MODELLING OF ELLIPTICALLY POLARIZING UNDULATORS

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Abstract

To investigate the effect of an elliptically polarized undulator (EPU) on the linear optics and dynamic aperture of storage ring a model is required for use in an optics code. An EPU can be modelled as an array of skew dipole magnets. The skew angle ranges from zero to ninety degrees depending on the degree of polarization. Crudely the EPU can be modelled using alternating skew dipole blocks. A model that better reproduces the sinusoidally varying fields can be achieved by slicing blocks into smaller subsets. Field roll-off produced by the limited transverse dimensions of the magnet blocks can be included as skew multipoles. For example the roll-off of the horizontal field in the vertical undulator mode is very nearly a skew sextupole. Corrections using multipole magnets are discussed.

INTRODUCTION

At present the Canadian Light Source (CLS) [1] has two Apple-II type undulators [2] in operation. Both these structures have undulator wavelengths of 0.075 m with total length just of about 1.6 m. With feed-forward tables to compensate steering and coupling at different gaps neither of these devices have an adverse feedback on the stored beam.

Future plans include the installation of EPUs with both longer wavelengths and larger overall length. These devices are expected to have a stronger effect on the stored beam in terms of tune shifts, especially in the vertical mode of operation. To assess in advance the possible effects an optics model is required.

There is no doubt that for studying the dynamic aperture at large amplitudes kickmaps [3] are best suited to the task. Kick maps, however, are both lattice and tune dependent. As well, they are not readily implemented in all optics codes. This paper reports on attempts to model EPU behaviour with standard magnetic elements using DIMAD [4]. The Apple-II EPU presently in service at the CLS is modelled in detail. Extra attention was given to the vertical mode where the effects on the stored beam are the largest.

Results from the model suggest corrections that can be implemented with multipole corrector magnets flanking the EPU.

EPU MODEL

Slicing

The insertion device (ID) fields are assumed to vary sinusoidally through the magnetic array. Modelling each pole can be done by converting each half sine wave to an "equivalent" square edge dipole magnet. For strong devices such as wiggler magnets this gives an incorrect tune shift. Tune shifts in close agreement with measurements can be achieved by "slicing" each sine wave into a number of hard edge dipoles. Slicing into eight segments gives good results for the on-axis tune shifts for even the strongest planar devices. Eight slices are used for modelling EPUs.

The amplitudes, B_n, of the eight slices are given by

$$B_n = B_{max} \sin(\pi/16(2n-1)),$$

where n is the slice number and B_{max} is the maximum field of the sine wave being modelled. (For the negative pole $B_{max} \rightarrow -B_{max}$.) The bend angles in each slice, θ_n, are proportional to B_n times $1/16^{th}$ the undulator wavelength giving a total bend angle $\theta_{tot} = \Sigma_n \theta_n$.

As set up by an entrance kick the entrance angle into the magnet array is $\theta_{tot}/2$. The entrance and exit pole face rotations, φ_{in} and φ_{out}, for each slice are given by

$$\varphi_{in,1} = \theta_{tot}/2$$
$$\varphi_{out,1} = \theta_1 - \varphi_{in,1} \;;\; \varphi_{in,2} = -\varphi_{out,1}$$
$$\varphi_{out,2} = \theta_2 - \varphi_{in,2} \;;\; \varphi_{in,3} = -\varphi_{out,2} \quad \text{.......etc.}$$

Similar considerations are given to the entrance and exit half strength poles.

It is the proper configuration of the pole face rotations angles that gives the proper tune shifts for planar IDs. Both on-axis and off-axis tune shifts for EPUs present a special problem. These are modelled by considering the effects of the magnetic field roll-offs derived from magnet models or field measurement as discussed later.

Skewing

For the various modes of operation in an EPU the magnetic fields will be at different orientations or skew angles. Four modes are considered and are: horizontal vertical, circular and 45°. All modes except the circular mode are at constant skew angles and are set accordingly. In the (ideal) circular mode there are equal horizontal and vertical fields that are 90° out of phase. Consequently the magnetic field is constant in each slice but each slice is skewed by a different angle, ψ_n. Assuming that the array starts at the maximum horizontal field with the vertical field 90° later, the skew angles for the circular mode are

$$\Psi_n\ (n=1,4) = \pi/8\ (9-2n);$$
$$\Psi_5 = -\Psi_4;$$
$$\Psi_6 = -\Psi_3;\ \text{etc.}$$

On-axis Roll-off Correction

The model as given above does not give the correct tune shifts for an EPU. The horizontal gap down the centre of the magnet array causes a transverse field roll-off. The roll-off can be used to estimate the on-axis tune shift caused by the gap for the linear modes of operation. In the immediate vicinity of the beam axis the field behaves like a sextupole with field strength B″. The field varies in sign from pole to pole. Since the electron beam is oscillating about the central field it experiences the same focusing (or defocusing) effect from each pole. The focusing effects accumulate and the EPU acts like an off-axis sextupole of length L, the total undulator length.

An off axis sextupole has a quadrupole component that is proportional to twice the displacement, X. Using the rms values (0.707) for the sinusoidally varying field and position the integrated quadrupole strength, in standard notation, is given by

$$k_1L = 2B''_{rms}X_{rms}\, 0.3/E\, L = 0.3\, B''_{max}X_{max}\, L\, /E \qquad (1)$$

where B″ is in T/m^2, X is in meters and E is the beam energy in GeV.

The field roll-off also produces amplitude dependent tune shifts. An estimate of the tune shift can be estimated by tracking off axis under the influence of the field changes produced by the roll-off. Each pole has even numbered multipole fields with strengths given by

$$k_2\, \lambda/2 = 2\ B''_{rms}\, 0.3/E\, \lambda/2 = 0.3\, B''_{rms}\, \lambda/E\ ;$$
$$k_4\, \lambda/2 = 4{\cdot}3{\cdot}2\ B''''_{rms}\, 0.3/E\, \lambda/2 = 3.6\, B''''_{rms}\, \lambda/E\ ;$$
$$\text{etc.} \qquad (2)$$

where λ is the undulator period.

75 mm APPLE II

Basic Parameters

The 75 mm APPLE II EPU in operation in the CLS 2.9 GeV storage is modelled. The characteristics for four operating modes of this device are given in Table 1. The modes are described by the electron motion and are: horizontal (H), vertical (V), circular (C) and Inclined (I) (The inclination is 45°.) R_{max} is the maximum orbit deviation (X in H mode, Y in V mode, etc.). B''_{max}, is the sextupole strength in the direction of interest.

Table 1: Parameters for the Modes of the SM EPU

	H	V	C	I	
λ (L)	0.075 (20.5 λ = 1.5375)				m
B_{max}	0.742	0.658	0.492	0.492	T
R_{max}	11.0	9.5	7.0	7.0	μm
B''_{max}	1464	–7393	–5929	--	T/m^2
k_1L	0.0026	–0.011	–0.006	--	T

On Axis Tune Shifts

The effective quadupole strengths, k1, for each mode are also shown in Table 1. The k_1 values for the H and V modes are calculated using eqn. 1. For the C mode the k_1 value was estimated from the H and V values by scaling the orbit amplitudes and taking the difference between the H and V modes. The I mode best modelled with no correction.

To calculate the on-axis tune shifts the k_1 values are included in the model as a single 1st order correction in the middle of the EPU. The resulting overall tune shifts are shown in figure 1. The tune shifts are compared with the tune shifts predicted by kick-map analysis and with actual measurements. Good agreement with both kick maps and measurements are seen with the uncorrected I mode giving the least favourable result. To a good approximation the on-axis tune shifts can be modelled with a single quadrupole correction.

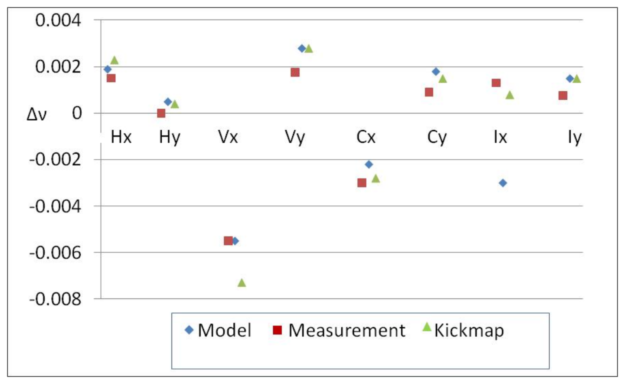

Figure 1: On-axis horizontal (x) and vertical (y) tune shifts for the various EPU modes. The model is compared to a kickmap predication and to measurements of the actual device.

Off-Axis Tune Shifts

The off-axis tune shifts are calculated by including the higher order multipoles, given by eqn. 2, at each pole in the EPU including appropriate multipoles for the half strength entrance and exit poles. The tune shifts are calculated by tracking over increasing amplitudes. [It should be noted that in evaluating the tune shifts the average absolute displacement off-axis should be considered. For example a particle with maximum amplitude of 3 mm has an average amplitude of 2 mm. At larger amplitudes the higher order distortion of phase space by tends to increase the average amplitude.]

The vertical mode, which gives the largest tune shifts, is modelled in detail. The horizontal tune shift with horizontal position is described here. [Other vertical tune shifts are not necessarily modelled correctly but are at any rate much smaller than the tune shifts considered.] The vertical mode is produced with a horizontal field, B_x. The field roll-off in the horizontal direction is shown in figure 2. Multipole components up to 6th order are shown. Magnet symmetry suppresses the odd harmonics. The plot represents the maximum field produced by the EPU when it is at its minimum gap. The vertical roll-off cannot be improved by a broadening of the EPU magnets.

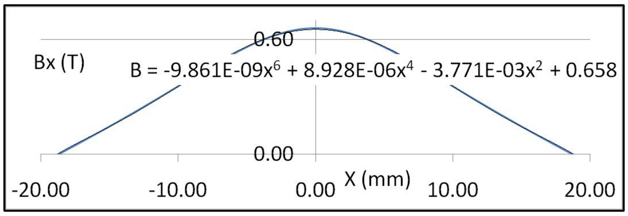

Figure 2: Roll off of B_x, vs horizontal position, X, in the EPU vertical mode (from magnet model).

For yet undiscovered reasons, the multipoles shown in figure 2 give off-axis tune shifts that are half those predicted by the kickmap. Agreement with the kickmap can be achieved by increasing the first two mulipoles by a factor of two. The highest order is adjusted to give the best fit. It is possible that more multipoles could extend the agreement beyond 10 mm. At these amplitudes the model becomes unstable and this region is best left for kickmap analysis. The model suggest however possible correction schemes for the EPU in the vertical mode.

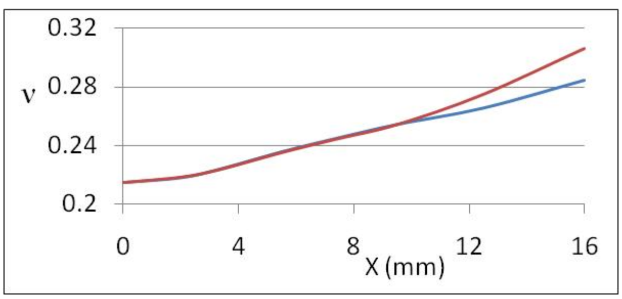

Figure 3: Horizontal tune shift vs horizontal amplitude in the EPU vertical mode. Top line: kickmap; bottom line: model.

CORRECTION

The 1st order effect of the EPU causes not only a tune shift but also ~10% variations of the betatron amplitudes from cell to cell. The largest tune shift is in the vertical mode. It can easily be corrected by a small change ($\Delta k_1L = 0.008\ m^{-1}$) to each of the two lattice quadrupoles flanking the EPU straight. Alternatively the tunes can be adjusted by applying a global tune correction using all the storage ring quadrupoles. In either case the betatron variations are not satisfactorily corrected.

New correction quadrupoles located at each end of the EPU can correct the tune with about half the quadrupole value ($k_1L = 0.004\ m^{-1}$). As well, this local correction reduces the betatron oscillations as shown in figure 4. The required correction could be achieved with two 0.05 m corrector quadrupoles with $k_1 = 0.16\ m^{-1}$.

The skew sextupole nature of the EPU in the vertical mode suggests that the off-axis tune shift could be corrected with skew sextupole corrector magnets. However, corrections with such magnets failed to produce satisfactory results. However, normal sextupoles flanking the EPU are able to reduce the off-axis tune shifts. Interestingly, the correction sextupoles were also able to reduce the off-axis tune shift seen with no EPU present. This suggests that the CLS lattice could benefit from addition sextupole magnets.

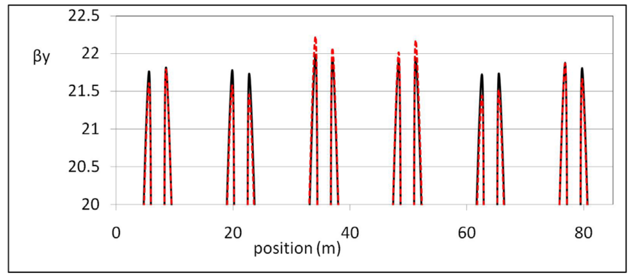

Figure 4: Variation in maxima of the vertical β functions for ½ the lattice. (Similar variations are seen over the full lattice.) Dashed line: uncorrected; solid line: corrected.

Preliminary modelling of a 0.2 m wavelength EPU suggests tune shifts as large as 0.01 or more in the vertical mode. In this case a quadrupole corrector of up to $k_1L = 0.01\ m^{-1}$ may be required. Furthermore, IDs require some small amounts of gap dependent steering correction. Horizontal and vertical steering, quadrupole correction and sextupole correction can be achieved with multi-function correction magnets [5].

CONCLUSION

An optics model considering the slices of magnetic field for every pole has been developed. Using information from the magnet model (or magnet measurements) to predict 1st order effects on-axis tune shifts are obtained in good agreement with kickmap models and with measurements. Off-axis tune shifts can be reproduced to amplitudes up to 10 mm but studies of larger amplitudes are best left to kickmap analysis. Tune shift corrections can be achieved with multipole corrector magnets of modest strength.

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