OPTIMAL TWISS PARAMETERS FOR TOP OFF INJECTION IN A SYNCHROTRON LIGHT SOURCE*

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Abstract

Injection into a storage ring requires that the injected beam be optimally matched to the storage ring lattice. For on axis injection this requires that the twiss functions of the transfer line match the twiss functions of the lattice. When injecting off axis, as is done in light sources for top off injection, the goal is to use the minimum phase space area in the storage ring. A. Streun has given an analytical method to compute the twiss functions for top off injection into the SLS where injection occurs at a beam waist. [1] We have extended his theory to include cases where there is no beam waist. A simple analytical formula is not possible in this case, however we give an algorithm to compute the twiss parameters of the injected beam given the storage ring lattice. We also compute the twiss functions for a variety of cases for the NSLS-II storage ring.

INTRODUCTION

Emitance growth during injection into a storage ring is a result of many factors. Among them is improper matching of the twiss functions between the transfer line and the storage ring. In the case of on axis injection this mismatch of twiss functions causes the phase ellipse of the injected beam to precess about the ellipse of the storage ring lattice leading to filamentation, emittance growth and possibly beam loss.

Modern synchrotron light sources operate in top off mode. In this mode the injected beam is injected off axis next to the circulating beam. The injected beam filaments and damps into the stored beam. The amount of phase space available to the injected beam to filament is often limited by the dynamic and physical apertures of the storage ring. In this case, equating the twiss functions between the transfer line and the storage ring may not be the optimal solution. However, the idea of minimizing the phase space needed by the injected beam remains. This was explored by Andreas Steun for the SLS where injection occurs at a beam waist.[1] In cases where the beam is injected at other locations or where a pulsed multipole is used for injection, the effects of having a non-zero α function for the injected and the stored beam need to be explored.[2]

In this paper we explore the effect of non-zero α when injecting off axis. First we develop a method for computing the necessary twiss functions for the injected beam in terms of the storage ring twiss functions and the injected beam location and angle. Then we discuss the optimal twiss functions for NSLS-II.

THEORY

The equation for the phase ellipse of a particle is

$$A = \frac{1}{\beta_s}[x^2 + (\alpha_s x + \beta_s x')^2] \qquad (1)$$

where x and x' are the coordinates in phase space, β_s and α_s are the twiss functions of the storage ring and A is the area of the ellipse divided by π. The coordinates of a particle on the "outermost" ellipse of the injected beam can be expressed as

$$x = x_0 + N_i\sqrt{\varepsilon_i\beta_i} \cos\varphi \qquad (2a)$$

$$x' = x_0' + N_i\sqrt{\frac{\varepsilon_i}{\beta_i}}(\sin\varphi + \alpha_i\cos\varphi) \qquad (2b)$$

where, β_i and α_i are the twiss functions of the injected beam, x_0 and x_0' are the central phase space coordinates of the injected beam relative to the stored beam, ε_i is the rms emittance, N_i is the number of rms beam sizes one wishes to consider and φ is the angle on the injected beam ellipse. This is shown in Figure 1.

The location where the acceptance is minimized is usually the end of the injection septum. However, in the case of pulsed multipole injection, a location after the multipole should be used since the multipole affects the twiss function of the injected beam and possibly the stored beam.
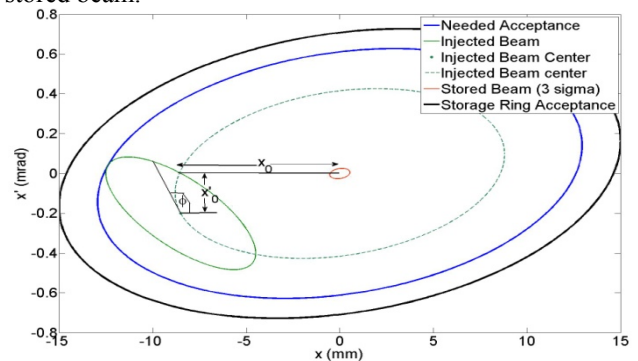

Figure 1: Phase space diagram showing an unoptimized phase space for injection.

The equation of the storage ring ellipse for a particle on the exterior of the injected beam is given by inserting Equations 2 into Equation 1:

$$A = \frac{[x_0^2 + (\alpha_s x_0 + \beta_s x_0')^2]}{\beta_s} + \frac{N_i^2\varepsilon_i\beta_s}{\beta_i}$$

$$-2N_i\sqrt{\frac{\varepsilon_i}{\beta_i}}(\alpha_s x_0 + \beta_s x_0')\sin\varphi$$

$$+\frac{N_i^2\varepsilon_i}{\beta_i}(\alpha_i\beta_s - \alpha_s\beta_i)\sin 2\varphi$$

$$+2N_i\sqrt{\varepsilon_i\beta_i}\left[\frac{1+\alpha_s^2}{\beta_s}x_0 + \alpha_s x_0' \right.$$
$$\left. -\frac{\alpha_i}{\beta_i}(\alpha_s x_0 + \beta_s x_0')\right]\cos\varphi$$
$$+N_i^2\varepsilon_i\left[\frac{1+\alpha_s^2}{\beta_s}\beta_i - \frac{1+\alpha_i^2}{\beta_i}\beta_s - 2\alpha_s\alpha_i\right]\cos^2\varphi$$

$$(3)$$

Each value of φ in equation 3 corresponds to an ellipse in the storage ring that an injected particle will precess around. The goal is to find the value of φ that returns the ellipse with the largest area and then minimize that area with respect to the injected beam twiss functions, injected beam position and injected beam angle. Taking the derivative of Equation 3 with respect to φ and setting it to zero give up to four solutions for φ mod 2π. One of these solutions is for the phase ellipse that encompasses the whole injected beam, which is the maximum area. The analytical solution is rather long and it is easier to solve for the phase ellipse numerically. Once the area has been determined, the area of the phase ellipse can be minimized. In the case of top off injection, the position and possibly angle is fixed by the geometry of the injection straight. This still allows one of optimize the twiss functions.

In the general case, it is easiest to do this procedure numerically. However, we now examine some specific cases where analytical results can be derived.

On Axis Injection

If one briefly considers on axis injection, then $x_0=x_0'=0$. The first and fourth terms in Equation 3 vanish. One can see by inspection that the remaining trigonometric terms are greatly simplified if $\beta_i=\beta_s=\beta$ and $\alpha_i=\alpha_s=\alpha$ giving

$$A = N_i^2\varepsilon_i(1 - 2\alpha^2\cos^2\varphi) \qquad (4)$$

Taking the derivative with respect to φ and setting to zero gives

$$\varphi = \frac{n\pi}{2}$$

$$(5)$$

where n is an integer. The maximum area occurs when cos φ=0 which give the expected result

$$A = N_i^2\varepsilon_i \qquad (6)$$

Off Axis Injection

When one considers off axis injection, the first term of Equation 3 is the area swept out by the center of the injected beam. Figure 1 shows that this can be a significant portion of the phase space that the injected beam sweeps out, so it is desirable to minimize this term. If the injected beam position is fixed with respect to the stored beam, then the injected beam should have an angle of

$$x'_0 = -x_0\frac{\alpha_s}{\beta_s}$$

$$(7)$$

with respect to the stored beam. Equation 7 can be inserted into Equation 3 to give

$$A = \frac{x_0^2}{\beta_s} + \frac{N_i^2\varepsilon_i\beta_s}{\beta_i} + \frac{N_i^2\varepsilon_i}{\beta_i}(\alpha_i\beta_s - \alpha_s\beta_i)\sin 2\varphi$$
$$+N_i^2\varepsilon_i\left[\frac{1+\alpha_s^2}{\beta_s}\beta_i - \frac{1+\alpha_i^2}{\beta_i}\beta_s - 2\alpha_s\alpha_i\right]\cos^2\varphi$$
$$+2\frac{x_0}{\beta_s}N_i\sqrt{\varepsilon_i\beta_i}\cos\varphi$$

$$(8)$$

If the injection point is a stored beam waist, then α_s is zero. Equation 7 then states that the injected beam should be parallel to the stored beam. By inspection of Equation 8, one can surmise that α_i should also be zero. Then equation 8 becomes

$$A = \frac{x_0^2}{\beta_s} + \frac{N_i^2\varepsilon_i\beta_s}{\beta_i} + 2\frac{x_0}{\beta_s}N_i\sqrt{\varepsilon_i\beta_i}\cos\varphi$$
$$+N_i^2\varepsilon_i\left[\frac{\beta_i}{\beta_s} - \frac{\beta_s}{\beta_i}\right]\cos^2\varphi$$

$$(9)$$

Equation 9 can be solved analytically for cos φ. First we introduce the following notation similar to Reference 1.

$$a = N_i^2\varepsilon_i$$

$$(10a)$$

$$b = \sqrt{\frac{\beta_i}{\beta_s}}$$

$$(10b)$$

$$\delta_0 = \frac{x_0}{N_i\sqrt{\varepsilon_i\beta_s}} = \frac{x_0}{\sqrt{a\beta_s}}$$

$$(10c)$$

We note that our definition of δ_0 is different than that in Reference 1. In their case, the objective was to move the beam centroid as close to the septum as possible, which leads to a slightly different definition of δ_0 and subsequently a different answer to the optimal b. In our case, we wished to keep the injected beam position constant, which would be necessary for a pulsed multipole.

Using the above definitions, Equation 9 becomes

$$A = \frac{a}{b^2}\left[1 + \delta_0^2 b^2 + 2\delta_0 b^3\cos\varphi - (1 - b^4)\cos^2\varphi\right]$$

$$(11)$$

Taking the derivative with respect to φ gives the following solution

$$\cos\varphi = \frac{\delta_0 b^3}{1 - b^4}$$

$$(12)$$

which provides the equation for the phase ellipse that encompasses the beam

$$A = \frac{a}{b^2}\frac{1 + \delta_0^2 b^2 - b^4}{1 - b^4}$$

$$(13)$$

To minimize this ellipse we take the derivative with respect to b

$$b^8 - 2\delta_0^2 b^6 - 2b^4 + 1 = 0 \qquad (14)$$

This can be rewritten in terms of cos φ using Equation 12 and gives the solution

$$\cos\varphi = \frac{1}{\sqrt{2}}$$

(15)

This can be reinserted into Equation 12 to give the equation for the optimal b

$$b_{opt}^{4} + \sqrt{2}|\delta_0|b_{opt}^{3} - 1 = 0 \qquad (16)$$

which can be solved numerically or as a quartic equation.

NSLS-II TOP OFF PARAMETERS

The NSLS-II injection system has been described in a number of publications.[3] It consists of a 200 MeV linac and a 3 GeV booster synchrotron. Top off injections are to consist of 80-150 bunches containing a total charge of 7.3nC every minute. The beam emittance from the booster is expected to be approximately 50 nm. The storage ring emittance will be approximately 1nm. The injection straight for the NSLS-II has been described in Reference 4.

Injection occurs in the horizontal plane. 4 bumps move the stored beam over 17 mm toward the septum. The injected beam will enter the septum 9.5 mm away from the bumped beam orbit parallel to the stored beam. The dynamic aperture of the storage ring is to be 15 mm in the horizontal plane at this point to allow for sufficient injection efficiency.

Table 1: Twiss functions for the storage ring and the injected beam at the end of the storage ring injection straight.

Storage Ring Parameter	Value
β_v	9.8 m
α_v	-1.4
β_s	21.4 m
α_s	-0.23
Horizontal Dynamic Aperture	15 mm
Injected Beam Parameter	
x_0	-9.5mm
x_0'	0 mrad (fixed)
β_i	10.7 m
α_i	-0.58
Minimum Required Horizontal Dynamic Aperture	12.5 mm

The storage ring twiss functions are shown in Table 1. The location we chose is the end of the injection straight section. The reason is that we had previously considered using a pulsed sextupole for injection and this would have been placed at the end of the injection straight.[4] The horizontal twiss functions were calculated numerically using MATLAB. The MATLAB code was tested against equations 6, 7, 13, and 16, where it gave identical results.

The injected beam vertical twiss functions are identical to the storage ring vertical twiss functions. The horizontal twiss functions are shown in Table 1. The resulting phase space is shown in Figure 3. We chose to have a $3\sigma_x$ clearance on the injected beam and to inject

the beam with no angle relative to the stored beam. Figure 3 shows that there is sufficient phase space to inject with 15 mm dynamic aperture.
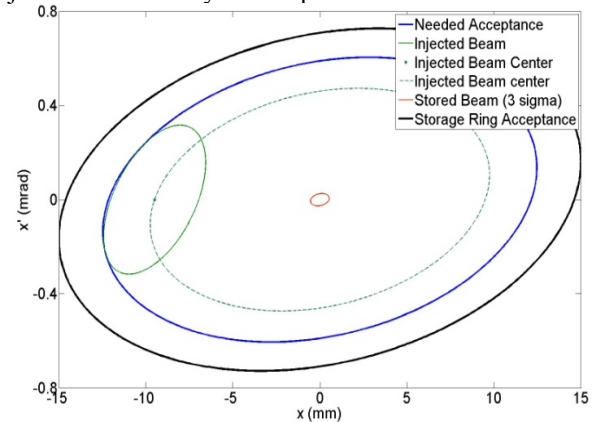

Figure 3: Phase space diagram optimized with the parameters in Table 1.

CONCLUSION

We have extended the formalism of A. Streun in determining the optimal twiss functions for injection into a storage ring. This formalism includes the effects of off axis and off angle injection as well as non-zero alpha for the storage ring and injected beam. We have used this formalism to compute the optimal twiss functions for the NSLS-II storage ring during top-off operation.

This formalism does fail to account for non-linear beam optics on the storage ring which distort the phase space at the injection point. In this case simulations will be necessary to account for the distortion.

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