HELLWEG 2D CODE FOR ELECTRON DYNAMICS SIMULATIONS

S.V.Kutsaev
Moscow Engineering-Physics Institute (State Univrsity), Moscow, Russian Federation.

Abstract

This paper introduces *"Hellweg 2D"* code, a special tool for electron dynamics simulation in waveguide accelerating structure. The underlying theory of this software is based on the numerical solutions of differential equations of particle motion. The effects considered in this code include beam loading, space charge forces, and external focusing magnetic field. "*Hellweg 2D*" is capable to deal with multi-sectional accelerators. Along with a manual input of electrodynamical parameters of the cells, for disk-loaded structures they can be calculated automatically with a help of experimental data tables. In order to obtain the maximum capture in the buncher section, the optimiser of phase velocity and electric field strength functions is developed. The comparison of U-1-M buncher beam dynamics simulations via *"Hellweg 2D"* and experimental data is provided.

INTRODUCTION

Today there are a lot of computer codes for beam dynamics simulations in particle accelerators. These programs posses a number of disadvantages. For example, some of them are based on a simplified model of a beam loading effect, which can be used only for a relativistic beam. Some use pre-calculated fields but real amplitude in each cell has to be estimated by the user. Some don't allow calculating dynamics in the bunching section of linac. Meanwhile, a rather simple and convenient method for intensive beam dynamics in travelling wave (TW) accelerating structures analysis has been proposed in 1980s [1]. This method has been realized in Moscow Engineering Physics Institute (MEPhI) via *DYNAM-1* code for M220 type computers. At present time, the necessity to advance tools for beam dynamics simulations has arisen due to a rapid computer engineering development.

Earlier efforts to create such a code for PC include the program *LINACSOLVER2D* code. Unfortunately, this work remained unfinished. The program wasn't able to consider the space charge effect or to generate the required initial distributions etc. This paper presents the description of "*Hellweg 2D*" code designed to replenish the mentioned disadvantages and to provide the user with synthesis tools along with analysis tools. This program's algorithm is based on the equations that can consider such effects as beam loading, space charge and solenoid field focusing. It is possible to simulate dynamics in multi-sectional accelerators. With the help of the reference data entered into this program, it becomes feasible to automatically determine the attenuation coefficient and the aperture size of the disk-loaded structure (DLS) type cells with a known phase velocity and normalized electrical field strength. The possibility to synthesize the accelerating structure with desired beam parameters can be a very useful feature of the "*Hellweg 2D*" code. It includes the bunching cells parameters optimiser for achieving the maximum coupling and the acceleration cells parameters optimiser for achieving the necessary output energy. In case of a constant gradient structure, the aperture radii of the cells are automatically adjusted to satisfy the condition so that the real electrical field strength in each cell remains constant.

NUMERICAL MODEL

The numerical model is based on the self-consistent equation system [1] describing the electrons motion in waveguide structures with variable dimensions. This system includes a 2D motion equation in an axial-symmetrical structure; an equation for a self-consistent RF-field amplitude created by the beam and an equation for a particle's phase in a self-consistent field.

If we consider the part of the beam with a wavelength width divided by N "large" particles, the dimensionless RF-filed amplitude $A=eE\lambda/W_0$ affecting each particle and the particle's phase, the ψ in this field can be calculated using the Eq.1:

$$\frac{dA}{d\xi}=A\left\{\frac{1}{2}\frac{d}{d\xi}(\ln R_b)-w\right\}-\frac{2B}{N}\sum_{n=1}^{N}I_o\left(\frac{2\pi}{\beta_w}\sqrt{1-\beta_w^2}\eta_b\right)\cos\psi_n \tag{1a}$$

$$\frac{d\psi}{d\xi}=2\pi\left(\frac{1}{\beta_w}-\frac{1}{\beta_\xi}\right)+\frac{2B}{AN}\sum_{n=1}^{N}I_0\left(\frac{2\pi}{\beta_w}\sqrt{1-\beta_w^2}\eta_b\right)\sin\psi_n \tag{1b}$$

Here and in the further text, the following symbols are used: z - longitudinal coordinate, x – transversal coordinate; λ – operating wavelength; β_w – phase velocity of wave; J_0 – accelerated current; P – input power; E – electrical field strength B_m – magnetic field induction; c – velocity of light; W_0 – electron's rest energy; W – particle's energy; α – attenuation of structure (if mentioned α stands for a Twiss parameter). Also the following expressions are assumed:

$$\xi=z/\lambda,\ \eta=x/\lambda,\ \gamma=W/W_0,\ w=\alpha\lambda,$$
$$B=eJ_0R_b/2W_0,\ R_b=\Lambda^2/2,\ \Lambda=E\lambda/P^{1/2} \tag{2}$$

The components of the dimensionless electric A and magnetic $H=ecB_m\lambda/W_0$ field amplitudes that affect each particle can be found using the Eq.3:

$$A_\xi=A(\xi)I_0\left(\frac{2\pi}{\beta_w}\sqrt{1-\beta_w^2}\eta\right)\cos\psi+A_\xi^{coul} \tag{3a}$$

$$A_\eta=-\frac{\beta_w}{2\pi\sqrt{1-\beta_w^2}}I_1\left(\frac{2\pi}{\beta_w}\sqrt{1-\beta_w^2}\eta\right)\times$$
$$\times\left\{\frac{dA}{d\xi}\cos\psi-\left[\frac{2B}{N}\sum_{n=1}^{N}I_0\left(\frac{2\pi}{\beta_w}\sqrt{1-\beta_w^2}\eta_n\right)\sin\psi_n+\frac{2\pi}{\beta_w}A(\xi)\right]\sin\psi\right\}+A_\eta^{coul} \tag{3b}$$

$$H_\theta=\frac{\beta_w A(\xi)}{\sqrt{1-\beta_w^2}}I_1\left(\frac{2\pi}{\beta_w}\sqrt{1-\beta_w^2}\eta\right)\sin\psi \tag{3c}$$

Apart from RF field, these expressions also consider a space charge field. To simulate the self-consistent dynamics of the particles, it is necessary to insert the Eq.3 in the equations of motion (Eq.4):

$$\frac{d\beta_\xi}{d\xi}=\frac{1}{\gamma\beta_\xi}\left(\left(1-\beta_\xi^2\right)A_\xi+\beta_\eta\left(H_\theta-\beta_\xi A_\eta\right)-\beta_\theta H_\eta^{EXT}\right) \tag{4a}$$

$$\frac{d\beta_\eta}{d\xi}=\frac{1}{\gamma\beta_\xi}\left(A_\eta-\beta_\xi H_\theta-\beta_\eta\left(\beta_\xi A_\xi+\beta_\eta A_\eta\right)\right)+\frac{\beta_\theta}{\beta_\xi\gamma}H_\xi^{EXT}+\frac{\eta\dot{\theta}^2}{\beta_\xi} \tag{4b}$$

$$\eta^2\gamma\beta_\xi\frac{d\theta}{d\xi}=\frac{1}{2}\left(C-\eta^2 H_\xi^{EXT}\right) \tag{4c}$$

SPACE CHARGE

For a space charge consideration the bunch is approximated as a uniformly charged ellipsoid [2]. In this case the corrections to the Eq.3 can be expressed as:

$$E_r^{coul}=\frac{1}{4\pi\varepsilon_0}\frac{3J\lambda}{c\gamma^2}\frac{(1-M)}{2R_x{}^2R_z}r \tag{5a}$$

$$E_z^{coul}=\frac{1}{4\pi\varepsilon_0}\frac{3J\lambda}{c\gamma^2}\frac{M}{R_x{}^2R_z}z \tag{5b}$$

Here R_x and R_z are the semiaxes of ellipsoid, J is an average current assuming that a bunch occurs every RF period and ε_0 is the permittivity of free space.

ACCELERATOR SYNTHESIS

DLS Parameters Computation

One of the "*Hellweg 2D*" convenient functions is automatic parameters evaluation for a disk-loaded structure (DLS). The program can find the normalized attenuation coefficient $\alpha\lambda^{3/2}$ and aperture radius a/λ by the given phase velocity β_w and normalized electrical filed strength Λ (Eq.2).

To compute these parameters the experimental data from the reference book [3] is taken. The values are given for several discrete points, so the double interpolation is used to obtain the solution.

This procedure is currently available only for DLS working on $\pi/2$ and $2\pi/3$ modes, but can be expanded on any structure by adding the necessary experimental or pre-calculated data. In future, the opportunity for user to add parameters for any structure would be available.

Buncher Optimization

Along with analysis if a given structure, "*Hellweg 2D*" can automatically adjust the input cell parameters (phase velocity and field strength); number of cells and input current if needed in order to obtain the necessary output parameters that are average energy; and capture the coefficient and\or the equable field amplitude distribution in structures with constant gradient.

The optimal capture can be achieved while varying the parameters of a buncher. One of the most popular buncher types are those with the variable phase velocity, because they have better bunching performance than the other ones. The electron dynamics in such a section depends on the particles initial energy, electrical field strength and an equilibrium phase values.

The buncher synthesis is a very complex task. Experimental functions [4] cannot guarantee the best results for the whole spectrum of initial conditions. These expressions are adequate for capture coefficient but not for energy spectrum width or phase length. The creation of the robust buncher-synthesizing module is one of the future development opportunities for this program.

Accelerating Structure Synthesis

Along with buncher optimisation, "Hellweg 2D" is capable to synthesize an accelerating section so that the output energy equals to a target value. Two types of optimisation are available: one for constant impedance structures and one for constant gradient structures. In first case the program simply adjusts the number of identical cells and, optionally, the value of an input current, to receive the exact value of necessary output energy. In the second case, every subsequent cell's aperture radius is additionally chosen so that the amplitude of the electrical field in this cell, considering RF power losses and beam loading, equals to the field's amplitude in the preceding cell.

For example, acceleration structure with a constant gradient for 2-sectional 20 MeV linac was synthesized via "*Hellweg 2D*". Each section is considered to be fed independently with 2 klystrons producing 5MW peak power at 2856 MHz operating frequency.

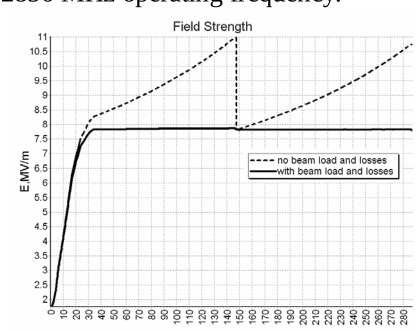


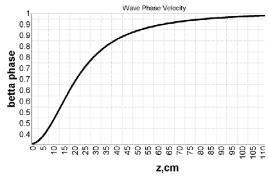
Figure 1: Electrical field amplitude distribution along generated structure with constant gradient

Considering the parameters of buncher cells are known and presented in the initial part of Fig.1 and the injected beam has average energy of 25 keV, the program was able to adjust the number of cells working on $2\pi/3$ mode and their aperture radii, so that the output energy after each section is 10 MeV (20 MeV total) and the electrical field strength in the regular part is constant what one can observe in Fig.1. As each section is fed separately so the jump in nominal electrical filed distribution can be observed. It's clear that the constant gradient condition is very well met and the synthesizer module has done a good job.

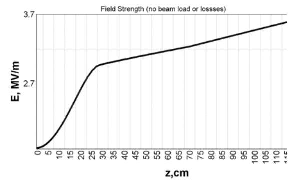
SIMULATION EXAMPLE

The examples of beam dynamics simulations by "*Hellweg 2D*" code are presents below. The point of these calculations was to compare the results of the program with theoretical, experimental and other codes simulated results. It also demonstrates the possibilities of this program on an example of a hybrid accelerator.

Waveguide buncher U-1-M [4] of the travelling wave electron linac has been developed and created in Moscow Engineering-Physics Institute (MEPhI). This device appears to be a 3MeV linac. The electrical field and phase velocity distributions along the buncher are presented in Fig.2.



a) Phase Velocity



b) Field Strength

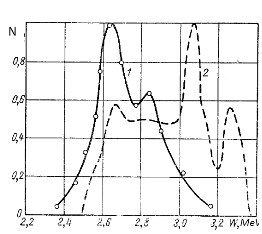
Figure 2: Accelerating Wave Parameters Distribution Along the Accelerator

RF feeding is carried out by a magnetron with output peak power 1.2 MW operating at 2797 MHz frequency. The beam is injected to the accelerator with energy 45 keV. Accelerating structure of the buncher is a DLS working on $\pi/2$-mode

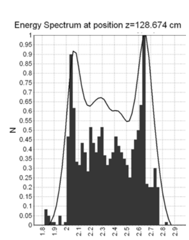
Its developers using an unnamed, self-made code have modelled the beam dynamics of U-1-M buncher. This code has been developed in the 1960s and based on a very simplified mathematical model, so its results are quite inaccurate. From this point, they will be called theoretical. The comparison of theoretical, experimental and simulated via *"Hellweg 2D"* results is presented in Table 1.

Table 1: Output Beam Parameters of U-1-M

Parameter	Hellweg 2D	Theory	Experiment
Output Energy, MeV	2.64	3.05	2.61
Beam Current, mA		100	
Particles Captured, %	86	95	90
Energy Spread, %	31	-	~20
Phase Length, deg	4.82	-	~6



a) 1 - experimental,
2 - theoretical



b) Hellweg 2D

Figure 3: Output Energy Spectrum

It is necessary to mention that the developers define the output energy not as an average value but as energy of the spectrum peak. All parameters are in a good accordance with experimental, except for the energy spread. This can happen because the spectrum is very rough and its width is very sensitive to any computational errors or because of a large measurement error. Fig.3 presents the energy spectrum at the end of accelerator.

FUTURE DEVELOPMENT

The following improvements to the code are planned:

- Adding the reference data for the cell types different from DLS will allow user to synthesize an accelerator based on the other structures. The possibility to add the reference data for any user defined structure is also panned.
- Development and implementation of the methods used for backward wave accelerators simulations.
- More robust functions for buncher optimization and synthesis.
- Improved coupler cell model and realistic field distribution inside drift tubes

CONCLUSIONS

"Hellweg 2D", a new code for electron beam dynamics simulations in TW accelerating structures was created. The program is based on the differential equations describing the particle's motion in waveguide sections. This code considers such effects as space charge, beam loading and solenoid focusing.

"Hellweg 2D" is a convenient self-sufficient tool for dynamics calculations which doesn't require pre-calculated RF-fields data. Nevertheless it can be used in conjunction with a particle-in-cell (PIS) based code for results accuracy adjustment The elements of accelerator synthesis make *"Hellweg 2D"* a very comfortable instrument for initial steps of accelerator development process as it doesn't require the user's presence during the optimisation.

The results provided with this code are in good agreement with an experimental data.

REFERENCES

[1] E.S.Masunov, Bunching and Accelartion of the Intensive Beam in the Waveguide Accelerating Structure with a Presense of an External Magnetic Field, Journal of Technical Physics, vol.49, #7, 1979, pp.1462-1463 (in Russian).

[2] K.R.Crandall, D.P. Rushtoi, Trace-3D Documentation, Third Ed., Los-Alamos National Laboratory, New Mexico, 1997

[3] O.A.Valdner, N.P.Sobenin, B.V.Zverev et al., Corrugated Waveguides, Reference Book, Third Ed., Energoatomizdat, Moscow, 1991 (in Russian)

[4] O.A.Valdner, E.G.Pyatnov, in: G.A.Tyagunov, Accelerators, vol.3, Gosatomizdat, Moscow, 1962, pp.121-133 (in Russian).