STUDY OF MULTIPACTING IN A COAXIAL COUPLER*

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Abstract

The performance of superconducting cavity, couplers and ceramic windows is greatly affected due to multipacting. The present paper describes the multipacting simulations carried out on the co-axial coupler. The equation of motion of electron in RF field is calculated numerically. The enhanced counter function (ECF) is calculated to find out whether a particular electron will give rise to the multipacting. The simulation was carried out for a co-axial coupler having the inner conductor diameter of ϕ34.78 mm and outer conductor diameter of ϕ80 mm at a RF frequency of 350MHz, 700MHz and 1050MHz.

INTRODUCTION

Accelerator and Pulse Power Division (APPD), Bhabha Atomic Research Center (BARC) is involved in the development of superconducting cavities [1, 2, 3]. The performance of superconducting cavity, couplers and ceramic windows is greatly affected due to multipacting. Beyond a certain threshold, cavity fields cannot be increased with increase in input power as incident power is absorbed by multipacting electrons. Evidence of multipacting has been observed experimentally in many RF devices. Thus it is essential to study the multipacting phenomena at various power levels and RF frequencies.

In practice, two types of multipacting phenomena are observed, viz., 'one-point' multipacing and 'two-point' multipacting. A RF power is fed to the superconducting cavity with the help of coaxial type of input coupler. The present paper deals with the multipacting analysis of the coaxial coupler. The study of multipacting is an involved process because of the complex geometries of the RF devices, time varying nature of the electromagnetic fields and susceptibility of the phenomena to the surface conditions. Thus the analysis is done using numerical methods.

THEORY

The study of multipacting essentially involves the tracing of electron trajectories in a electromagnetic field and calculation of generated secondary electrons upon the impact of primary electrons with the physical boundary of the RF structure.

The co-axial coupler operates in a transverse electromagnetic (TEM) mode. It has only two non-zero components viz., radial electric field (E_r) and azimuthal magnetic field (H_ϕ). Consider a coaxial coupler having inner conductor radius 'a' and outer conductor radius 'b'with characteristics impedance, 50Ω. If 'P' is the power flowing through the coupler then peak electric field, E_0 and peak magnetic field, H_0 can be written in terms of voltage and current.

In practice coaxial coupler is used to feed power to the load e.g. cavity. Thus it has finite length and due to the mismatch between the load impedance and characteristics impedance, a reflected wave exists. Thus electric and magnetic field inside a coupler at any position, (r,z) and time, t is a vector sum of incident and reflected wave. The standing waves are generated and electric/magnetic nodes and antinodes are generated along the length of the coupler. The amount of impedance mismatch is represented by voltage standing wave ratio (VSWR). In general E(r,t) and H(r,t) can be written as,

$$E\left(\vec{r},t\right)= E_0 \sin\left(\omega t - kz\right)\left(\frac{2\,vswr}{1+vswr}\right) \quad (1)$$

$$H\left(\vec{r},t\right)= H_0 \cos\left(\omega t - kz\right)\left(\frac{2\,vswr}{1+vswr}\right) \quad (2)$$

The general equation of motion of electron is given by,

$$\ddot{r} = \left(e/m\right)\left(\vec{E} + \vec{v}\times\vec{B}\right) \quad (3)$$

Code Development

Since the structure is azimuthally symmetric, the problem is solved in r-z plane (2-dimensional geometry).The programs are developed in 'C'language. Upon resolving equation (3), the resulting radial and axial components are the coupled equations that are solved numerically using Euler's method for small time steps, Δt. The initial conditions like the position, energy and the phase of the electron at time t=0 are provided by user. The new velocity and the position of the electron after time 'Δt' is calculated. The calculations are continued till the electron hits the physical boundary of the RF device (i.e. r=a or r=b in this case). When it hits the wall the final energy, position and time is recorded. The secondary emission coefficient, δ is calculated if the energy of the electron lies between certain energy range provided by the user. The δ is given by, [4]

$$\delta = \delta_m \left[f\, Exp\left(1-f\right)\right]^a \quad (4)$$

where, $f = E/E_m$

'E' is the emitted electron energy and 'E_m' is the energy at which $\delta\sim\delta_m$. The parameter 'a' has value 0.62 for f<1

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and 0.25 for f ≥1. The maximum secondary emission coefficient, δ_m is 1.2 and 1.3 for niobium and copper respectively [5].

The energy and the direction of secondary electrons is assigned using Monte Carlo method. It is assumed that secondary electrons follow Maxwell Distribution [6]. The normalized probability distribution function is given by,

$$f(E) = \left(2/\sqrt{\pi}\right)\left(2\overline{E}\right)^{-3/2}\sqrt{E}\,\exp\left(\frac{-E}{2\overline{E}}\right) \qquad (5)$$

The maximum of the probability distribution curve, occurs at $E = \overline{E}$ .The energy is assigned to the secondary electron using the rejection method. Two series of uniform random numbers lying between 0 and 1 are generated using Park and Miller algorithm [7]. Let μ_1 and μ_2 be the two random numbers. The energy is given by,

$$E = \mu_1 E_{max} \qquad (6)$$

A *f(E)* is calculated using equation (5). Using second random number $f'(E) = \mu_2 f_{max}$ is calculated. If *f'(E)* < *f(E)* then random number, μ_1 is accepted and corresponding energy is assigned to the secondary electron. Otherwise the number is rejected and a next pair of random number is generated.

It is assumed that the electrons are generated isotropically about a surface normal. Thus the angle made by an emitted secondary electron with the surface normal is proportional to $cos^{-1}\eta$, where η is the random number.

The resonant trajectories are tracked for maximum 20 impacts. A Enhanced Counter Function, ECF is used to find out whether a particular electron will give rise to the multipacting. ECF is the total number of secondary electrons generated after 20 impacts.

The total number of electrons generated due to single electron after 20 impacts is,

$$N_i = \prod_{k=1}^{20} \delta_k \qquad (7)$$

$$ECF = \sum_{i=1}^{N_0} N_i \qquad (8)$$

The program calculates the ratio, ECF/N_0 after 20 impacts, where N_0 is the total number of primary electrons that undergo 20 impacts. If the ratio, $ECF/N_0 > 1$ at a given power level then it is a multipacting level. The order of multipacting depends on the time interval between two successive impacts.

RESULTS AND DISCUSSION

The measured secondary electron yield consists of the true secondary, rediffused primary and the elastically scattered electrons. The experiments show that true secondary electron yield is ~58.9 % for primary electron energy between 0 to 40 eV and follows Maxwell like distribution with peak at 2.9 eV [8,9]. Thus to generate secondary electrons with Monte Carlo method, an average electron energy of 2.9eV and maximum electron energy of 40 eV is assumed.

The simulations were carried out for two coaxial couplers, viz. Coupler 1 (ID: ϕ34.78 mm, OD: ϕ80 mm) and Coupler2 (ID: ϕ17.38 mm, OD: ϕ40 mm) at a RF frequency of 350MHz, 700MHz and 1050MHz under matched condition (VSWR = 1) and under full power reflection (VSWR=10000). Large number of primary electrons (=730) with various initial positions, energy and phases were generated. Fig. 1 shows the one point, 3rd order multipacting trajectory in Coupler 2 at a RF frequency of 1050 MHz of a primary electron started at phase of 0.957 rad. Fig. 2 shows the same trajectory of secondary electrons between successive impacts as a function of time which clearly indicates that multipacting order is 3. Fig. 3 shows the variation of (ECF/N_0) with RF power for Coupler 1 under various load or VSWR conditions at a frequency of 700 MHz. At higher VSWR, multipacting is likely to occur at lower power level that is evident from the results. Fig. 4 shows the multipacting bands at a frequency of 1050 MHz for Coupler 1 and Coupler 2, initial electron position is chosen at the electric field maximum. Fig. 5 shows the multipacting bands at various frequencies for Coupler 1, no multipacting was observed at frequency of 350 MHz at these power levels. From the Fig. 4 and fig. 5, it is clear that multipacting power level scales with the frequency and the coupler dimensions. Fig. 6 shows the multipacting bands for the Coupler 1, at a frequency of 1050 MHz with initial primary electron position, Z0 chosen at electric and magnetic field maximum.

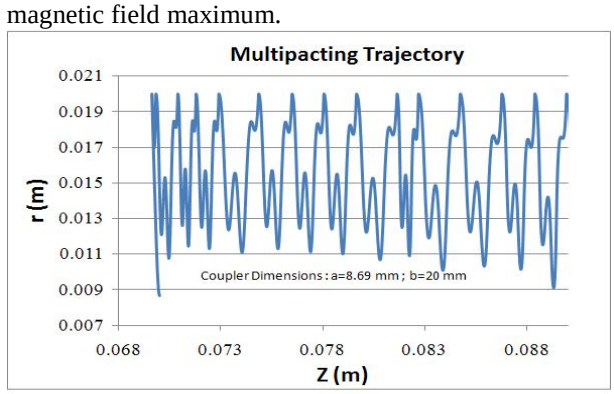

Figure 1: One Point, 3rd order trajectory.

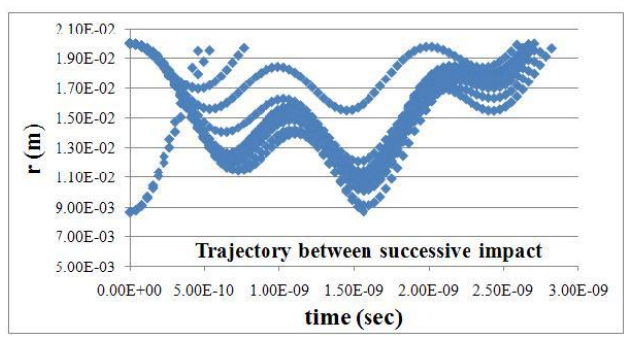

Figure 2: Trajectory as a function of time.

CONCLUSION

In conclusion, the results are in agreement with the scaling law [10]. Thus, multipacting zones at various power levels can be calculated using scaling law. For other coupler components like door-knob transition, ceramic window 3-d analysis is necessary.

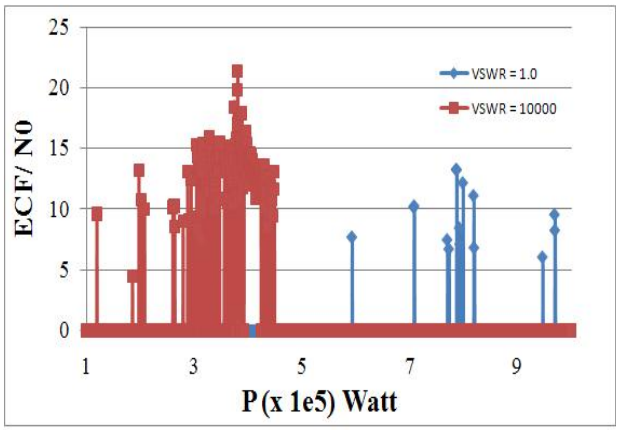

Figure 3: Multipacting bands for different loads.

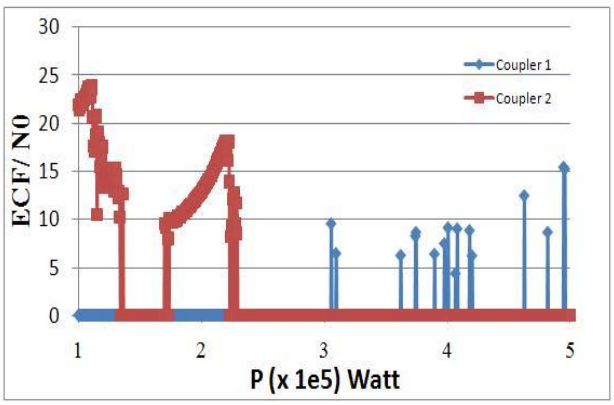

Figure 4: Multipacting bands for different coupler size.

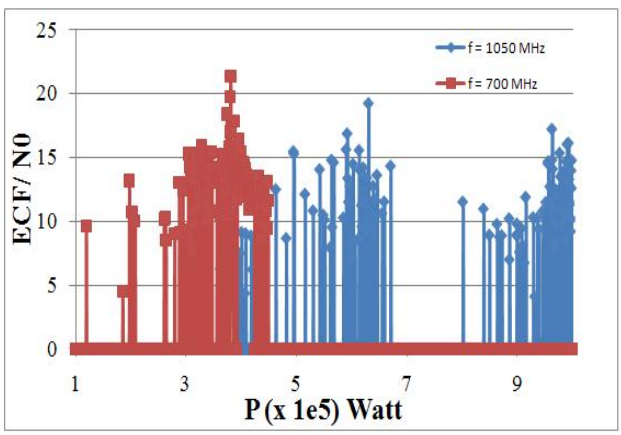

Figure 5: Multipacting band for various frequencies.

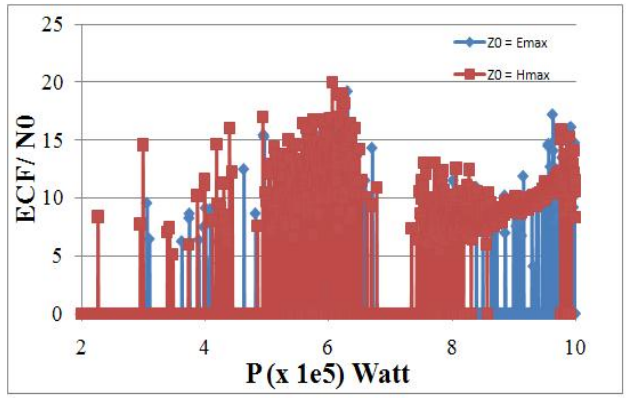

Figure 6: Multipacting band for different intial position.

REFERENCES

[1] A.Roy, A.S.Dhavale, J.Mondal, K.C.Mittal, "RF Design of a Single Cell Superconducting Elliptical Cavity with Input Coupler", 12th Workshop on RF Superconductivity, Cornell University, USA (2005)

[2] A.Roy, J. Mondal, K.C.Mittal, "Superconducting Elliptical 1050 MHz β=0.49 Cavity Design for High Current Proton Accelerator", InPAC'06, Mumbai, India, November 1-4, (2006)

[3] A.S.Dhavale and K.C.Mittal,"Evaluation of external Q of the superconducting cavity using Kroll-Yu method", Review Of Scientific Instruments 77, 066101 (2006)

[4] J.R.M.Vaughan, "A New Formula for Secondary Emission Yield", Transactions on Electron Devices, Vol. 36, No. 9, p 1963-1967(1989)

[5] H.Padamsee, J.Knobloch, T. Hays, "RF Superconductivity for Accelerators", edited by John Wiely & Sons, p. 183(1998)

[6] S. Humphries, "Modeling Secondary Emission in a Finite – element Multipactor code", 9th Workshop on RF Superconductivity, Los Almos, California, USA, November 1-5(1999)

[7] W.H. Press, S.A. Teukolsky et. al., "Numerical receipes in C", Second Edition, Cambridge University Press, p. 278-279(1992)

[8] R.E.Kirby, F.K.King, "Secondary Electron Emission Yields from PEP II Accelerator Materials", SLAC-PUB-8212, October (2000)

[9] R.E.Kirby, F.K.King, "Secondary Electron Emission from Accelerator Materials", SLAC-PUB-8380, February(2000)

[10]E.Somersalo, P.Yla-Oijala, "Analysis of Multipacting in Coaxial lines", PAC'95, Dallas, USA(1995)