SPECTRUM PROPERTY ANALYSIS OF A WIGGLER-LIKE UNDULATOR

S. D. Chen, T.M. Uen, NCTU, Hsinchu, Taiwan
C. S. Hwang, NSRRC, Hsinchu, Taiwan

Abstract

A wiggler with the property of low total radiation power and that maintains a large photon flux in the hard x-ray region, 5-20 keV, which is necessary for the special demand of users, was investigated to decrease the difficulty of the design of optical components in the beam line and to decrease the load of RF cavity power. Such an insertion devise is called a wiggler-like undulator. The spectrum of this wiggler-like undulater was investigated with a code of which the algorithm is based on a compromising between useful photon flux and total radiation power of insertion devices for spectrum optimization. The properties of the spectrum of the wiggler-like undulator are discussed herein. Furthermore, the power distribution is somehow also discussed.

INTRODUCTION

Synchrotron light in the hard x-ray region, 5-20 keV, is required by particular users. Most users prefer a continuous spectrum, for which purpose one within 5-20 keV, in the middle-energy storage ring, is generated with a wiggler magnet that is an insertion devise (ID) with a strength parameter much greater than unity. The accompanying disadvantage of use of an ID with a large strength parameter is a large total radiation power, which enhances the difficulty of designing the optical device for the beam line and causes a load on the RF cavity power. To solve these two problems, we investigated obtaining a large photon flux within the hard x-ray region with a small total radiation power.

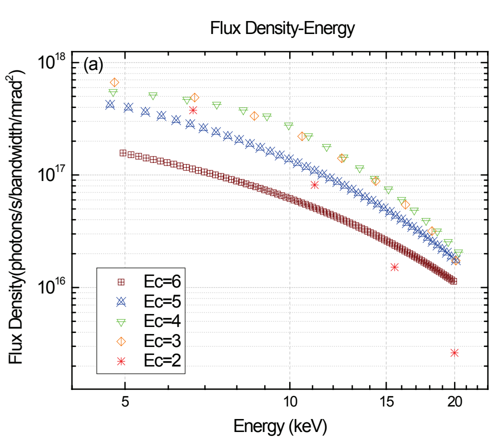
An algorithm for optimization has been developed [1]. The central magnetic field of ID should be set appropriately to a small value to obtain a small total radiation power, and a wiggler-like undulator was considered. Decreasing the central magnetic field causes the photon flux somehow to be sacrificed slightly, and the energy spectrum tends to become discontinuous. For a wiggler-like undulator, a continuous spectrum can be generated in two ways: one involves altering the strength parameter slightly, and another involves tapering the undualtor. Based on a developed auto-optimization program code [1], other spectrum properties are described here, including calculation of the flux density, the spectrum from adjusting the strength parameter, and the spectrum of a tapered undulator. An issue about the power distribution was also discussed.

CALCULATION OF THE FLUX DENSITY

The spectrum of wiggler-like undulator is expected to be understood. An initial question is which formula to calculate the spectrum is suitable – that for a wiggler or an undulator. After study of the derivation and properties of some formulas, this formula for the flux density of an undulator [2] was chosen for a stander calculation,

$$\frac{dN_{\rm ph}(\omega)}{d\Omega} = \alpha\gamma^2 {N_p}^2 \frac{\Delta\omega}{\omega}\frac{I}{e}\sum_{i=1}^{\infty} i^2 \mathrm{Sinc}({F_\sigma}^2 + {F_\pi}^2) \quad (1)$$

which includes information about the interference pattern of the σ- and π-modes and also the interference result of the SINC-function on frequency space. The formulae for the wiggler, which are pole numbers multiplied by the spectrum of the bending magnet, exclude the interference



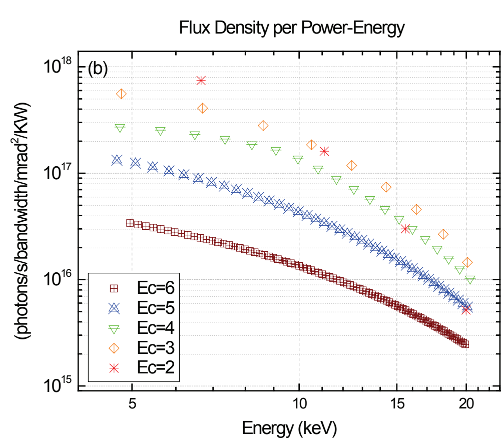


Figure 1: Maxima of harmonics within 5-20 keV are drawn with points to indicate the maximal intensity of each harmonic. Five cases of critical energy -- 2, 3, 4, 5 and 6 keV -- are shown. (a) flux density, (b) flux density per total radiation power.

Table 1: Parameters for five cases. The numbers of harmonics within 5-20 keV, from first to last, are shown.

critical energy /keV	harmonics # (first/last)	B/T	period /mm	K	total power /KW	shift K	Δ gap /mm
2	3/9	0.33	28	0.875	0.507	-	-
3	5/21	0.51	36	1.73	1.19	0.5	4.08
4	11/47	0.67	43	2.69	2.02	0.29	1.44
5	25/107	0.84	51	3.99	3.18	0.18	0.64
6	59/237	1.01	60	5.66	4.61	-	-

information, so they are not identified in simulating the spectrum of a wiggler-like undulator. The formula for the brilliance of an undulator includes information about the electron beam, but for simplicity, the beam information is preferably omitted. The formula for the total flux of an undulator [2] is

$$N_{\text{ph}}(\omega_i)|_{\theta=0} = 1.431 \times 10^{17} I \,\text{Np}\, \frac{\Delta\omega}{\omega} \frac{i\,K^2}{1+\frac{1}{2}K^2} \times [J_{\frac{i-1}{2}}(x) - J_{\frac{i+1}{2}}(x)]^2 \quad (2)$$

which excludes information about the spatial distribution, also was not preferred.

The formula for the flux density of an undulator should be modified for a wiggler-like undulator. The harmonic number in the main spectrum region of a wiggler-like undulator is greater than one for the well known undulator, and it is greater from roughly the eleventh to the fiftieth. For these high harmonics, the bandwidth, generally set with 0.1 per cent accuracy, should be simply modified to

$$\frac{\Delta\omega}{\omega} = 0.1\% \left(\frac{1}{i\text{Np}} > 0.1\%\right); \frac{\Delta\omega}{\omega} = \frac{1}{i\text{Np}} \left(\frac{1}{i\text{Np}} < 0.1\%\right) \quad (3)$$

In where i is the number of the harmonic, and N_p is the number of periods.

The modified formula was used in an auto-optimization program code developed for low-power-high-flux-wiggler by Mathematica [1]. Several cases with adding a critical energy step by step were simulated. There spectra appear in Fig. 1; the parameters are compared in table 1. The beam energy is 3 GeV, beam current 0.4 A, gap of ID 15mm, and length of ID 2 meter. When the critical energy is set to a smaller value, the flux density and flux density per power in energy range 5-20 keV increase substantially, but discontinuities appear in the spectra.

DISCONTINUITY OF THE SPECTRUM

As the critical energy is decreased for a smaller total radiation power, the continuous spectrum as wiggler spectrum becomes harmonic peaks as undulator spectrum, but users of a synchrotron lamp prefer a continuous spectrum. Of two ways to solve the problem of a discrete spectrum, one is to alter slightly the strength parameter of the wiggler-like undulator, as in operating the familiar undulator, which can shift the energy of the harmonics. For a hybrid permanent ID structure, altering the strength parameter means adjusting the gap of an ID. The variation of the strength parameter and corresponding gap adjustment are shown in table 1; the spectra with the altered strength parameter appear in Fig. 2. The case with critical energy 3 keV has greater flux density per power. This method gives a continuous spectrum for users who do not require white light.

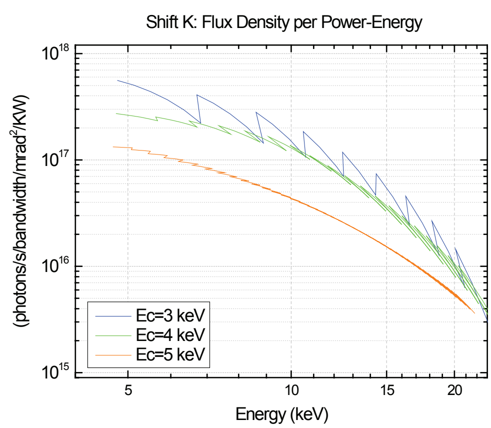


Figure 2: Simulations of flux density per power with shifted K.

Another approach is to produce a continuous spectrum with a tapered undulator. A tapered undulator has multiple central fields, each of which is decreased or increased pole by pole, and is generated on decreasing or increasing the gap of an undulator from head to end. This method increases the r.m.s. widths of harmonic lines, and causes the lines to overlap, so as to generate white light. One case for critical energy equal to 4 keV in Fig. 1 was tapered, and was simulated by B2E associated with Igor Pro [3]. A extent 1.5 mm of gap difference from head to end of the undulator for this case is needed to make the spectrum continuous in the energy range 5-20 keV, but the intensity of the spectrum is sacrificed, yielding about 0.021 times the initial intensity. This method produces a continuous spectrum for users who require white light.

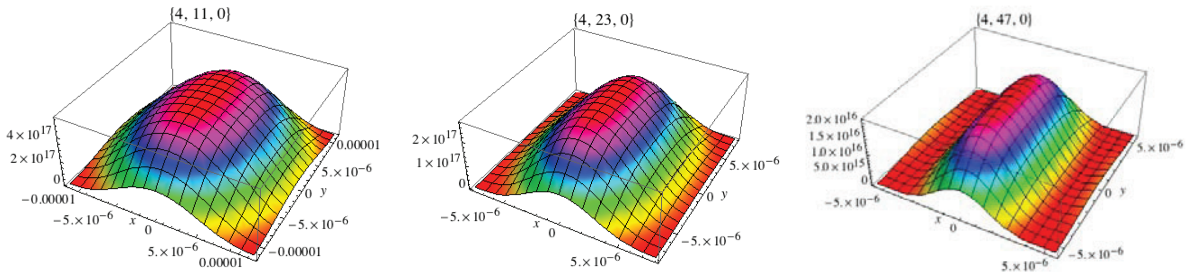


Figure 3: Patterns of the distribution of flux density in transverse directions of the e-beam. Three cases -- 11th, 23rd and 47th harmonics – with E_c= 4 keV in Fig. 1 are shown. The scale of transverse axis has unit rad; The plot range is the opening angle of each harmonic, which varies case by case.

POWER DISTRIBUTION

The power distribution at a specific frequency of a wiggler-like undulator was investigated. One issue about the total flux is remarkable. The total flux (cf. Eq. 2) is derived on multiplying a solid angle by flux density s(cf. Eq. 1)

$$N_{\mathrm{ph}}(\omega)|_{\theta=0} = 2\pi\sigma_{\mathrm{rms}}{}^2 \frac{dN_{\mathrm{ph}}(\omega)}{d\Omega}|_{\theta=0} \quad (4)$$

In which θ=0 means that the observation point is on the axis. The r.m.s. opening angle of undulator radiation is

$$\sigma_{\mathrm{rms}} = \frac{1}{\gamma}\sqrt{\frac{1+\frac{1}{2}K^2}{2iN_p}} \quad (5)$$

in which γ is a relativistic factor, i is the number of harmonic, K is strength parameter, and N_p is number of periods. The case E_c=4 keV in Fig. 1 is taken as an example. Some patterns of the flux density within the region of the r.m.s. opening angle of undulator radiation are simulated in figure 3. The simulation result in figure 3 indicates that a calculation of the total flux of the undulator is questionable for a large harmonic number. The intensity decreases more rapidly than expected as the point of observation moves from the axis. Equation (4) indicates that the intensity of the flux density should be maintained in the range of the r.m.s. opening angle.

CONCLUSION

A wiggler-like undulator was obtained on optimizing an ID with properties of large flux and small total radiation. The bandwidth according to a spectrum formula of an undulator should be modified for use on a wiggler-like undulator. Of two ways to treat the discontinuity of spectrum, one is by slightly altering the strength parameter, which is suitable for users who do not require white light, and another involves using a tapered undulator, which is preferable for users who require white light. The formula for the total flux of an undulator is questionable for use at a large harmonic number.

REFERENCES

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