BEAM-INDUCED ELECTRON LOADING EFFECTS IN HIGH PRESSURE CAVITIES FOR A MUON COLLIDER*

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Abstract

Ionization cooling is a critical building block for the realization of a muon collider. To suppress breakdown in the presence of the external magnetic field, an idea of using an RF cavity filled with high pressure hydrogen gas is being considered for the cooling channel design. One possible problem expected in the high pressure RF cavity is, however, the dissipation of significant RF power through the beam-induced electrons accumulated inside the cavity. To characterize this detrimental loading effect, we develop a simplified model that relates the electron density evolution and the observed pickup voltage signal in the cavity, with consideration of several key molecular processes such as the formation of the polyatomic molecules, recombination and attachment. This model is expected to be compared with the actual beam test of the cavity in the MuCool Test Area (MTA) of Fermilab.

INTRODUCTION

High Pressure RF (HPRF) cavity can be a very effective solution for the development of a compact muon ionization cooling system, such as a Helical Cooling Channel (HCC). Initial experiments of the HPRF cavity in the absence of the beam have demonstrated that higher field gradients are achievable compared with the case of a conventional evacuated cavity. When the beam is propagating through the HPRF cavity, however, there is some concern in relation to the beam-induced electrons. If the beam-induced electrons are not removed properly, theory expects that a significant amount RF power is dissipated by Ohmic heating, and the cavity is heavily loaded. A detailed theoretical understanding and a precise measurement of such beam loading effects are critical in evaluating the feasibility of the HPRF cavity concept for a future muon collider.

EQUIVALENT CIRCUIT MODEL

The sources of the loading effects in the HPRF cavity can be divided into two parts: the beam-induced image charges in the cavity surface, and the beam-induced free electrons generated inside the cavity. Representing the effect of the image charges by a current generator ($-I_b$) and the effect of the accumulated electrons by an additional shut resistance (R'), we obtain an approximate equivalent circuit

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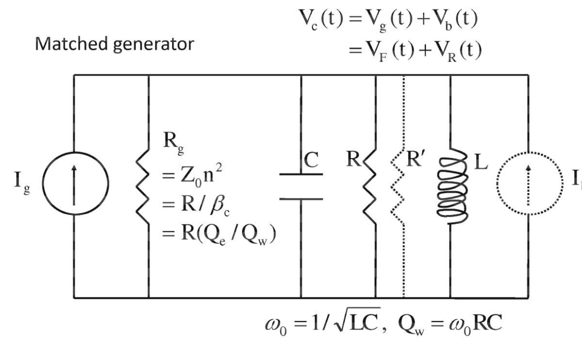

Figure 1: Equivalent circuit for the beam-loaded cavity transformed into the resonator circuit [2].

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equation (see Fig. 1) as follows [1]:

$$\left\{\frac{d^2}{dt^2} + \omega_0\left(\frac{1}{Q_L} + \Delta\left[\frac{1}{Q}\right]\right)\frac{d}{dt} + \omega_0^2\right\} V_c = 2\frac{\omega_0}{Q_e}\frac{dV_F}{dt} - \frac{\omega_0}{2}\left[\frac{R}{Q}\right]\frac{dI_b}{dt}, \quad (1)$$

where $\omega_0 = 1/\sqrt{LC}$ is the resonant frequency of the cavity, $Q_L(Q_e)$ is the loaded (external) quality factor of the cavity, and $V_c = V_F + V_R$ is the cavity voltage which is the sum of the forward and reverse voltages. Here, $\Delta[1/Q]$ denotes the change in the quality factor due to Ohmic dissipation, and $[R/Q]$ is the beam-coupling parameter. Negative sign in front of the beam current I_b implies that the induced voltage from the beam current will decelerate the beam. Let's take the each term to be varying at roughly the driving frequency ω and express it in terms of phasor according to $V = \mathrm{Re}\left[\tilde{V}(t)e^{j\omega t}\right]$. Here, we allow for some slow variation in the envelopes. In the slowly-varying envelope approximation, $|d\tilde{V}/dt| \ll |\omega\tilde{V}|$, the cavity responds to the generator and beam currents according to

$$\frac{d\tilde{V}_c}{d\tau} + (1 - j\tan\psi + \gamma)\tilde{V}_c = \frac{1}{2}Q_L\left[\frac{R}{Q}\right]\left(\tilde{I}_g - \tilde{I}_b\right). \quad (2)$$

Here, $\tau = t/T_f$ is time measured in units of filling time $T_f = 2Q_L/\omega_0$, and $\gamma = Q_L\Delta[1/Q]$ is a damping coefficient. The difference in driving frequency from resonant frequency is characterized by tuning angle ψ, which is given by ' $\tan\psi = Q_L(\omega_0/\omega - \omega/\omega_0)$. In most operating condition of the HRPF cavity ($I_b \approx I_{\mathrm{DC}} \approx 32$ mA), it is expected that the loading effects from the image charges are small compared with the Ohmic dissipation through the beam-induced electrons. Therefore, in the rest of this paper, we will focus only on the beam-induced electron loading effects.

ELECTRON DENSITY EVOLUTION

During propagation through the hydrogen gas, the incident beam generates electrons by beam-impact ionization (mostly, $\mathrm{p} + \mathrm{H}_2 \to \mathrm{H}_2^+ + \mathrm{e}^-$). The electrons ejected from the primary ionization also produce some secondary electron/ion pairs. The resultant increase in the electron density after one microbunch ($N_b \approx 10^9$ protons/bunch) passes through the cavity can be conveniently expressed by [3, 4]

$$\Delta n_e \approx \frac{1}{\pi r_b^2} \frac{\rho dE/dx}{W_i} \times N_b, \tag{3}$$

where r_b is the radius of a uniform density beam, ρ is the mass density of hydrogen gas at room temperature T_{room}, dE/dx is the stopping power, and $W_i(= 35~\mathrm{eV})$ is the effective average energy to produce an electron/ion pair. Most electrons are thermalized quickly by elastic and inelastic collisions with the background hydrogen gas, and drift with the applied RF field until annihilated through recombination, attachment, or some diffusion processes. At the very high pressure considered in this study, however, diffusion is indeed negligible. A simplified rate equation for the electron density evolution in thermal equilibrium can be written as

$$\frac{dn_e}{dt} = S - \beta_r(T_e) n_e^2 - k_a(T_e, T_g) n_e n_g. \tag{4}$$

Here, $S \approx \Delta n_e / T_b$ is the average source term, where T_b is the bunch spacing which is a sub-harmonic of the fundamental frequency. Furthermore, β_r is the *total* rate coefficient for various dissociative recombination (DR) processes, including

$$\begin{aligned} \mathrm{e}^- + \mathrm{H}_2^+ &\to \mathrm{H} + \mathrm{H}, \\ \mathrm{e}^- + \mathrm{H}_3^+ &\to \mathrm{H} + \mathrm{H} + \mathrm{H}, \text{ or } \mathrm{H}_2 + \mathrm{H}. \end{aligned}$$

Due to the complicated reaction chains involving various polyatomic molecules (H_3^+, H_5^+, H_7^+, etc.) under an unusually high hydrogen pressure, a precise theoretical estimation of β_r is somewhat difficult so far. On the other hand, k_a represents the rate coefficient for the dissociative attachment (DA) to the background neutral hydrogen gas, $\mathrm{e}^- + \mathrm{H}_2 \to \mathrm{H} + \mathrm{H}^-$. Note that k_a depends not only on the electron temperature T_e, but also on the background gas temperature T_g. In particular, it is well-known that k_a increases dramatically with the increase of the vibrational energy of H_2. At room temperature, however, only the lowest vibrational level is indeed populated. In Eq. (4), we assume overall charge neutrality, i.e., $n_e \approx n_{\mathrm{H}_2^+} + n_{\mathrm{H}_3^+} + n_{\mathrm{H}_5^+} + \cdots$. We also note that $n_e \ll n_g = p/k_B T_{\mathrm{room}}$.

PERTURBATION FROM ELECTRONS

For the nominal operating conditions of the HPRF cavity ($f = \omega/2\pi \approx 805$ MHz), ions can be assumed immobile. On the other hand, electrons not only interact with the external RF field, but also suffer collisions with the background gas molecules. The total electron-neutral collision frequency for momentum transfer is approximately given by $\nu_m \approx 2.3 \times 10^{11} p[\mathrm{psi}]$ for $4 < E_0/p\,[\mathrm{V/cm/torr}] < 30$ and $T_{\mathrm{room}} = 300$ K [4], and the response of plasma electrons to the external RF field is described by the complex conductivity:

$$\sigma(w) = \sigma_{DC} \left[\frac{\nu_m^2}{\nu_m^2 + \omega^2} - j \frac{\omega \nu_m}{\nu_m^2 + \omega^2} \right], \tag{5}$$

where $\sigma_{DC} = n_e e^2 / m_e \nu_m$ is the DC conductivity. When the cavity is (partially) filled with electrons of the complex conductivity (5), the imaginary part causes the upshift in the resonance frequency and the real part results in the decrease of the Q value. They are given by Slater's perturbation equations [4] as

$$\Delta f_0 \approx \frac{f}{2} \left(\frac{\omega}{\nu_m} \right) \Delta \left[\frac{1}{Q} \right], \tag{6}$$

$$\Delta \left[\frac{1}{Q} \right] \approx \frac{\int_V \frac{1}{2} \sigma_{DC} E_0^2(r,z) dV}{\omega \int_V \frac{1}{2} \epsilon_0 E_0^2(r,z) dV} = \left(\frac{\omega}{\nu_m} \right) \frac{\langle n_e \rangle}{n_c}, \tag{7}$$

where $\langle n_e \rangle = \int_V n_e E_0^2 dV / \int_V E_0^2 dV$ is the average electron density weighted over initial spatial electric field distribution $E_0(r,z)$, and $n_c = \epsilon_0 m_e \omega^2 / e^2$ is the critical density. If the electron swarm consists of a small radius uniform column ($r \approx r_b$), much smaller than the cavity radius r_w, then we can further approximate $\langle n_e \rangle \approx n_e (\xi r_b^2 / r_w^2)$, where the geometric factor ξ indicates that the effects of the electron accumulation is significant where E_0 is large. For example, $\xi = 1$ when the electric fields are uniform across the cavity, whereas $\xi \approx 10$ for the actual HPRF cavity (calculated from the SUPERFISH code), in which the electric fields are highly concentrated near the cavity center where most electrons are expected to be accumulated.

CHANGES IN THE PICKUP SIGNAL

Assuming no image current ($\tilde{I}_b = 0$), negligible resonance frequency shift ($\tan\psi = 0$ and $\Delta f_0 / f_0 \ll 1$), and small temperature change ($T_e \approx const.$), Eq. (2) is further simplified as

$$\frac{d\hat{V}_c}{d\tau} + \left[1 + \gamma(\tau)\right] \hat{V}_c = 1, \tag{8}$$

with a proper normalization [$\hat{V}_c(\infty) \to 1$, for $\gamma \to 0$]. Note that γ is directly proportional to the evolution of the electron density, and thus we can rewrite it as $\gamma = \mu n_e(t)$.

In Figs. 2-5, we plot the changes in the pickup signal during the beam-cavity interaction for several different operation scenarios. Here, the baseline conditions are beam pulse length = 20 μs, beam intensity = 10^9 protons/bunch, $\beta_r \sim 10^{-8}~\mathrm{cm}^3/\mathrm{s}$, $k_a \sim 0~\mathrm{cm}^3/\mathrm{s}$, and the gas pressure $p = 1000$ psi. The RF pulse length is 50 μs, and the forward RF power is incident on the cavity 10 μs before the beam's arrival. Figure 2 indicates that the pickup voltage decays parabolically right after the beam is incident at $t = 0$. Since $n_e(t) \sim St$ near $t = 0$, we can fit the parabola

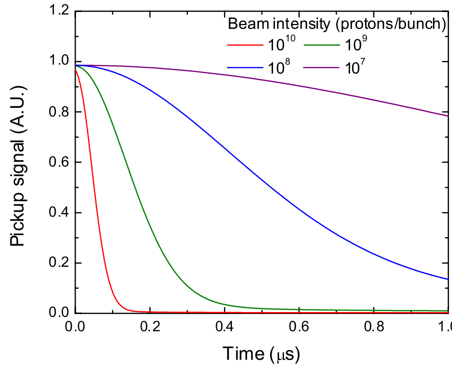

Figure 2: Temporal evolutions of the pickup signal for several different beam intensities. To see the decay of the pickup signal more clearly, we plot from 0 to 1 μs only.

by $\hat{V}_c \approx 1-(\mu S T_f/2)\tau^2$, and get an estimation of S. After the beam passes through, the pickup signal recovers almost linearly as shown in Fig. 3. When the recombination is dominant, the slope is given by $d\hat{V}_c/d\tau \sim \beta_r T_f/\mu$. If the recombination rate is high enough then the recovery of the pickup signal is no longer linear, but rather affected by the filling time (loaded Q) of the cavity [see Fig. 4]. When T_e (or equivalently E_0/p) is sufficiently high, or there is significant population of the vibrationally excited H_2, then the attachment process should be taken into account. In this case, the pickup signal recovers somewhat exponentially ($d\ln\hat{V}_c/d\tau \sim k_a n_g T_f$) as shown in Fig. 5. In Figs. 4 and 5, we note that the pickup signal can have a finite saturation level, $\hat{V}_c \sim 1/(1+\mu n_e(\infty))$, which gives some information on $n_e(\infty)$. For the case with negligible attachment, $n_e(\infty) = \sqrt{S/\beta_r}$, whereas for the case with significant attachment, $n_e(\infty) = S/k_a n_g$.

CONCLUSIONS

Combining the equivalent circuit equation (2), the electron density rate equation (4), and the complex plasma con-

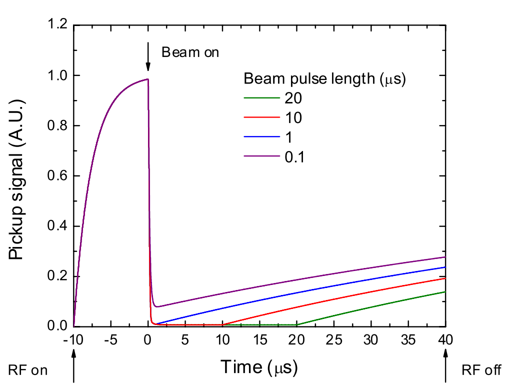

Figure 3: Temporal evolutions of the pickup signal for several different beam pulse lengths.

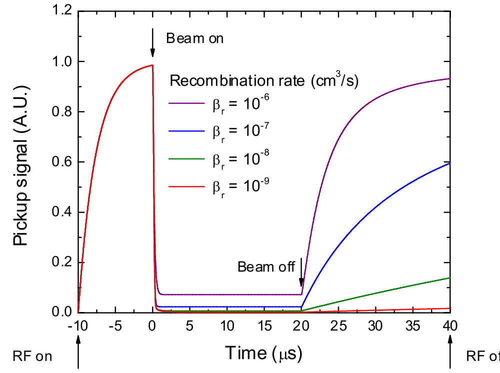

Figure 4: Temporal evolutions of the pickup signal for several different recombination rates.

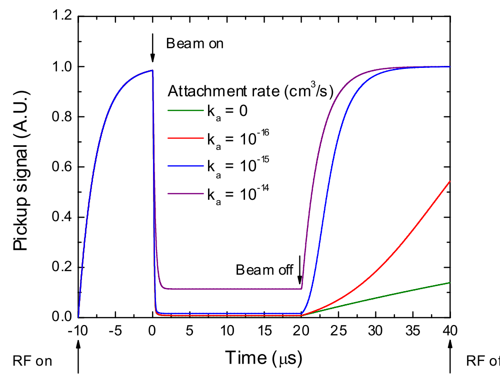

Figure 5: Temporal evolutions of the pickup signal for several different attachment rates.

ductivity (5), we develop a simplified model to estimate the changes in the pickup signal of the HPRF cavity, when an intense ionizing beam is passing through. Some self-consistent effects such as changes in the electron temperature (which in turn affects β_r and k_a), and the detuing of the resonance frequency (which causes a phase shift in the pickup signal) will be included in the future work. The model developed here is expected to give a practical guidance for the actual beam test of the HPRF cavity in the MuCool Test Area (MTA) of Fermilab.

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