WAKE FIELD ANALYSIS BY TIME DOMAIN BEM WITH INITIAL VALUE PROBLEM FORMULATION

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Abstract

This paper presents a new formulation of the Time Domain Boundary Element Method (TDBEM), an initial value problem formulation, for particle accelerator wake fields analysis. Conventional formulation of the TDBEM had some difficulties in practical uses, instabilities caused by interior resonance, larger memory requirement, initial value setting, etc. To solve these problems, the initial value problem formulation is introduced here. In addition, it will be shown that the initial value problem formulation will be applied to 4D domain decomposition method to reduce required memory.

INTRODUCTION

A Time Domain Boundary Element Method (TDBEM) has advantages of grid dispersion free property, treatment of electron bunch with curved trajectories, etc. in particle accelerator wake field analysis [1,2]. On the other hand, the TDBEM has also serious problems of heavy calculation cost, large required memory and initial value setting which were main reasons why the TDBEM can not be widely used yet in the wake field analysis. For the large memory problem, the scattered field formulation and the moving window scheme were introduced into the TDBEM and it was shown that the TDBEM can be applied to very long accelerator structures [3-5].

This paper presents a new formulation of the TDBEM, an initial value problem formulation. To use the initial value problem formulation of the TDBEM, a new type of moving window scheme, which can be applied to curved trajectory or electron motion with smaller velocity than the speed of light, will be introduced.

FORMULATION OF TDBEM

To clarify purpose of the new formulation, the conventional formulations of the TDBEM are summarized at the first part, and after that, the initial value formulation is introduced in this section.

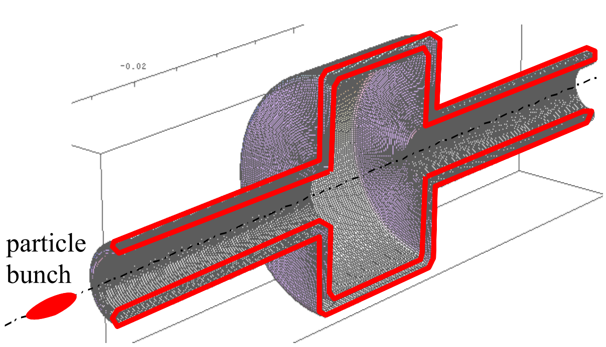

Figure 1: Numerical model of conventional TDBEM

Conventional TDBEM

The conventional TDBEM was formulated based on the following time domain magnetic field integral equation (MFIE) ,

$$\mathbf{B}(ct,\mathbf{x}) = \mathbf{B}_{ext}(ct,\mathbf{x}) + \frac{1}{4\pi}\int_S \left(\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t'} \right) \times (\mathbf{n}\times\mathbf{B}(t_{ret},\mathbf{x}')) \right) dS' \qquad (1)$$

where $\mathbf{n}$ is a unit normal vector of boundaries, t_{ret} is retarded time and $\mathbf{B}_{ext}$ is external fields produced by the particle bunch. Then the all boundaries are assumed to be perfect electric conductor (PEC). In this formulation, application of this scheme to particle accelerators faces to difficulty in initial value setting at infinitely long accelerator tube because numerical domain have to be finite size. To avoid this difficulty, wake field phenomena are formulated as open boundary problems as in Fig.1, that is, the initial value can be taken to be zero if the particle bunch starts the motion at upstream outside of the accelerator structure. However the torus shape and thin layer numerical model of Fig.1 results in bad condition number of the matrix and often induces numerical instability caused by interior resonance [3]. On the other hand, another solution to the difficulty of the initial value setting was the scattered field formulation of the TDBEM (S-TDBEM) [3],

$$\mathbf{B}_{scat}(ct,\mathbf{x}) = \frac{1}{4\pi}\int_S \frac{\dot{\mathbf{E}}_{scat}(t_{ret},\mathbf{x}')\times\mathbf{n}}{c^2|\mathbf{x}-\mathbf{x}'|^2}dS' + \frac{1}{4\pi}\int_S \left(\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t'} \right) \times (\mathbf{n}\times\mathbf{B}_{scat}(t_{ret},\mathbf{x}')) \right) dS' \qquad (2)$$

To adopt only the scattered field components $\mathbf{B}_{scat}$ as unknown value, the initial value at the upstream uniform

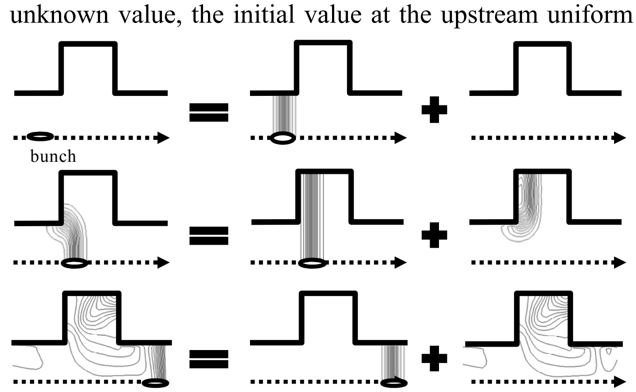

Figure 2: Scattered field component of total em fields

tube section can be taken to be zero if the particle bunch travels on the axis. (see Fig.2) In the scattered field formulation, the matrix condition number is improved and there are no instabilities caused by interior resonance, and in addition, so-called moving window scheme can be used since time domain simulation is performed by the explicit scheme. However the S-TDBEM can be used only for the case that the initial position of the particle bunch is located on axis in the uniform accelerator tube, and there were no way to avoid reflection of backscattering wave from the cavity at the upstream tube end.

Initial value problem formulation of TDBEM

In the conventional TDBEM, all initial values in the time domain integral equation were set to zero. To include the influences of the initial value in time domain simulation (see Fig.3), the integral equation (1) is expanded to more generalized formulation as follows,

$$\begin{aligned}\mathbf{B}(ct,\mathbf{x}) &= \mathbf{B}_{ext}(ct,\mathbf{x}) + \frac{1}{4\pi}\int_{V_0}\left(\frac{\dot{\mathbf{B}}}{c}r' + \frac{\partial\mathbf{B}}{\partial r'}r' + \mathbf{B}\right)do' \\ &+ \frac{1}{4\pi}\int_S\left(\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t'}\right)\times(\mathbf{n}\times\mathbf{B}(t_{ret},\mathbf{x}'))\right)dS' \\ &- \frac{1}{4\pi}\int_S\left(\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t'}\right)(\mathbf{n}\cdot\mathbf{B}(t_{ret},\mathbf{x}'))\right)dS' \\ &+ \frac{1}{4\pi}\int_S\frac{\dot{\mathbf{E}}(t_{ret},\mathbf{x}')\times\mathbf{n}}{c^2|\mathbf{x}-\mathbf{x}'|^2}dS'\end{aligned} \tag{3}$$

$$\begin{aligned}\mathbf{E}(ct,\mathbf{x}) &= \mathbf{E}_{ext}(ct,\mathbf{x}) + \frac{1}{4\pi}\int_{V_0}\left(\frac{\dot{\mathbf{E}}}{c}r' + \frac{\partial\mathbf{E}}{\partial r'}r' + \mathbf{E}\right)do' \\ &+ \frac{1}{4\pi}\int_S\frac{\mathbf{n}\times\dot{\mathbf{B}}(t_{ret},\mathbf{x}')}{c^2|\mathbf{x}-\mathbf{x}'|^2}dS' \\ &- \frac{1}{4\pi}\int_S\left(\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t'}\right)(\mathbf{E}(t_{ret},\mathbf{x}')\cdot\mathbf{n})\right)dS' \\ &- \frac{1}{4\pi}\int_S\left(\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t'}\right)\times(\mathbf{E}(t_{ret},\mathbf{x}')\nu\mathbf{n})\right)dS'\end{aligned} \tag{4}$$

In this formulation, the initial field volume integral term (second terms of the right hand side of (3) and (4)) and the surface boundary integrals for non PEC boundaries are additionally included comparing with the conventional equation (1). In addition, the electric field integral equation (EFIE) is also used in the time domain simulation. Although the formulation becomes more complicated than the conventional one (1), the generalized integral equations (3) and (4) are applied to various kind problems. One of useful applications of the initial value problem formulation is 4D domain decomposition. To use this formulation, we can start the time domain simulation at any timing, that is, the time domain simulation can be decomposed in time and space, and each sub-domain calculation can be downsized. (see Fig.4)

NUMERICAL EXAMPLES

In the wake field analysis, the 4D domain decomposition is modified to a moving window technique. To use the moving window technique, long accelerator structure can be divided into small parts which can be calculated by smaller memory. A numerical example of 2D cross section of 7 cell cavities model is shown in Fig.5. In this numerical model, upstream and downstream surfaces are assumed to be absorbing boundary, and the cavity wall is PEC. Then the bunch size is taken to be 1cm. The figure 6 shows time domain behaviour of surface current density induced by the particle bunch. The dotted line in Fig.6 indicates each sub-domain calculation portion. To avoid singularity in the fields which is caused by particle bunch motion across the sub-domain boundary, the sub-domains have overlapped regions each others. We can confirm that sub-domain calculations are smoothly connected each others in the 4D domain decomposition method.

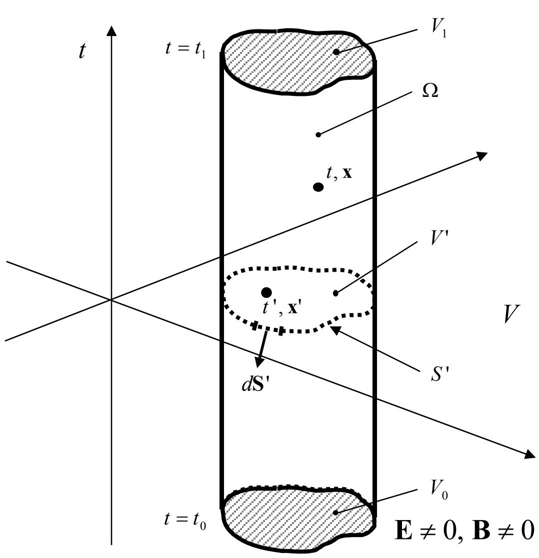

Figure 3: Finite 4D domain for time domain integral eq.

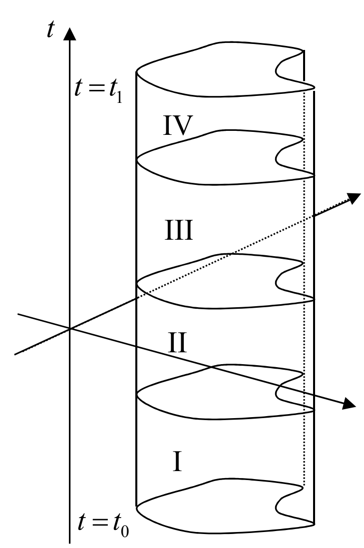

Figure 4: 4D domain decomposition

SUMMARY

This paper has presented the initial value problem formulation of the TDBEM as an expansion of the conventional TDBEM to more general cases. This formulation enables us to adopt 4D domain decomposition method in the time domain simulation. In particle accelerator wake field analysis, the 4D domain decomposition can be used as a new type of moving window techniques. Especially this moving window technique can be applied to curved trajectories combining with 3D formulation.

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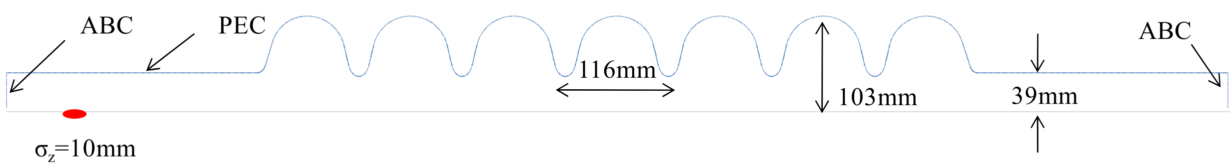

Figure 5: Cross section of 7 cell cavities

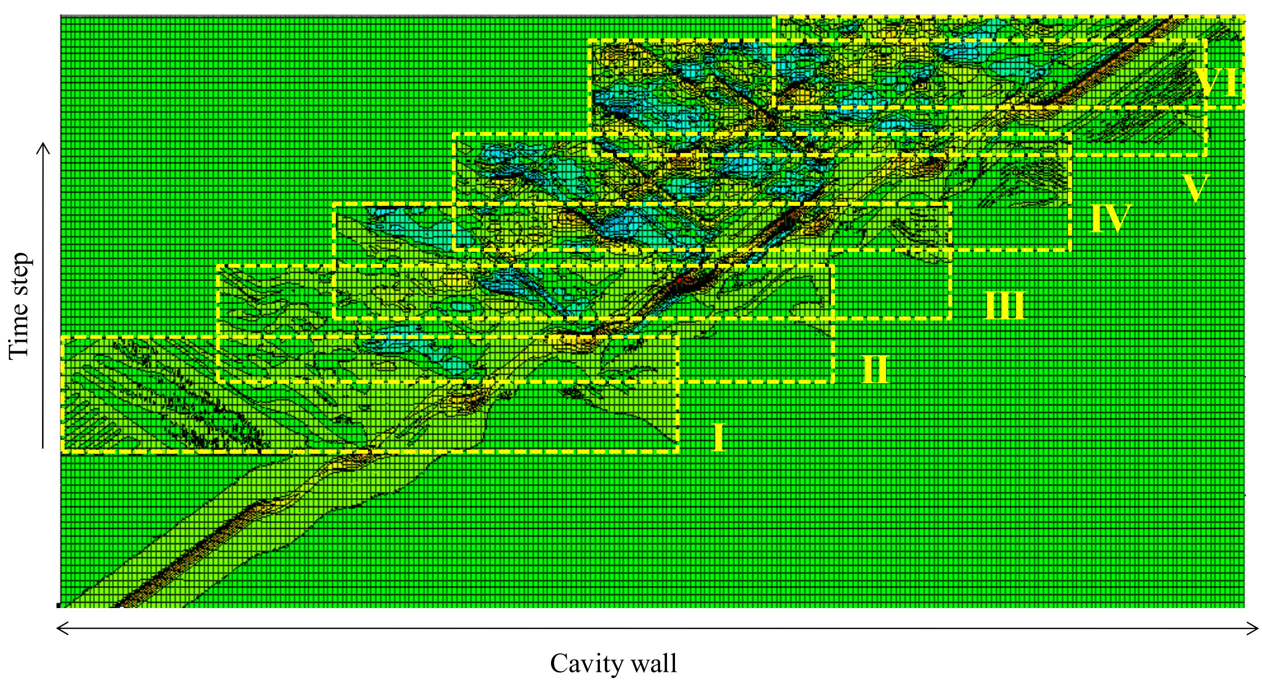

Figure 6: Time domain behaviour of induced surface charge and calculation sub-domains