ELIMINATION OF HALL PROBE ORIENTATION ERROR IN MEASURED MAGNETIC FIELD OF THE EDGE-FOCUSING WIGGLER

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Abstract

The experimental model of edge-focusing (EF) wiggler has been fabricated to evaluate its performance rigorously with the magnetic field measurement. It is a 5-period planar wiggler with an edge angle of 2° and a period length of 60 mm. The magnetic field is measured using Hall probes at four different wiggler gaps. The field gradient of the EF wiggler is derived from the magnetic field measured at several transverse positions along the longitudinal position. It is experimentally confirmed that a high field gradient of 1.0 T/m is realized, as designed, along the beam axis. We found that the small orientation errors of the Hall probes and of the linear stages for them lead to large errors in the measured magnetic field. We analyzed the relation between the orientation errors of the measurement system and the measured magnetic field or field gradient using a model magnetic field of the EF wiggle. We corrected the measurement magnetic field based on this analysis. Detailed description of error sources and corrections are given in this paper.

INTRODUCTION

We are conducting free electron laser (FEL) experiment in a terahertz region using L-band electron linac at the Institute of Scientific and Industrial Research (ISIR), Osaka University [1]. As a method for the integrated-focusing to make small beam size over a whole wiggler, we proposed the EF wiggler, which can produce the high magnetic field gradient superimposed on the normal wiggler field. It can be used not only for SASE experiments but also for FEL in order to enhance the gain. In order to evaluate the magnetic performance of the EF wiggler, the first experimental model of the EF wiggler has been fabricated and its magnetic fields have been measured at KEK using Hall probes [2]. In the magnetic measurement, we turned out that the setting error of the Hall probes and of the 3-axis linear stage for them with respect to the wiggler induces large errors in the magnetic field and the field gradient because the magnetic field varies appreciably to all the three directions.

EDGE-FOCUSING WIGGLER

The EF wiggler is basically a Halbach type wiggler made only of permanent magnet blocks. The shape of the magnet blocks with vertical magnetization is an isosceles-trapezoidal right prism with an edge angle ϕ, while that with longitudinal magnetization is a parallelepiped with the same angle. The EF wiggler can produces the strong transverse focusing field incorporated with the normal wiggler field. The model magnetic field of the EF wiggler B^{model} can be written as follows;

$$\begin{aligned} B_x^{\mathrm{model}} &= Gy \\ B_y^{\mathrm{model}} &= B_0 \cosh(ky)\cos(kz) + Gx \qquad (1) \\ B_z^{\mathrm{model}} &= -B_0 \sinh(ky)\sin(kz) \end{aligned}$$

where B_0 the peak magnetic field of the ideal planar wiggler, $k = 2\pi/\lambda w$, and G is the field gradient. Equation 1 is satisfies the Maxwell equations. In actuality, the field gradient has a small z-dependence with a period of $\lambda_w/2$ determined by the mechanical structure of the EF wiggler.

The model EF wiggler consists of five-periods with the period length $\lambda_W = 60$ mm and the edge angle $\phi = 2°$. The permanent magnet is Nd-Fe-B with the residual induction of 1.32 T, and the standard dimensions of the magnet blocks are $2a \times 2b \times 2c = 100 \times 20 \times 15 mm^3$. The peak magnetic field of the wiggler at g = 30mm is calculated using 3D program to be 0.43T [3].

MEASUREMENT SYSTEM

The Hall probes for the vertical and the horizontal field measurements are separately mounted on two small copper plates. They were calibrated against a NMR sensor, so that the magnetic field can be measured with an accuracy of 1×10^{-5} T. Figure 1 shows the experimental setup for the magnetic field measurement. Each Hall probe is attached at the tip of an arm equipped with two tilt stages for adjusting the orientation of the Hall probe, and a rotation stage for adjusting the horizontal direction of the arm. The two arms are mounted on a 3-axis linear stage with two vertical linear stages for independent adjustment of their heights and the magnetic field is measured simultaneously in the horizontal and the vertical directions along the wiggler axis, which is referred to as the z-axis, at various transverse positions, designated as

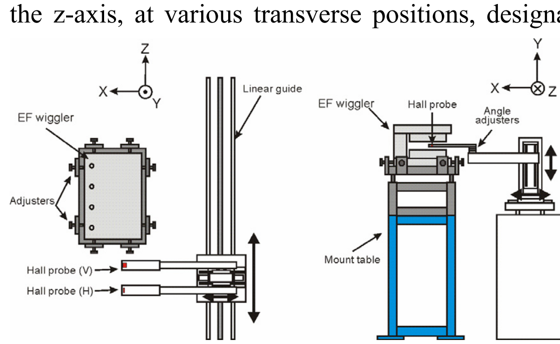

Fig. 1: Schematic drawing of the magnetic field measurement system for EF wiggler. Left is top-view and right is side-view.

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the x-axis in the horizontal direction and as the y-axis in the vertical direction so that the field mapping in the wiggler is made. The origin of the coordinate system is set at the mechanical center of the wiggler, which is the middle point in the longitudinal and the horizontal directions on the median plane. In first set up, the z-axis of the EF wiggler is set parallel to the z-axis of the 3-axis linear stage with an angular accuracy of 50 μrad. The transverse positions of the two Hall probes are adjusted and made equal with respect to the EF wiggler.

ELIMINATION OF HALL PROBE ORIANTAION ERROR

Coordinate system for defining orientation error

The orientation error of a Hall probe may be defined as shown schematically in Fig. 2 (a). A Hall probe measures a component of the magnetic field normal to its plane and it is insensitive to its rotation around the normal axis, so that the orientation error may be defined by the polar angle and the azimuthal angle (θ, ϕ). The white circle with black outline represents a Hall probe with its normal axis a_3 ideally aligned to one of the axes defined for the wiggler, X, Y, or Z, and the gray circle with red outline represents a Hall probe with an orientation error. Two Hall probes are used for measurement of the B_x and B_y, and their orientation errors may be specified by (θ_x, ϕ_x) and (θ_y, ϕ_y), respectively. The orientation errors of the 3-axis linear stage on which the Hall probes are mounted may be defined similarly as shown in Fig. 2 (b). The axis s_3 denotes the ideal direction of motion and the orientation error is defined by the polar angle ψ_3 and the azimuthal angle ζ_3. Three linear stages comprising the 3-axis linear stage may have independent and different orientation errors, so that they are denoted by (ψ_x, ζ_x), (ψ_y, ζ_y) and (ψ_z, ζ_z). The origin of the coordinate system is calibrated to be the magnetic center of the EF wiggler and the (s_1, s_2, s_3) stands for one of the cyclic permutations of (X, Y, Z) defined for the magnetic field measurement.

Magnetic field

We first consider correction for the measured magnetic field against the orientation errors. The measured magnetic fields B_x^{mes} and B_y^{mes} using the Hall probes with the orientation errors may be given by

$$B_x^{mes} = B_y \sin\theta_x \cos\phi_x + B_z \sin\theta_x \sin\phi_x + B_x \cos\theta_x \cong B_x \pm B_z \theta_x \quad (2\text{-}1)$$

$$B_y^{mes} = B_z \sin\theta_y \cos\phi_y + B_x \sin\theta_y \sin\phi_y + B_y \cos\theta_y \cong B_y \quad (2\text{-}2)$$

where **B** = (B_x, B_y, B_z) is the magnetic field in EF wiggler. For measurement of the horizontal magnetic field, the orientation of the Hall probe is adjusted at a position of the highest B_y on the median plane, where B_x and B_z are zero owing to the structural symmetry, so that intrusion of B_y becomes minimum and B_x^{mes} becomes zero. Because this adjustment makes $\phi_x = \pm\pi/2$ but θ_x is indefinite, B_z mingles in the measured field B_x^{mes}. Figure 3(a) shows a result of simulated measurement of the horizontal magnetic fields with and without the

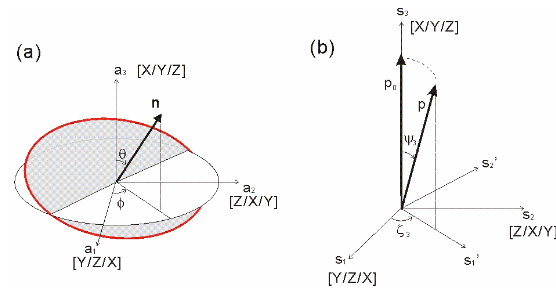

Fig 2: Coordinate system for defining the orientation error of a Hall probe (a) and of a linear stage and the position of the Hall probe (b).

orientation error. The horizontal magnetic fields of the EF wiggler are calculated off the median plane at (x, y) = (0, 2mm) along the longitudinal position and an orientation error of the Hall probe (θ_x = 5 mrad, $\phi_x = \pi/2$) is assumed. The B_x^{mes} without the error, which is plotted with the solid line, shows an oscillation with the period $\lambda_w/2$, while the peaks of the magnetic field with the error, plotted with the dashed line, becomes alternatively high and low because of the mixing of the longitudinal magnetic field B_z, which changes its direction with the period λ_w as can be seen in Eq. (2-1). The polar angle of the orientation error θ_x may be estimated with Eq. (2-1) and B_z^{model} given in Eq. (1) so that this alternative variation is diminished. The horizontal magnetic fields shown in Fig. 3(b) are those corrected for the orientation error of θ_x = 0.7 mrad and $\phi_x = \pi/2$ estimated by the method. Because it is not possible to estimate the azimuthal angle ϕ_x uniquely by this method, it is assumed to be $\phi_x = \pi/2$, which gives the minimum value of θ_x.

For measurement of the vertical magnetic field, the orientation error of the Hall probe is corrected at a position on the median plane where B_y is the highest, as in the case of the horizontal magnetic field. Since B_x and B_z are zero on the median plane and accordingly the first and the second terms of the center of Eq. (2-2) become zero, the polar angle θ_y may be uniquely set to be zero by maximizing the measured magnetic field, so that the vertical magnetic field can be measured without the orientation error.

The orientation error of the 3-axis linear stage may affect the measured magnetic field. Let (a, b, c) be the spurious position measured with linear scales attached to the linear stages and (x, y, z) be the actual position, then

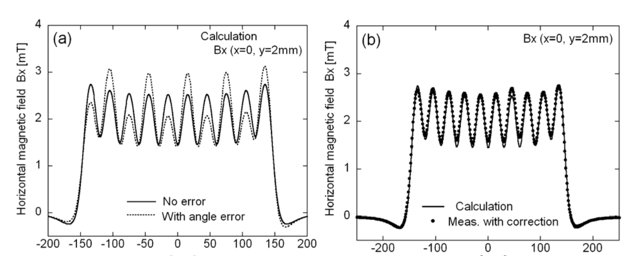

Fig. 3: (a) Horizontal magnetic fields at x=0, y=2 mm calculated along the z-axis with no orientation error (solid lines) and with an orientation error (θ_x=5.0 mrad, ϕ_x=-π/2) of the Hall probe (dashed lines). (b) Measured horizontal magnetic field with correction with θ_x =0.7 (dots) and calculation (solid line).

they are related as

$$\begin{aligned} x &= a+\delta x_x+\delta x_y+\delta x_z = a\cos\psi_x + b\sin\psi_y\sin\zeta_y + c\sin\psi_z\cos\zeta_z \\ y &= b+\delta y_x+\delta y_y+\delta y_z = a\sin\psi_x\cos\zeta_x + b\cos\psi_y + c\sin\psi_z\sin\zeta_z \\ z &= c+\delta z_x+\delta z_y+\delta z_z = a\sin\psi_x\sin\zeta_x + b\sin\psi_y\cos\zeta_y + c\cos\psi_z \end{aligned} \quad (3)$$

The horizontal and the vertical magnetic fields are given respectively by

$$\begin{aligned} B_x(x,y,z) \cong B_x^{mes}(a,b,c) &+ \frac{\partial B_x(a,b,c)}{\partial x}\left(a(\cos\psi_x - 1) + b\sin\psi_y\sin\phi_y + c\sin\psi_z\cos\zeta_z\right) \\ &+ \frac{\partial B_x(a,b,c)}{\partial y}\left(a\sin\psi_x\cos\zeta_x + b(\cos\psi_y - 1) + c\sin\psi_z\sin\zeta_z\right) \\ &+ \frac{\partial B_x(a,b,c)}{\partial z}\left(a\sin\psi_x\sin\zeta_x + b\sin\psi_y\cos\zeta_y + c(\cos\psi_z - 1)\right) \end{aligned} \quad (4\text{-}1)$$

$$B_y(x,y,z) \cong B_y^{mes}(a,b,c) + \cdots \text{abbrev.} \quad (4\text{-}2)$$

The magnetic fields are measured along the central axis of the EF wiggler, so that we may set a = b = 0. If the partial derivatives of the model magnetic field are substituted to Eqs. (4-1) and (4-2), then the magnetic fields are given by

$$B_x(0,0,z) \cong B_x^{mes}(0,0,c) + Gc\psi_z\sin\zeta_z \quad (5\text{-}1)$$

$$B_y(0,0,z) \cong B_y^{mes}(0,0,c) + Gc\psi_z\cos\zeta_z \quad (5\text{-}2)$$

The linear stage for the z direction can be precisely aligned to the EF wiggler with an orientation error of the polar angle ψ_z less than 50 μrad, so that the second terms of the right side in Eqs. (5-1) and (5-2) can be eliminated. Thus, the orientation errors of the 3-axis linear stage do not affect the measured magnetic field under the present experimental conditions.

Field Gradient

The horizontal and the vertical field gradients with the orientation errors of the Hall probes are given by

$$\frac{\partial B_x^{mes}}{\partial y} = \frac{\partial B_y}{\partial y}\sin\theta_x\cos\phi_x + \frac{\partial B_z}{\partial y}\sin\theta_x\sin\phi_x + \frac{\partial B_x}{\partial y}\cos\theta_x \cong \frac{\partial B_x}{\partial y} \pm \frac{\partial B_z}{\partial y}\theta_x\sin\phi_x \quad (6\text{-}1)$$

$$\frac{\partial B_y^{mes}}{\partial x} = \frac{\partial B_z}{\partial x}\sin\theta_y\cos\phi_y + \frac{\partial B_x}{\partial x}\sin\theta_y\sin\phi_y + \frac{\partial B_y}{\partial x}\cos\theta_y \cong \frac{\partial B_y}{\partial x} \quad (6\text{-}2)$$

By the adjustment of the Hall probes in the way described above section, the orientation errors is reduced to be $\phi_x = \pm\pi/2$ and $\theta_y = 0$, so that the field gradients are approximately given by the right sides of Eqs. (6-1) and (6-2). There is no orientation error in $\partial B_y/\partial x$, while the polar angle θ_x remains in $\partial B_x/\partial y$. The field gradients with and without the orientation error shown in Fig. 4 (a) are exactly similar to the horizontal magnetic fields in Fig. 3 (a). If the partial derivative of the model field is substituted to Eq. (6-1), the horizontal gradient along the central axis is given by

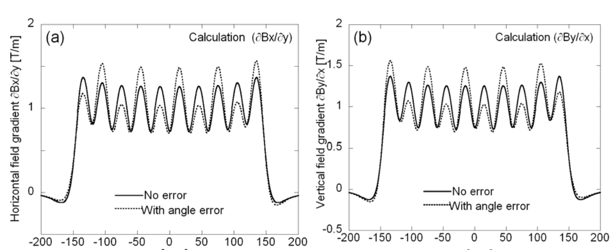

Fig 4: (a): Horizontal field gradients at x=0 calculated along the z-axis with no orientation error (solid lines) and with orientation error (θ_x =5.0mrad, $\phi_x = -\pi/2$) of the Hall probe (dashed lines). (b): Vertical field gradients ($\partial By/\partial x$) calculated along the wiggler axis with no error (solid line) and with an orientation error of ψ_x = 5.0mrad and $\zeta_x = \pi/2$ in the linear stage for the x-direction.

$$\frac{\partial B_x^{mes}(0,0,z)}{\partial y} \cong \frac{\partial B_x(0,0,z)}{\partial y} \mp B_0 k\sin(kz)\theta_x\sin\phi_x . \quad (7)$$

The orientation error θ_x may be easily estimated with the horizontal field gradient derived from the measured magnetic field and the model magnetic field in the same way as it is used for the horizontal magnetic field. The orientation errors derived from the horizontal field and the horizontal gradient should be equal to each other. The orientation error of θ_x = 0.7 mrad and $\phi_x = \pi/2$ estimated by the method.

The field gradients produced by the orientation errors of the 3-axis linear stage may be given by

$$\begin{aligned} \frac{\partial B_x(x,y,z)}{\partial y} &\cong \cos\psi_y\frac{\partial B_x(x,y,z)}{\partial b} \cong \frac{\partial B_x^{mes}(a,b,c)}{\partial b} + \frac{\partial B_x(a,b,c)}{\partial x}\psi_y\sin\zeta_y + \frac{\partial B_x(a,b,c)}{\partial z}\psi_y\cos\zeta_y \\ &\cong \frac{\partial B_x^{mes}(a,b,c)}{\partial b} \end{aligned} \quad (8\text{-}1)$$

$$\begin{aligned} \frac{\partial B_y(x,y,z)}{\partial x} &\cong \cos\psi_x\frac{\partial B_y(x,y,z)}{\partial a} \cong \frac{\partial B_y^{mes}(a,b,c)}{\partial a} + \frac{\partial B_y(a,b,c)}{\partial y}\psi_x\cos\zeta_x + \frac{\partial B_y(a,b,c)}{\partial z}\psi_x\sin\zeta_x \\ &\cong \frac{\partial B_y^{mes}(a,b,c)}{\partial a} - B_0k\sinh(kb)\cos(kc)\psi_x\cos\zeta_x - B_0k\cosh(kb)\sin(kc)\psi_x\sin\zeta_x \end{aligned} \quad (8\text{-}2)$$

where the partial derivatives of the model fields are substituted in the Eqs. (8-1) and (8-2). The horizontal field gradient is not affected by the orientation error of the linear stage, while the vertical field gradient is. Figure 4(b) shows results of simulation for vertical field gradients with and without the orientation error of the linear stage. The vertical field gradient in the EF wiggler is calculated along the central axis at x = 0 and y = 0 and an orientation error of the Hall probe (θ_x = 5 mrad, $\phi_x = -\pi/2$) is assumed. The vertical field gradient with the error varies with the longitudinal position exactly same as the horizontal field gradient with the orientation error of the Hall probe shown in Fig. 4(a). The orientation error of the horizontal linear stage mixes ($\partial B_y/\partial z$) in the vertical field gradient, while the orientation error of the Hall probe for the horizontal direction mixes ($\partial B_z/\partial y$) in the horizontal field gradient. The polar angle of the orientation error ψ_x for $\zeta_x = \pm\pi/2$ may be estimated from the measured horizontal gradient along the central axis of the EF wiggler at a = b = 0 using

$$\frac{\partial B_y(0,0,z)}{\partial x} \cong \frac{\partial B_y^{mes}(0,0,c)}{\partial a} \mp B_0k\sin(kc)\psi_x \quad (9)$$

and ($\partial B_y^{model}/\partial z$) or the partial derivative calculated from the measured vertical magnetic field B_y so that this alternative variation is diminished. The measured vertical field gradients are corrected against an orientation error of the linear stage of ψ_x = 0.9 mrad and $\zeta_x = \pi/2$ so that the symmetry is restored.

We found that the measurement error caused by the small orientation errors of the Hall probes and of the 3-axis linear stages can be eliminated by the correction method. For the magnetic field measurement using Hall probe, this method might be very useful to correct the measurement error that it is difficult to remove in initial setting.

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