COMPUTATION OF ELECTROMAGNETIC MODES IN THE TRANSVERSE DEFLECTING CAVITY

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Abstract

The new generation X-ray Free Electron Laser (SwissFEL) under development at the Paul Scherrer Institut (PSI) will employ a transverse deflecting cavity [3] for beam diagnostics. Since this cavity design breaks the symmetry, a complete 3-dimensional eigenmodal analysis is indispensable. The 3-dimensional eigenmodal solver Femaxx has been developed in a collaboration between PSI and the Swiss Federal Institute of Technology (ETH) Zurich. Femaxx [2] aims at large-size generalized eigenvalue problems, therefore it has been optimized for distributed memory parallel compute clusters. We use Femaxx to analyze the transverse deflecting cavity, i.e., to compute electromagnetic eigenmodes corresponding to the lower eigenfrequencies. For further usage in the beam dynamics code OPAL [1], we sample the eigenmodal fields on a 3-dimensional Cartesian grid.

DESCRIPTION OF THE PROBLEM

The Femaxx code solves the Vector Wave Equation: Let Ω be a closed domain describing a large accelerator structure. Neither do we assume that external fields, sources or charges are present, nor do we consider loss mechanisms. The perfect boundary condition applies on the surface Γ of the domain. Then the electromagnetic eigenmodes and eigenfrequencies can be computed by solving the eigenvalue problem.

$$\mathbf{curl}\,\mathbf{curl}\,\mathbf{E}(\mathbf{x}) = \lambda \mathbf{E}(\mathbf{x}), \quad \mathbf{x} \in \mathbf{\Omega}, \quad \lambda = \omega^2/c^2, \tag{1}$$

$$\mathbf{div}\,\mathbf{E}(\mathbf{x}) = 0, \quad \mathbf{x} \in \mathbf{\Omega} \tag{2}$$

$$\mathbf{n} \times \mathbf{E}(\mathbf{x}) = 0, \quad \mathbf{x} \in \mathbf{\Gamma} \tag{3}$$

(2) imposes the divergence-free constraint in the source-free domain, and (3) is the infinite conductivity boundary condition on the surface. The magnetic field $\mathbf{H}$ is then calculated by the relation.

$$\mathbf{H}(\mathbf{x}) = \frac{1}{-i\omega\mu_0}\mathbf{curl}\,\mathbf{E}(\mathbf{x}) \tag{4}$$

SOLVER

In the current version of Femaxx, two eigensolvers have been developed and parallelized. They are the *real symmetric* Jacobi-Davidson algorithm [2] and the LOBPCG. The solvers are based on the Trilinos software framework which defines basic parallel objects like vectors and sparse matrices. Trilinos provides a wide range of solvers like iterative solvers complemented with numerous preconditioners, for instance, multilevel and incomplete factorization preconditioners. Trilinos communicates through MPI.

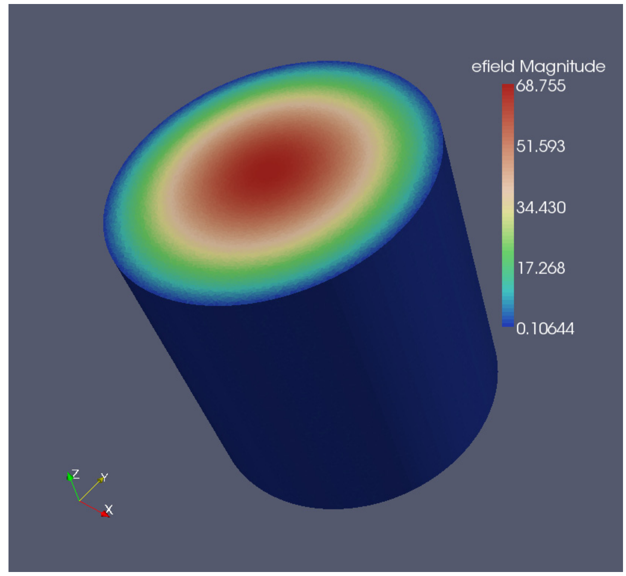

Figure 1: Pillbox cavity: the electrical field distribution of the $TM010$ mode.

Table 1: Pillbox Cavity: analytical vs. numeric solution

	analytical solution	numeric solution
TM010 mode - Resonance frequency	$2294.9MHz$	$2295.2MHz$
TM010 mode - Quality factor	24153	24140

AN EXAMPLE: PILLBOX CAVITY

Before simulating the transverse deflecting cavity, we validated the correctness and reliability of Femaxx, by means of the pillbox cavity. For this geometry, the eigenmodes are known analytically [4]. We also run Femaxx to get the numerical simulation of the TM010 mode (Figure 1). Then we can compare the numerical result with the analytical solution, which should be close to each other (Indeed, the resonant frequency and the quality factor that depend on the eigenmode are close (Table 1)).

In a further comparison of the analytical and the numerical eigenmodes we set up a test beam and let it pass through both field distributions by using OPAL. The analytical eigenfield is sampled in a cylinder and the numerical eigenfield is sampled (interpolated) on nodes of a Cartesian grid. The results of this simulation, energy and emittance,

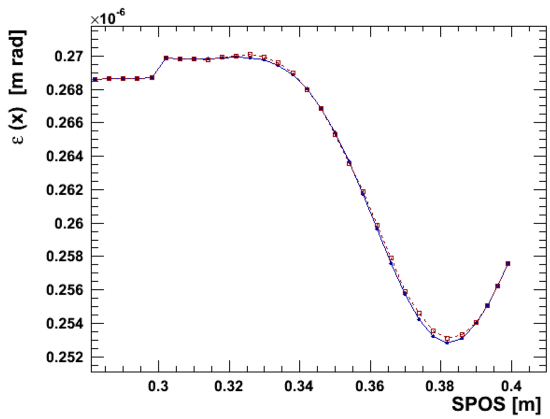

Figure 2: The comparison of the emittance in x.

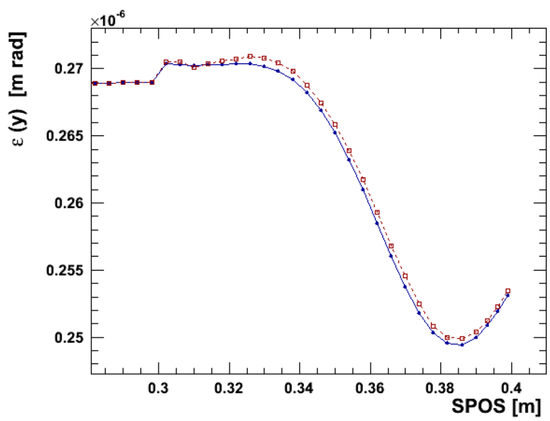

Figure 3: The comparison of the emittance in y.

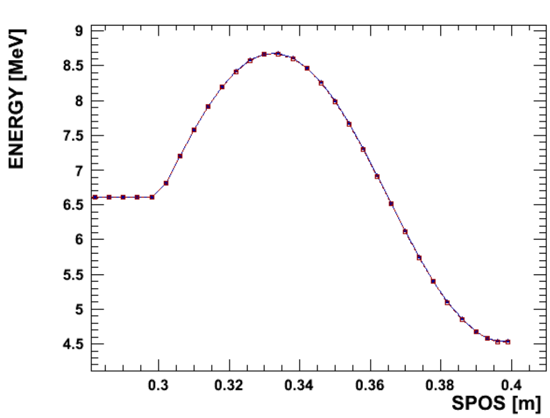

Figure 4: The comparison of the Energy.

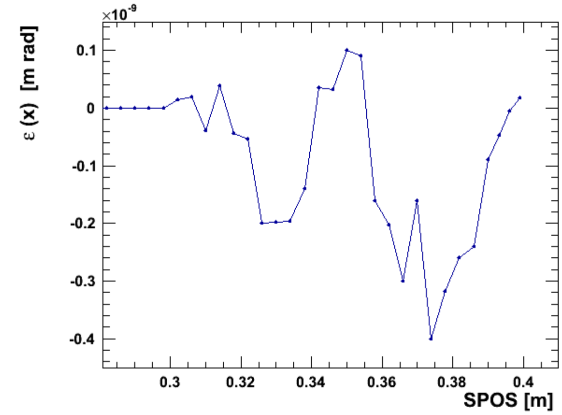

Figure 5: The difference of the emittance in x is at the order of 10^{-9} m rad.

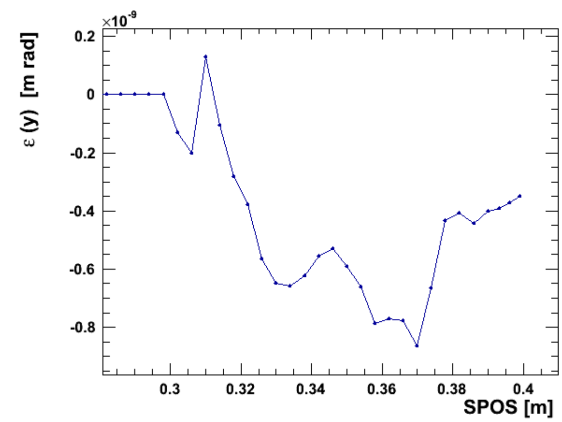

Figure 6: The difference of the emittance in y is at the order of 10^{-9} m rad.

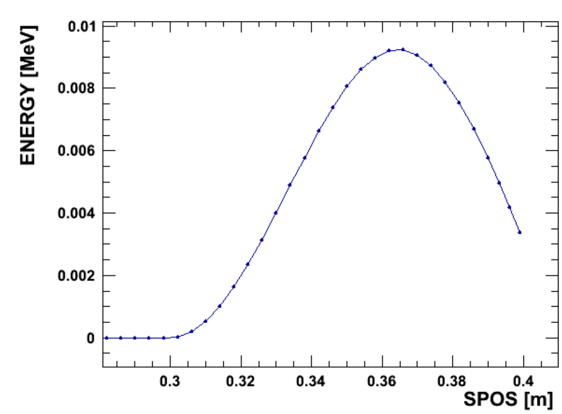

Figure 7: The difference of the Energy is at the order of 10^{-2} MeV.

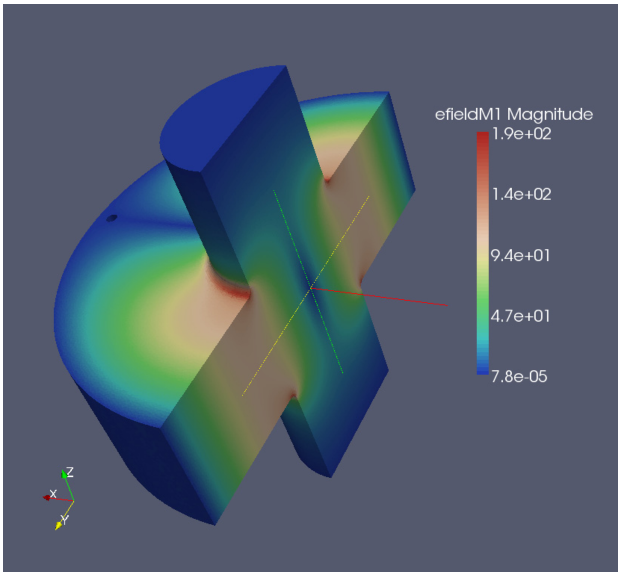

Figure 8: Transverse deflecting cavity: the Electrical field distribution of the TM110 mode (in half of the domain). The TM110 mode is the working mode. The computed resonant frequency is $3.0031\ MHz$ (The design parameter is $2.9979\ MHz$), and the computed Quality factor is 16200 (The design Parameter is 15800).

are displayed in Figures 2–4. The differences are given in Figures 5–7. They are small (below 1%) from which we conclude that the eigenmodes computed by Femaxx are accurate.

TRANSVERSE DEFLECTING CAVITY

Now that the Femaxx code is validated, we can simulate the transverse deflecting cavity. The electrical field distribution (in half of the domain) and the Cartesian sampling of the field are presented in Figures 8 and 9, respectively.

Some numerical details for this problem are listed here:

1. The mesh: $8'191'482$ tetrahedrons.
2. Quadratic Nédélec Elements (with 20 degrees of freedom per element).
3. The eigenvalue problem $Ax = \lambda Mx$ and the solver.
   - Matrix order: $51'214'510$.
   - Eigensolver: Jacobi-Davidson method
   - Linear Solver for the correction equation: QMR
   - Preconditioner: Hierarchical Basis Preconditioner combined with Algebraic Multigrid Preconditioner
   - Timing: computing the basic two eigenmodes takes about 2 hours (using 1024 processors at the Swiss Super Competing Center).
   - Memory usage: the maximum dimension of the search space is 15.

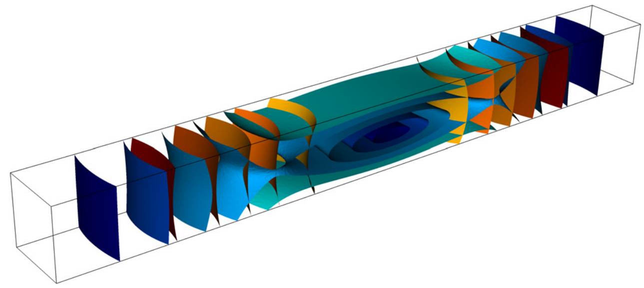

Figure 9: Electrical and Magnetic fields of the transverse deflecting cavity (visualized by G. Weber (LBNL)).

4. Description of the Cartesian Sampling
   - Sampling domain: $1\,cm \times 1\,cm \times 10\,cm$
   - Grid side: $100 \times 100 \times 1000$

CONCLUSION

With Femaxx we are able to solve large-size eigenvalue problems and export the eigenfield distribution sampled on a Cartesian grid. The sampling is accurate and reliable and can be employed by OPAL for particle tracking and beam analysis. The successful simulation of the transverse deflecting cavity is an important step toward the design and manufacturing of the new generation X-ray Free Electron Laser at PSI.

REFERENCES

[1] A. Adelmann et al., *The OPAL (Object Oriented Parallel Accelerator Library) Framework* Tech. Report, Paul Scherrer Institut, 2008. (Available at URL `http://amas.web.psi.ch/docs/index.html`).

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