2}{\omega U} \quad (2)$$

and using the Panofsky Wenzel Theorem [5] given by

$$\left[ \frac{R}{Q} \right]_T = \frac{\left| \int_{-\infty}^{+\infty} \vec{E}_z(z, x=x_0) e^{\frac{j\omega z}{c}} dz \right|^2}{(kx_0)^2 \omega U} \quad (3)$$

where  $k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$  and  $x_0$  is the offset from the beam axis.

Since the  $[R/Q]$  and  $Q_{0,n}$  of each mode are properties of the cavity, damping of the modes are achieved by additional couplers added to the design and the new decay time is given by  $\tau = Q_{L,n}/\omega_n$  where  $Q_{L,n}$  is the loaded quality factor. The resultant longitudinal and transverse wake impedances are given by,

$$Z_Z = \left[ \frac{R}{Q} \right] Q_{L,n} \quad (4)$$

$$Z_T = \frac{\omega}{c} \left[ \frac{R}{Q} \right] Q_{L,n} \quad (5)$$

The longitudinal impedances are generated by the particles moving along the beam axis due to longitudinal electric field, where the resultant transverse impedances are generated by the particles moving off axis. Therefore it is important to determine the  $[R/Q]$ ,  $Q_L$  and wake field impedances for each HOM present in the cavity to identify the modes that requires damping.

## HOM ANALYSIS

The  $[R/Q]$  values are determined for the 499 MHz and 400 MHz parallel-bar cavity designs using CST Microwave Studio. Since the cavity structure doesn't have the circular symmetry, it is not practical to categorize the HOMs as monopole, dipole etc. as in a conventional cavity with circular symmetry. The HOMs are identified by the field orientation in the cavity along the beam axis ( $z$  axis), and are grouped in to 4 main types of modes as shown in Table 2. The deflecting mode with the field orientation of  $E_x, H_y$  gives the resultant deflection in  $x$  direction. The other field orientation of  $E_y, H_x$  gives a net deflection in  $y$  direction. The longitudinal magnetic field present does not contribute to any HOMs.

Table 2: Types of HOMs

Field on Beam Axis	Type of Mode
$E_x, H_y$	Deflecting
$E_z$	Accelerating
$E_y, H_x$	Deflecting
$H_z$	Does not couple to the beam

For the 499 MHz design all the modes up to 2.5 GHz are calculated which is above the beam aperture cutoff frequency of 2.2 GHz for the  $TE_{11}$  mode with a beam aperture radius of 40 mm. For the 400 MHz the all the modes up to 2.0 GHz are calculated. The upper limit in frequency is way above 1.14 GHz, the cut off frequency in  $TE_{11}$  mode for a beam aperture of radius 100 mm. The corresponding  $[R/Q]$  values are shown in Fig. 2. It clearly shows that for the parallel-bar cavity geometry the LOMs do not exist. The longitudinal  $[R/Q]$  is determined using Eq. (1) and the transverse  $[R/Q]$  is determined using Eq. (2) and (3) considering an off set of  $x_0 = 5$  mm off from the beam axis. The transverse  $[R/Q]$  calculated using direct integration method and Panofsky Wenzel Method are in agreement within  $<1\%$ . The calculated longitudinal  $[R/Q]$  values for all the deflecting modes are insignificant.

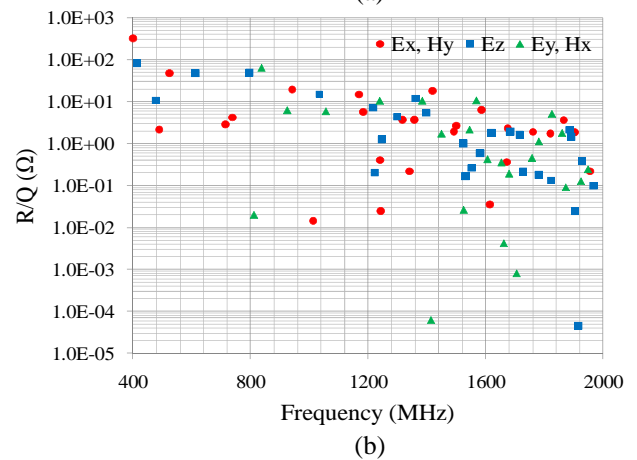
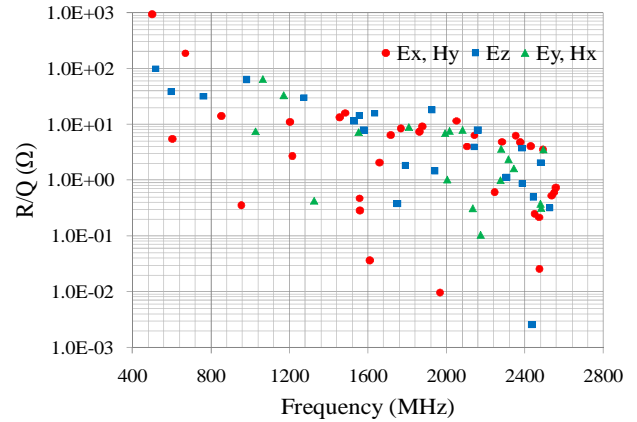


Figure 2:  $[R/Q]$  for (a) 499 MHz and (b) 400 MHz parallel-bar cavities.

The fundamental deflecting mode in the 499 MHz design has the highest shunt impedance of 934  $\Omega$  and 318  $\Omega$  in the 400 MHz design compared to all other modes for

both designs. Most of the HOMs in the higher frequency range have lower  $[R/Q]$  values compared to the fundamental mode  $[R/Q]$ . The lower  $[R/Q]$  values contribute to low wake impedances given that the  $Q_L$  is low.

### Analysis of Field Profiles

The beam-induced voltage for the 499 MHz parallel-bar cavity will be relatively minimal due to the low average beam current of 100 mA. However the effects of HOMs are significant in the 400 MHz parallel-bar cavity proposed for the LHC luminosity upgrade, due to the high beam current ( $\sim 2A$ ). The modes with highest  $[R/Q]$ s identified for the 400 MHz are shown in Table 3.

Table 3: Modes with highest  $[R/Q]$

Mode Type	Frequency (MHz)	$[R/Q]$ ( $\Omega$ )
$E_x, H_y$	400.7	318.0
$E_z$	412.9	81.1
$E_y, H_x$	836.7	65.0

The parallel-bar cavity geometry has two fundamental modes as 0 (accelerating) mode and  $\pi$  (deflecting) mode that are degenerate. Inclusion of beam aperture and rounding of edges on top and bottom surfaces eliminates the degeneracy, in which the deflecting mode has the lowest frequency [2]. However the rounded edges introduce a vertical magnetic field along the beam axis, perpendicular to the horizontal electric field between the two parallel bars. The electric and magnetic field orientation of the fundamental deflecting mode is shown in Fig. 3. The resultant deflection from the magnetic field is  $V_r^* = 0.033$  MV as to the deflection of  $V_r^* = 0.408$  MV from the electric field ( $E_r = 1.0$  MV/m). The relative contribution from the magnetic field is small compared to that from the electric field.

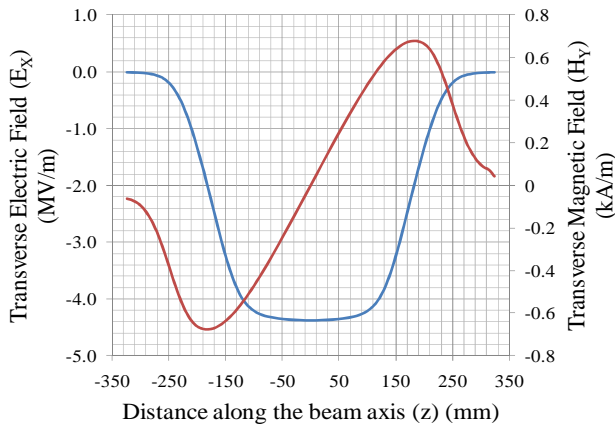


Figure 3: Transverse electric (blue) and magnetic (red) fields along the beam axis for the fundamental deflecting mode in 400 MHz parallel-bar cavity.

The field orientation of the accelerating mode for the 400 MHz design is shown in Fig. 4 that has the next highest  $[R/Q]$ . The vertical deflecting mode with field orientation  $E_y, H_x$  for the same design is shown in Fig. 5 that also has a relatively higher  $[R/Q]$ .

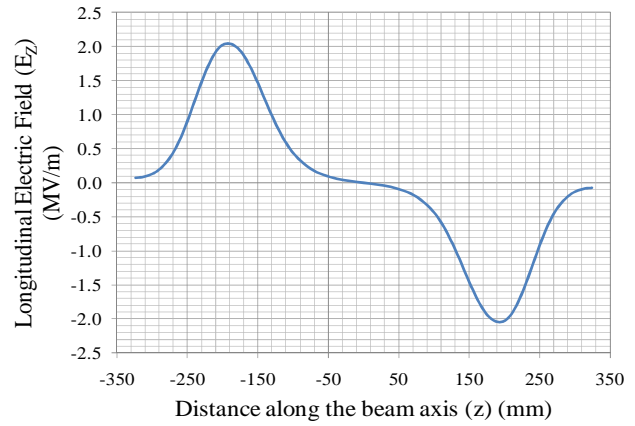


Figure 4: Longitudinal electric field along the beam axis for the fundamental accelerating mode.

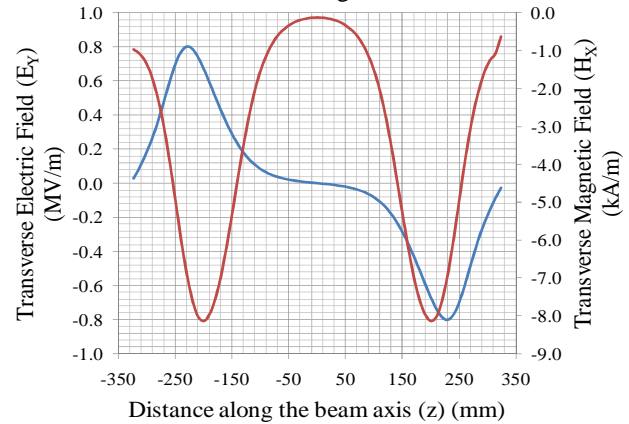


Figure 5: Transverse electric (blue) and magnetic (red) fields along the beam axis for the vertical deflecting mode.

## CONCLUSION

A detailed study of the HOM properties for the parallel-bar geometry with frequencies 499 and 400 MHz is presented in terms of frequency,  $[R/Q]$  and field profiles. One of the advantages of the geometry is the non existence of any LOMs, which only requires higher mode filters in terms of damping.

A beam dynamics study is required to determine the threshold impedances for both designs.  $Q_L$  and wake impedances can be determined with proper HOM couplers placed on the cavity structure. Furthermore damping of HOMs by reducing  $Q_L$  is under way.

## REFERENCES

- [1] J.R. Delayen and H. Wang, Phys. Rev. ST Accel. Beams 12, 062002 (2009).
- [2] S.U. De Silva and J.R. Delayen, Proceedings of SRF'09, Berlin, Germany, p. 589 (2009).
- [3] Z. Li *et al.*, Compact 400-MHz Half-wave Spoke Resonator Crab Cavity for the LHC Upgrade, These proceedings.
- [4] K. Hosoyama *et al.*, Proc. of the 7th Workshop on RF Superconductivity, p.547 (1998)
- [5] W.K.H. Panofsky and W.A. Wenzel, Rev. Sci. Instrum. 27, 967 (1956).