UNDULATOR HARMONIC FIELD ENHANCEMENT ANALYSIS*

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Abstract

The enhancement of arbitrary odd harmonic field is analyzed for pure permanent magnet undulator. The two dimensional analytical formula of the magnetic field is given for a modified Harbch type undulator, in which the magnet blocks have different size. It is shown that the odd harmonic field can be enhanced by optimizing the length ratio of the vertical magnetized magnet blocks and the horizontal magnetized magnet blocks, the 3rd harmonic field can exceeded 20% of the fundamental field and 7th harmonic field can exceeded 3 % of the fundamental field for magnet gap-period ratio equal to 0.1.

INTRODUCTION

Using the higher harmonic is one of ways for FEL to develop towards the shorter wavelength ranges. [1-3] For a planar undulator, an ideal sinusoidal periodic magnetic field in the direction perpendicular to the direction of the electron motion causes the electrons to oscillate at odd harmonics frequency in the transverse direction, thus cause the odd harmonics radiations on-axis.[4] For actual planar undulators, the magnetic field is non-sinusoidal and with harmonics field components. The harmonic radiation can be enhanced by aptly increase the harmonic field component [5]. Therefore some methods for this purpose were proposed, such as adding high permeability shims inside the undualtor. [6] In this paper, we present a modification for Standard Harbch type undulator, so as to increases the harmonic magnetic fields components.

ANALYSIS

In actual planar undulators, the magnetic field is non-sinusoidal, when expanded in Fourier series the field itself includes odd spatial harmonics due to the symmetry of the magnetic structure. For a pure permanent magnet undulator, the magnetic fields in two dimensional approximate (the width of magnetic pole is assumed to be much large than the magnet gap) is [7]

$$B_y(y,z) = 2B_r \sin c(\frac{\pi}{M})* \sum_{j=0}^{\infty} Ch(nk_u y)*Sin(nk_u z)*\frac{(-1)^j}{n}(1-e^{-nk_u h})e^{-nk_u g/2} \quad (1)$$

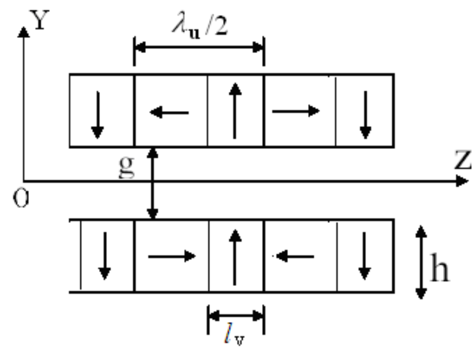

Figure 1: The configuration of modified Harbch type pure permanent magnet undulator.

Where $k_u=2\pi/\lambda_u$, λ_u is the magnetic field period, g is the magnet gap, h is height of magnet blocks, $n=1+j*M$ is the harmonic number, M is the number of magnet blocks in one periods of one side magnetic pole. For a standard Harbch type undulator, four magnet blocks with same size was take in one periods，namely M=4, thus only odd harmonic fields n=1,5,9,13,… components exist from Eq.(1). Now we consider the magnetic field of a modified Harbch type undulator, for which the magnet blocks have different size. We analyze the case that the length of the vertical magnetized magnet blocks (is l_v) is different with that of the horizontal magnetized magnet blocks (is l_H, and $l_v+l_H=\lambda_u/2$, Fig.1). After some lengthy deduction, we obtain the following two dimensional analytical formula for the magnetic field

$$B = \sum_{n=0}^{\infty} B_n Ch(nk_u y) Sin(nk_u z)*Sin\frac{n\pi}{2} \quad (2)$$

$$B_n = \frac{4\sqrt{2}B_r}{n\pi}(1-e^{-nk_u h})e^{-nk_u g/2}\cos(n\pi\frac{l_v}{\lambda_u}-\frac{\pi}{4}) \quad (3)$$

One can see that generally all the odd harmonic field components exist. When $l_v=l_H=\lambda_u/4$, it gives the Standard Harbch type case (M=4 in Eq.(1)). From Eq.(3), the peak magnetic field decrease with the magnet block height-period ratio, and the higher harmonic number is, the slower decrease. (see Fig.2) Though the larger harmonic field fractions can be obtained for the smaller height-period ratio, but the magnetic field intensities become low. To compare with the Standard Harbch type case, we take the magnet block height is as $h=\lambda_u/4$.

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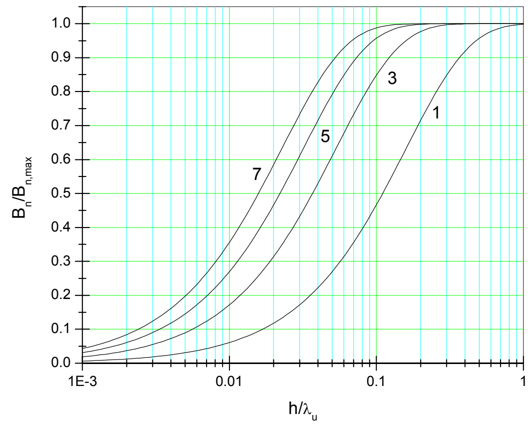

Figure 2: The relative harmonic undulator field with the magnet block height.

The variation of harmonic undulator field B_n along with the length of vertical magnetized magnet block is given in Fig.3. It is shown that as the length of the magnet block changes, the higher harmonic number is, the larger the field varies, and the variation of the fundamental field is least. For a given harmonic number, the corresponding field component can be maximized by optimizing the length ratio of vertical magnetized magnet blocks and horizontal ones. From Eq.(3), we can get the maximum nth odd harmonic field

$$\frac{B_{n,\max}}{B_1} = \frac{(-1)^m (1-e^{-n2\pi h/\lambda_u}) e^{-(n-1)\pi g/\lambda_u}}{n(1-e^{-2\pi h/\lambda_u})\cos[(4m-n+1)\pi/4n]} \quad (4)$$

by taking

$$\frac{l_v}{\lambda_u} = \frac{(4m+1)}{4n} \quad (m < \frac{2n-1}{4}) \;, \quad (5)$$

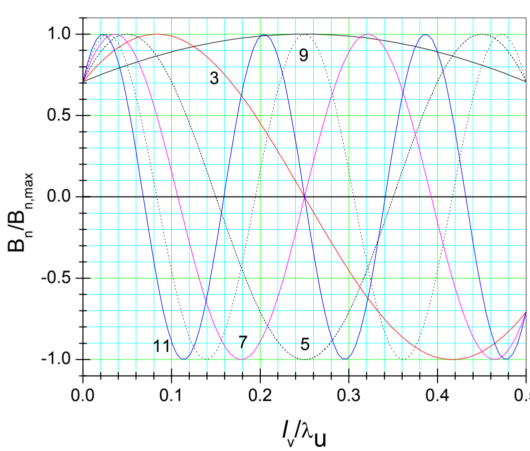

Figure 3: The relative harmonic undulator field with the length of vertical magnetized magnet block.

From Eq.(3)(4), one can see that it has the larger harmonic field fractions for the smaller gap-period ratio. The maximum 3rd and 7th harmonic fields as function of the magnet gap-period ratio are shown in Fig 3 and Fig 4, respectively. For the magnet gap-period ratio g/λ_u=0.1, the 3rd harmonic field over 0.2T can be achieved with l_v/λ_u=5/12, that exceeded 20% of the fundamental field; and the 7th harmonic field can exceed 3 % of the fundamental field with l_v/λ_u=5/28. While the fundamental field decreased about 13.4% and 2.5% respectively.

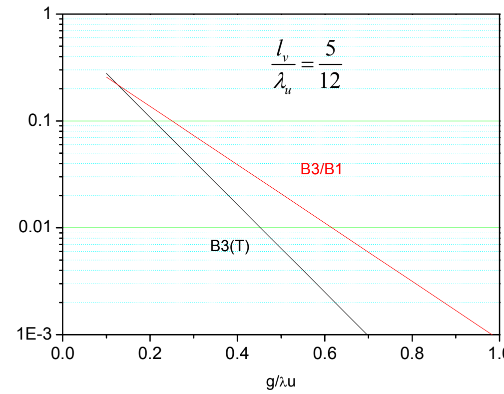

Figure 4: The maximum 3rd harmonic field with the magnet gap-period ratio. ($h=\lambda_u/4$).

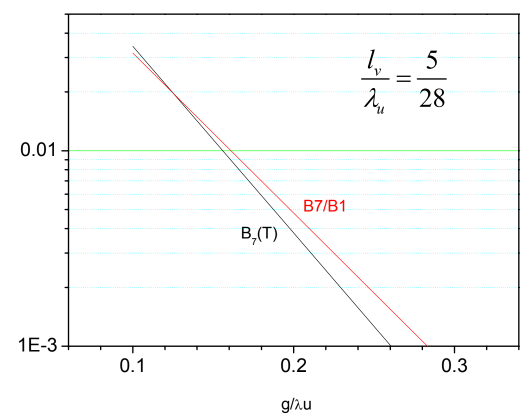

Figure 5: The maximum 7th harmonic field with the magnet gap-period ratio. ($h=\lambda_u/4$).

SUMMARY

In summary, a simple modification to Standard Harbch type undulator was proposed to enhance the harmonic magnetic fields component. The pure permanent magnet undulator with the different size magnet blocks is analysed, and the two dimensional analytical formula of the magnetic field is given. The result show that the harmonic field component can be increased obviously, by optimizing the length ratio of vertical magnetized magnet block and horizontal magnetized magnet block, especially for the smaller gap-period ratio case. The method is easy to implement. Its effect on the fundamental field is small.

For the next work, the effect of undulator harmonics field on the spontaneous radiation and Free-Electron Laser harmonic generation will be analysed.

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