NEW MULTICONDUCTOR TRANSMISSION-LINE THEORY AND THE ORIGIN OF ELECTROMAGNETIC NOISE

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Abstract

We study the mechanism of electromagnetic noise in standard two-conductor transmission-line circuit in appearance due to its coupling to the circumstance through grounding and radiation/absorption by using the newly developed multiconductor transmission-line theory. We discuss then how to reduce the electromagnetic noise by taking in the circumstance line as a middle line in the electric circuit and by arranging all the electric loads symmetrically around the middle line.

INTRODUCTION

We start with a statement that usual electric circuits have two transmission-lines in appearance and face to electromagnetic (EM) noise through EM radiation and absorption and through the grounding to the earth. Synchrotron accelerators with power supplies of switching type use high electric power and they are very sensitive to the EM noise. In fact, the use of standard electric circuit provides large EM noise to accelerator magnets. Hence, it was not easy to make stable operation of synchrotron at J-PARC. The main ring (MR) synchrotron accelerator introduced then the Sato-Toki (ST) symmetrization method and were able to reduce largely the noise level for successful synchrotron operation [1].

In order to understand the generation of the electromagnetic noise, we have to consider the influence of the circumstance on electric circuit. Starting from an elementary electro-magnetism, the electric field produced by a charge placed in vacuum is totally changed when the charge is placed near the earth. This is true also for a current going through a transmission line. Hence, the performance of two-line electric circuit should get the influence from the circumstance. In addition, electric circuits are usually grounded to the earth, through which EM noise comes in to the electric circuit. In order to simplify the problem and understand the influence of the circumstance, we consider a three-conductor transmission-line system, where two lines are the main lines and the third line is considered as the circumstance line. The circumstance line is considered to carry the EM noise. We further impose a condition that the total current is zero so that the system is closed [2]. It is also interesting to study the case without this restriction of the total current being zero, which is being studied, and is mentioned briefly towards the end of this paper [3].

We formulate a new three-conductor transmission-line theory by constructing coupled differential equations, where the coefficients of differential equations are obtained explicitly. We utilize similar expressions of electric field and magnetic field due to charge (Coulomb's law) and current (Ampere's law), and determine the coefficients of potential and inductance. With the coupled differential equations, we can show that the common and normal modes decouple from each other when the first and second lines have the same geometrical shape and the same distance to the third line. On the other hand, when the symmetry of the first and second lines is broken, a coupling of the normal and common modes appears. This coupling produces the EM noise of the circumstance in the normal mode. If we consider this third line to be the middle line as used in the HIMAC system, we can confine all the EM noise and energy within the three transmission-line system as shown in the paper [4], which is based on lumped circuit loads and perfect conductors for wiring. We are then able to avoid the EM noise in the normal mode by symmetrization around the middle line.

We shall discuss the construction of a new multiconductor transmission-line theory. We discuss then how we can avoid the EM noise in the normal mode.

MULTICONDUCTOR TRANSMISSION-LINE THEORY

We start writing the standard coupled differential equations for multiconductor transmission-line (MTL) system [5, 6].

$$\frac{\partial V_i(x,t)}{\partial x} = -\sum_j^N L_{ij}\frac{\partial I_j(x,t)}{\partial t} \quad (1)$$

$$\frac{\partial I_i(x,t)}{\partial x} = -\sum_j^N C_{ij}\frac{\partial V_j(x,t)}{\partial t}$$

These equations for currents $I_i(x,t)$ and potentials $V_i(x,t)$ for each transmission line $i = 1, 2, ..N$ at position and time x, t are derived from the Maxwell equation for the TEM mode. The coefficients of capacity C_{ij} and the coefficients of inductance L_{ij} are defined as

$$Q_i(x,t) = \sum_{j=1}^N C_{ij}V_j(x,t) \quad (2)$$

$$A_i(x,t) = \sum_{j=1}^N L_{ij}I_j(x,t)$$

Here, the charge $Q_i(x,t)$ is related with the scalar potential $V_i(x,t)$ through the coefficients of capacity. The vector potential $A_i(x,t)$ is related with the current through

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the coefficients of inductance. It should be noted that the coefficients L_{ij} are written by the geometries of the transmission-lines directly but the coefficients C_{ij} are determined by an indirect manner.

The first important step toward a new MTL theory is to reverse the relation for the charge and the scalar potential by using P_{ij} for $[C^{-1}]_{ij}$ as

$$V_i(x,t) = \sum_{j=1}^{N} P_{ij} Q_j(x,t) \tag{3}$$

Here, P_{ij} are called the coefficients of potential. To be specific, we consider the case of three parallel conductor transmission-lines (parallel cables) and explicitly work out the behavior of this coupled system. We write electric potentials, V_1, V_2 and V_3, and the corresponding currents, I_1, I_2 and I_3, for the three lines. To be concrete in our discussion, we impose the condition that the sum of the three currents is zero, $I_1(x,t) + I_2(x,t) + I_3(x,t) = 0$. This relation for the currents ensures that the three transmission-lines form a closed system.

We have introduced the coefficients of potential P_{ij} as the inverse of the C-matrix, $P = C^{-1}$. We can then write the above equation in terms of the coefficients of potential as

$$\frac{\partial V_i(x,t)}{\partial t} = -\sum_{j}^{N} P_{ij} \frac{\partial I_j(x,t)}{\partial x} , \tag{4}$$

with the condition that the sum of the three currents zero. The next important step is to introduce the concept of the normal and common modes.

$$\begin{aligned} V_n &= V_1 - V_2 \\ V_c &= \frac{1}{2}(V_1 + V_2) - V_3 \\ I_n &= \frac{1}{2}(I_1 - I_2) \\ I_c &= I_1 + I_2 \end{aligned} \tag{5}$$

Here, those quantities with the suffix n are for the normal-mode and the other with the suffix c are for the common-mode. We then obtain the relation $I_3 = -I_c$ owing to the imposed condition. We note here that normal-mode quantities are usually used for electric circuit. However, the common-mode quantities, which carry EM noise, have not been discussed clearly in the literature.

We obtain then the coupled differential equations for the normal and common modes as

$$\begin{aligned} \frac{\partial V_n}{\partial t} &= -P_n \frac{\partial I_n}{\partial x} - P_{nc} \frac{\partial I_c}{\partial x} \\ \frac{\partial V_c}{\partial t} &= -P_{cn} \frac{\partial I_n}{\partial x} - P_c \frac{\partial I_c}{\partial x} . \end{aligned} \tag{6}$$

The coefficients P_n, P_c and P_{nc} are written as

$$\begin{aligned} P_n &= P_{11} + P_{22} - 2P_{12} \\ P_c &= \frac{1}{4}(P_{11} + P_{22} + 2P_{12} - 4P_{13} - 4P_{23} + 4P_{33}) \\ P_{nc} &= P_{cn} = \frac{1}{2}(P_{11} - P_{22}) - P_{13} + P_{23} . \end{aligned} \tag{7}$$

We can work out the other differential equations for the normal mode and the common mode.

$$\begin{aligned} \frac{\partial V_n}{\partial x} &= -L_n \frac{\partial I_n}{\partial t} - L_{nc} \frac{\partial I_c}{\partial t} \\ \frac{\partial V_c}{\partial x} &= -L_{cn} \frac{\partial I_n}{\partial t} - L_c \frac{\partial I_c}{\partial t} \end{aligned} \tag{8}$$

We have expressions for the L coefficients similar to the P coefficients, which are omitted to write here [2]. In general, therefore, the normal and common modes are coupled. This means that the normal mode is influenced by EM noise through the coupling to the common mode, where the EM noise is present.

We want to understand first the most symmetric case in which the coupling terms are zero: $P_{nc} = 0$ and $L_{nc} = 0$. This condition is fulfilled when the sizes and shapes of lines 1 and 2 are the same ($P_{11} = P_{22} : L_{11} = L_{22}$), and the relations of lines 1 and 2 to 3 are the same ($P_{13} = P_{23} : L_{13} = L_{23}$). In this case, the normal and common modes are completely decoupled. We have then differential equations for the normal mode as

$$\begin{aligned} \frac{\partial V_n}{\partial t} &= -P_n \frac{\partial I_n}{\partial x} \\ \frac{\partial V_n}{\partial x} &= -L_n \frac{\partial I_n}{\partial t} . \end{aligned} \tag{9}$$

This expression with the coefficients P_n and L_n corresponds to the one written in textbook [5, 6]. We can eliminate I_n and obtain the conventional wave equation for V_n as

$$\frac{\partial^2 V_n}{\partial x^2} = \frac{L_n}{P_n} \frac{\partial^2 V_n}{\partial t^2} = L_n C_n \frac{\partial^2 V_n}{\partial t^2} , \tag{10}$$

where we have defined the capacity for the normal mode as $C_n = \frac{1}{P_n}$. Hence, the capacity for the normal mode is written as the inverse of the special sum of the coefficients of potential. We can work out these coefficients and the results are $\frac{L_n}{P_n} = L_n C_n = \frac{1}{c^2}$, where c is the velocity of light. This means that the normal-mode propagates through the transmission-lines at the speed of light. The common mode also propagates at the speed of light, where the coefficients P_c and L_c are determined newly [2].

We can work out the coefficients of potential and inductance in the similar manner in the quasi-stationary approximation using the Neumann's formula [2]. It is important to note that the coefficients of potential can be calculated in exactly the same method as the coefficients of inductance. We write only the results.

$$\begin{aligned} L_{ij} &= \frac{\mu}{2\pi}(\ln \frac{2l}{\tilde{a}_{ij}} - 1) , \\ P_{ij} &= \frac{1}{2\pi\varepsilon}(\ln \frac{2l}{\tilde{a}_{ij}} - 1) . \end{aligned} \tag{11}$$

We use here the geometrical mean distance (GMD) as $\tilde{a}_{ij}$, where we are able to consider the surface skin effect of alternating current.

$$\ln \tilde{a}_{ij} = \frac{1}{S_i S_j} \int \int \ln |\vec{s}_i - \vec{s}_j| ds_i ds_j \tag{12}$$

Here, l is related with the length of the transmission-line. The large S_i is the cross section of the transmission line i and the vector $\vec{s}_i$ is the vector in the cross section area.

We calculate now all the physical coefficients of inductance and potential for the three conductor lines. We write only L_n to save space [2].

$$L_n = L_{11} + L_{22} - 2L_{12} = \frac{\mu}{2\pi} \ln \frac{\tilde{b}^2}{\tilde{a}_1 \tilde{a}_2} \qquad (13)$$

We see that the coefficient of inductance for the normal mode excludes the length of the conductor transmission-lines suited for transmission-line theory. We mention further that all the coefficients of potential and inductance have the proportional relation $P_{ij}/L_{ij} = c^2$. We obtain the coefficients of characteristic impedance through

$$Z_{ij} = \sqrt{P_{ij} L_{ij}}\ , \qquad (14)$$

for all the coefficients of inductance and potential so that the characteristic impedance of the i-th line itself is determined by Z_{ii}. We have the same relation for the normal and common mode coefficients as $Z_n = \sqrt{P_n L_n}$. We can write coupled differential equations for the normal and common modes in terms of the characteristic impedances. We can solve completely the coupled equations with any boundary condition. We can calculate how signals and noise in the normal mode transfer to the common mode and vise-versa. We skip writing all the details and ask readers to refer our paper [2].

SATO-TOKI SYMMETRIZATION AROUND THE THIRD LINE

The common mode in the three-conductor transmission-line system provides a current going through the third line, $I_3 = -I_c$. If this third line represents the circumstance, where the EM noise is present, the coupling of the normal mode with the common mode indicates that the EM noise is unavoidable for the performance of electric circuit. We can construct two main transmission-lines as we wish, but we cannot control the circumstance. Hence, we cannot fulfill the symmetric condition to decouple the normal and common modes in the usual arrangement. Hence, we take the third line (circumstance line) within the electric circuit where the third line is directly connected to the middle point of the power supplies by our will as shown in Fig.1.

If we further consider that the inserted line is the middle line of the electric circuit of the symmetrically arranged circuit as the HIMAC system, we can control electromagnetic field and hence can suppress electromagnetic noise in the normal mode [4]. The concept of the three-line electric circuit with the Sato-Toki symmetrization around the middle line is shown in Fig.1.

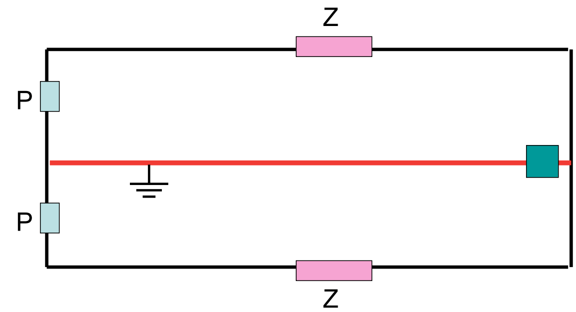

Figure 1: Three-line electric circuit with the arrangement of power supplies and electric loads arranged symmetrically around the middle line.

SUMMARY AND ANTENNA MODE

We have discussed the performance of symmetrically arranged circuit with the third line used as the central line. The central line is then used for the symmetrization to reduce the EM noise in the normal mode by decoupling from the common mode. This symmetrization of the transmission-lines are necessary in addition to the symmetrization of power supplies and loads based on the assumption of perfect conductors for wiring [4]. We may remove the condition of having the condition that the sum of all the currents is zero. In this case of $I_1 + I_2 + I_3 \neq 0$ there appears antenna mode. We are then able to describe the coupling of the normal, common and antenna modes. The extension of the present model to include the antenna mode requires two important ingredients. One is the introduction of the retardation terms in the relation of scalar and vector potentials to charge and current. The other is the introduction of the resistance term for the flow of current through transmission-lines [3]. It is interesting to point out that the symmetry construction of the three transmission-lines provide the condition that the normal mode decouples from the common and antenna modes simultaneously. On the other hand, the common mode never decouples from the antenna mode.

We should conclude this paper by saying that the condition of the normal mode without EM noise is to make symmetrization around the third line (middle line) to decouple the normal mode from the common mode. We should introduce then all the electric loads symmetrically around the third line and the electric loads should use only the normal mode [4].

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