

# THEORETICAL EXAMINATION OF RADIATION SPECTRUM FROM THE QUASI-PERIODIC UNDULATOR

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## Abstract

Recently, new type QPUs that generate radiation spectra different from those by conventional type QPU were proposed. In principle, the shape of radiation spectrum from a new QPU is determined by magnetic field distribution having different quasi-periodic pattern. However, calculated spectra using a realistic magnetic field are often different from those of theoretical expectation. In this paper, a comparative study is made to examine differences between theoretical results and numerical calculations. Also, it is shown how to correct magnetic field to get predicted spectra that fit to the theory. In addition, magnetic field distributions to achieve the best performance are presented.

## INTRODUCTION

Conventional periodic undulator is a bright light source that can generate high flux quasi-monochromatic light. However, its radiation contains higher harmonics at rational harmonic positions and those are not easy to eliminate with conventional optical components. For this reason, the quasi-periodic undulator (QPU) that can radiate irrational harmonics instead of rational harmonics was proposed[1]. It suits with experiments that need highly monochromatic light after passing through the monochromator. The QPU is realized by the application of 1D quasi-periodic lattice to the light-and-electron phase slip sequence in an undulator.

Recently, new type QPUs that generate radiation spectra different from those by conventional type QPU were proposed [2,3]. However, calculated spectra using a realistic magnetic field are often different from those of theoretical expectation derived from a simple model.

In this paper, a detailed comparative study is conducted to examine why there are such differences, how to correct magnetic field to generate predicted spectra that fit to the theory.

## RADIATION SPECTRUM EXPECTED FROM SIMPLE MODEL

In a QPU, the feature of light-and-electron phase slip is described by using 1D quasi-periodic lattice. One of the way to create QP lattice is to project 2D periodic lattices in a band onto the line inclined with  $\tan\alpha$  (fig.1). In a new QPU,  $r (=b/a)$  and  $\alpha$  need to satisfy the equation:  $r \tan\alpha > 1$  [2].

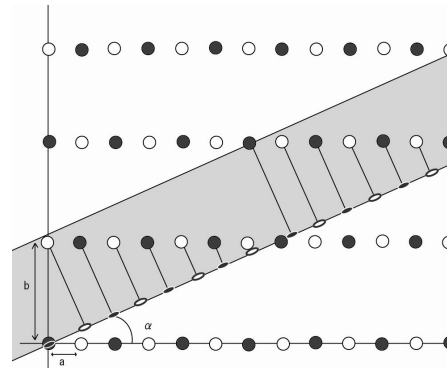


Figure 1: Example of projection method for creating 1D-quasi-periodic lattice applied to QPU

The theoretical prediction of spectrum can be found by Fourier transformation of the 2D lattice into reciprocal lattice and projection of reciprocal lattice onto an irrationally inclined line. The distance from the origin to a projected point is proportional to on-axis spectral peak position. And distance between reciprocal lattice point and the inclined line is related to intensity [1].

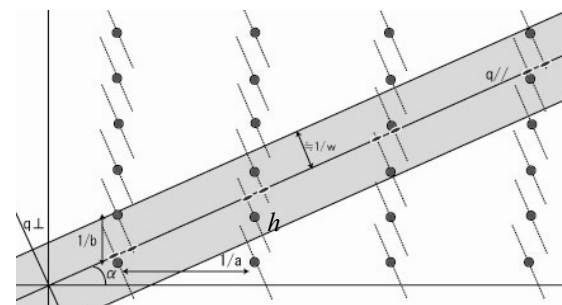


Figure 2: Example of theoretically expected spectrum.

In fig.2, the point projected from  $h$  corresponds to third harmonic. Therefore, in order to avoid the contamination by third harmonic (contribution from  $h$ ), the inclination should be changed in an appropriate irrational angle. This is the reason  $\alpha$  need to be chosen as follows:

$$\alpha = \tan^{-1}(0.5826) \approx 0.5275 \text{ [rad]}.$$

## DIFFERENCES AND CAUSES

Figure 3 is the magnetic field of new type QPU. Unlike previous QPUs, a new QPU has stronger magnetic field at characteristic positions of quasi-periodicity.

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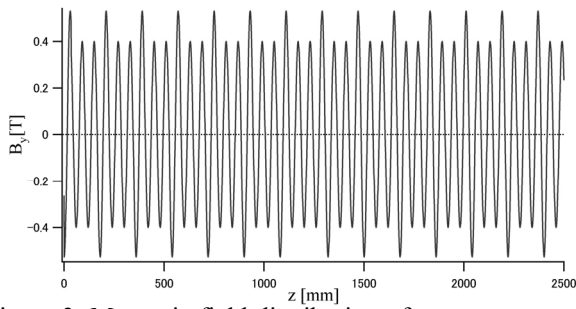


Figure 3: Magnetic field distribution of a new type quasi-periodic undulator.  $L=2.5$ [m],  $\lambda_u=60$ [mm],  $r=3$ ,  $\alpha=0.5275$ [rad] and  $B_{0q}=0.4$ [T].

Figure 4 shows the on-axis radiation spectrum calculating by fig.3 (computed) and the theoretical expectation by a simple model (theoretical). These spectral positions are normalized by first harmonic intensity and position. The accelerator parameters are assumed to be the parameters of HiSOR-II that Hiroshima Synchrotron Radiation Center plans to build as the next generation VUV light source [4].

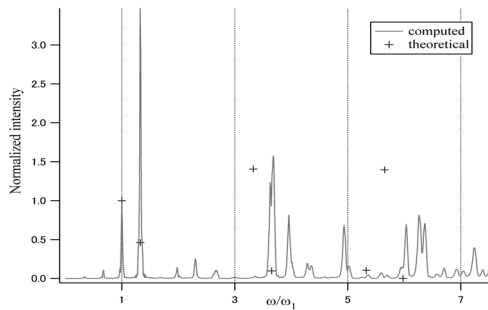


Figure 4: The on-axis radiation spectrum calculated by the realistic magnetic field (computed) and by the theoretical expectation (theoretical)

In the new QPU spectrum, positions of higher harmonics shift toward higher-energy from positions of rational harmonics. That is one of the feature points of new QPU. But, as seen in fig.4, there is a difference between theoretical spectrum and computed one. A cause of the difference between two spectra is excess phase lag arisen from stronger magnetic field at around quasi-periodic lattice points (dotted line in fig.5).

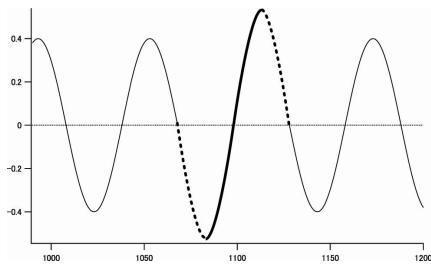


Figure 5: Magnet field distribution of new QPU near the QP position.

## HOW TO CORRECT DIFFERENCES

As shown in the previous section, the cause of difference is excess phase lag. Therefore, the way to correct difference is to change magnetic field to avoid unnecessary phase lag in the dotted line area.

According to this recipe, the corrected magnet field is shown as the solid line in fig.6. At around quasi-periodic lattice points, corrected magnet field has slightly shifted peak position (about 1[mm] outward) and reduced peak intensity ( $0.528 \rightarrow 0.464$  [T]) as compared with that before correction (dotted line).

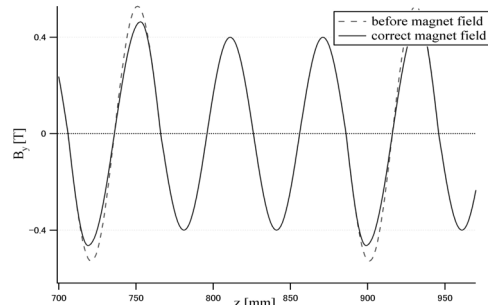


Figure 6: Expanded view of magnet field distribution of new QPU.

The spectrum from corrected magnetic field distribution is shown in fig.7. One can see a good agreement between computed spectrum and theoretical one.

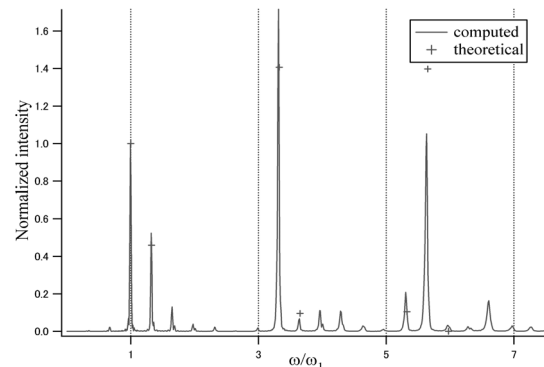


Figure 7: The on-axis radiation spectrum calculated with the corrected magnetic field distribution (computed) and that of theoretical prediction (theoretical).

## SIMPLER METHOD OF CORRECTION

Correction using the method in the previous section is not so easy in a realistic magnetic field correction procedures for an actual undulator. Therefore, some other methods of the correction without changing the peak position are desirable. For this reason, the possibility to realize alternative simpler method for adjusting the phase lag at the QP-position was investigated. As the result, it was found that the phase lag could be corrected by just reducing the peak intensity of magnetic field at QP-position.

The corrected magnetic field by using a new method is shown as the solid line in fig.8. At around quasi-periodic lattice a point, corrected magnet field has reduced peak intensity (from 0.528 T to 0.464 T) compared with that before correction (dotted line) but without moving the peak position.

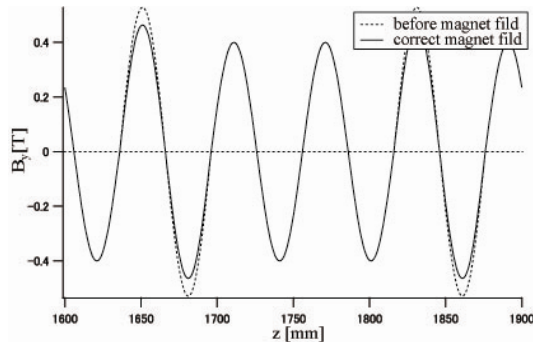


Figure 8: Expanded view of magnet field distribution for the simpler method case.

Spectrum from corrected magnet field by using simpler method is shown in fig.9.

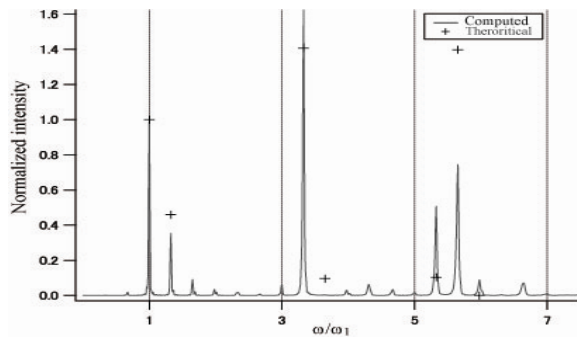


Figure 9: The on-axis radiation spectrum calculated with the corrected magnetic field (computed) and that of theoretical prediction (theoretical).

This spectrum is similar to that of ideal QPU. So, it seems rather easy to make QPU that can generate an optimal spectral radiation by using this method [5].

## CONCLUSION

In this paper, we showed a guideline for how to correct magnetic field to realize ideal spectra with a realistic magnetic field that fit with spectra predicted by the theory with a simple model. This method may be useful to create various types of QPUs that generate arbitrary designed and desired irrational radiation spectra.

## REFERENCES

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