MICROBUNCHING AND RF COMPRESSION*

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Abstract

Velocity bunching (or RF compression) represents a promising technique complementary to magnetic compression to achieve the high peak current required in the linac drivers for FELs. Here we report on recent progress aimed at characterizing the RF compression from the point of view of the microbunching instability. We emphasize the development of a linear theory for the gain function of the instability and its validation against macroparticle simulations that represents a useful tool in the evaluation of the compression schemes for FEL sources.

INTRODUCTION

The velocity bunching technique to compress the beam for FEL applications [1] represents an appealing alternative to magnetic chicanes due to its immunity to radiation effects and to the possibility of achieving the compression without emittance degradation.

In this paper we present a linear theory for the gain function of small amplitude density perturbations in the longitudinal phase space through an rf compressor. We develop a method similar to that used for the study of the microbunching instability in magnetic compressors [2]. We first use the linear approximation for the single-particle motion of an electron in the rf structure, which limits the model to low or moderate compression factors (up to a factor 4). We also adopt a 1D model for the space charge impedance describing collective effects, meaning that at small wavelenghts and low energy the theory may not work properly [3]. Finally we assume a coasting beam approximation thus limiting the study to beam dynamics in the longitudinal core of the physical electron bunches.

We conclude with comparisons against macroparticle simulations showing a reasonable agreement over a wide spectrum of perturbation wavelengths.

BEAM DYNAMICS THROUGH AN RF COMPRESSOR

We start by writing the longitudinal motion of an electron in a travelling wave rf structure by means of the Hamiltonian $H = \sqrt{m^2c^4 + p_s^2c^2} - e\phi(s,t)$ where $-e$ is the electron charge, s the longitudinal coordinate, and $\phi = (E_0/k_{rf}) \cos(k_{rf}s - \omega_{rf}t + \psi_0)$ the electric potential, yielding the longitudinal electric field $E_s = -\partial\phi/\partial s = E_0 \sin(k_{rf}s - \omega_{rf}t + \psi_0)$. For simplicity, in the following we assume a phase velocity $\omega_{rf}/k_{rf} = c$ for the travelling wave.

The canonical equations of motion for a single particle can be written in terms of the space separation $\Delta z = z - z_r$

between the electron and the reference particle, and $\Delta\gamma = \gamma - \gamma_r$ with γ_r the reference particle relativistic factor [4]

$$\frac{d\Delta z}{ds} = \frac{\gamma_r}{\sqrt{\gamma_r^2 - 1}} - \frac{\gamma_r + \Delta\gamma}{\sqrt{(\gamma_r + \Delta\gamma)^2 - 1}}$$

$$\frac{d\Delta\gamma}{ds} = \alpha k_{rf}\left[\sin\left(k_{rf}s - \omega_{rf}t_t + \psi_0\right) - \sin\left(k_{rf}s - \omega_{rf}t_t + k_{rf}\Delta z + \psi_0\right)\right] \quad (1)$$

with $\alpha = eE_0/(mc^2k_{rf})$

The first-order solution of the above linear system is expressed in terms of the transfer matrix $\mathbf{M}$: $\mathbf{x}(s) = \mathbf{M}\, \mathbf{x}_0$, where $\mathbf{x}(s) = (\Delta z(s), \Delta\gamma(s))$ and $\mathbf{x}_0 = \mathbf{x}(s_0)$, with matrix $\mathbf{M}$ obeying $d\mathbf{M}/ds = \mathbf{A}\mathbf{M}$, with initial condition $\mathbf{M}(s_0) = 1$ and matrix $\mathbf{A}$ defined by

$$\mathbf{A}(s) = \begin{pmatrix} 0 & \left[\gamma_r^2 - 1\right]^{-3/2} \\ -\alpha k_{rf}^2 \cos\left(k_{rf}s - \omega_{rf}t_r(s) + \psi_0\right) & 0 \end{pmatrix} \quad (2)$$

We assume a beam distribution at the entrance of the rf compressor $s = s_0$ consisting of a zero-order smooth density, uniform in Δz and Gaussian in $\Delta\gamma$ with a chirp h_0

$$f_0\left(\mathbf{x}_0; s_0\right) = \frac{1}{\sqrt{2\pi}\sigma_\gamma} e^{-(\Delta\gamma_0 - h_0\Delta z_0)^2 / 2\sigma_\gamma^2} \quad (3)$$

and a first-order perturbation $f_1(\mathbf{x}_0; s_0)$. The normalization of the distribution function is chosen so that $n_0 d\Delta z\int d\Delta\gamma_0 f(\mathbf{x}; s)$ represents the number of particles in the interval $d\Delta z$, where n_0 is the beam line density.

The beam density function $f_0(\mathbf{x}_s; s)$ at s is related to the beam density $f_0(\mathbf{x}_0; s_0)$ at s_0 by $f(\mathbf{x}_s, s) = f(\mathbf{M}^{-1}\mathbf{x}_s; s_0)$. The normalized charge density evolves from $\rho(\Delta z_0; s_0) = \int d\gamma_0 f_0(\mathbf{x}_0; s_0) = 1$ at $s = s_0$ to

$$\rho(\Delta z_s; s) = \int f_0(\mathbf{x}_s; s)d\Delta\gamma_s = \frac{1}{|M_{11}(s) + h_0 M_{12}(s)|} = C(s) \quad (4)$$

The last equality in the above equation identifies the compression factor $C = C(s)$.

The effect of collective forces is to change the particle energy according to

$$\frac{d\Delta\gamma}{ds} = F(\tilde{\rho}, \Delta z_s; s) = -\frac{4\pi I_0}{I_A Z_0} \int dk e^{ik\Delta z_s} Z(k; s)\tilde{\rho}(k; s) \quad (5)$$

where $I_0 = ecn_0$ is the electron beam current, $I_A = ce/r_e \simeq 17kA$ the Alfvén current, Z_0 the vacuum impedance, and $\tilde{\rho}(k; s)$ the Fourier transform of the normalized charge density at s.

Since collective effects in this case are largely dominated by space charge, we model them by means of an effective impedance [5] relating the longitudinal

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component of the electric field and longitudinal charge-density fluctuations in the frequency domain. In our model we neglect the possible dependence of the longitudinal component of the electric field on the radial coordinate and adopt a 1D expression for the space-charge impedance of the form [6]

$$Z(k,s) = \frac{iZ_0}{\pi\gamma r_b} \frac{1 - 2I_1(\xi)K_1(\xi)}{\xi}\Bigg|_{\xi = kr_b/\gamma} \quad (6)$$

where $Z_0 = 120\pi$ is the vacuum impedance, and I_1 and K_1 are the modified Bessel functions of first and second kind.

Expression (6) is obtained from transverse averaging of the longitudinal component of the electric filed of an infinitely long beam with circular cross-section of radius r_b (and uniform transverse density) perturbed by a small longitudinal modulation of wavenumber k.

The starting point of our analysis is the linearized Vlasov equation expressed in the integral form [2]

$$f_1(\mathbf{x}_s;s) = f_1(\mathbf{x}_0;s_0) - \int_{s_0}^s ds' F(\tilde{\rho}_1; \Delta z_{s'};s') \frac{\partial f_0(\mathbf{x}_s;s')}{\partial \Delta\gamma_{s'}} \quad (7)$$

with the collective force F depending on the Fourier transform of the first-order density perturbation.

Starting from (7), after some mathematical manipulations and assuming an initial perturbation consisting of a sinusoidal perturbation to the charge density of the kind

$$f_1(\mathbf{x}_0;s_0) = Ae^{ik_0\Delta z_0} \frac{e^{-(\Delta\gamma_0 - h_0\Delta z_0)^2/2\sigma_\gamma^2}}{\sqrt{2\pi}\sigma_\gamma} \quad (8)$$

we derive the Fourier components of the first-order density perturbation

$$\tilde{\rho}_1(k;s) = e^{-[C(s)k_0M_{12}\sigma_\gamma]^2/2} A\delta(k_0 - k/C(s)) + \\ + \int_{s_0}^s ds' \hat{K}(s',s)\tilde{\rho}_1(k';s') \quad (9)$$

with kernel

$$\hat{K}(s',s) = 4\pi i \frac{I(s')}{I_A} kM_{12}(s',s) \\ \frac{Z(k',s')}{Z_0} e^{-[k'M_{12}(s') - kM_{12}(s)]^2\sigma_\gamma^2/2}\Bigg|_{k' = kC(s')/C(s)} \quad (10)$$

where $I(s') = I_0C(s')$ is the beam current at s' and $M_{12}(s',s)$ is the matrix advancing from s' to s. The solutions of eq. (9) have the form

$$\tilde{\rho}_1(k;s) = b(k;s)C(s)\delta(C(s)k_0 - k) \quad (11)$$

with the function b(k;s) obeying the equation

$$b(C(s)k_0;s) = Ae^{-[C(s)k_0M_{12}\sigma_\gamma]^2/2} + \\ + \int_{s_0}^s ds' \hat{K}(s',s)b(C(s')k_0;s') \quad (12)$$

with kernel K(s',s) obtained from (10) with the substitution k'→k₀C(s'). Eq. (12) turns out to be formally identical to the equation describing bunching in magnetic compressors [2]. The linear gain is defined as the ratio of the amplitude of the perturbation at s to the initial one

$$g(k_0;s) = |b(C(s)k_0;s)/A| \quad (13)$$

Because of space charge, an initial density modulation will induce energy modulation along the beam. By following the same methods used to determine the linear gain, we obtain also the amplitude of the energy modulation in the form

$$\Delta\tilde{\gamma} = \frac{4\pi I(s')}{Z_0 I_A} \delta(k - k_0 C(s))C(s) \int_{s_0}^s ds' b(k';s') \times \\ Z(k',s')e^{-[k'M_{12}(s',s)\sigma_\gamma]^2}\left\{-[k'M_{12}(s',s)\sigma_\gamma]^2\right\}_{k'=k_0C(s')} \quad (14)$$

VALIDATION AGAINST MACROPARTICLE SIMULATIONS

The numerical model

As a way to validate the model presented in the previous section, we carried out macroparticle simulations using the code TSTEP [7], a derivative of PARMELA [8]. We considered the evolution of a 1 nC ideal transversely uniform, flat beam spanning a 10° RF phase at 2856 MHz (corresponding to a length of ~3 mm and a peak current of 100A) in a range of density modulation wavelengths between 50 and 300µm with an initial amplitude of the sinusoidal perturbation of ±10% and no energy modulation. The beam starting with an energy of 5.6 MeV (the typical output energy with a field 120 MV/m for 1.6 cells RF gun used in some FEL facilities such as LCLS or SPARC) with vanishing initial energy-phase correlation and uncorrelated energy spread, is transported in a beamline consisting in a 0.6 m long drift followed by a SLAC-type 3m long TW section used as a RF compressor (RFC). The drift length has been chosen to correspond to roughly a quarter of wavelength of longitudinal plasma oscillation in order to maximize the amplitude modulation at the exit of the compressor. The compression is controlled by moving the linac RF phase away from the crest toward the zero crossing of the RF field. To highlight the importance of a full account of space-charge effects we compare in fig.1 the gain obtained turning on and off the collective forces in the RFC (while they are included through the leading drift in both cases) for $\psi_0 = -82°$ (C=2),[ψ_0 differs by 90° and sign from eq. (1)].
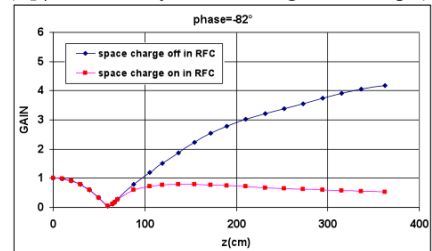

Figure 1 Gain vs z for $\psi_0 = -82°$ with space charge on and off in the RFC.

Comparisons

As the beam size variation affects in a sensitive way the results, a high order polynomial interpolation for the

computed rms transverse size as a function of z was introduced in the model. It was found that the best agreement between the analytical model and simulations is obtained when in the expression of the impedance we use the relationship $r_b = a \cdot \sigma_x$ with $a = 1.95 - 0.001\lambda_m[\mu m]$ instead of $a = 2$ (uniform beam). The good agreement between simulations and theory including the empirical adjusted factor a is shown in fig.2.
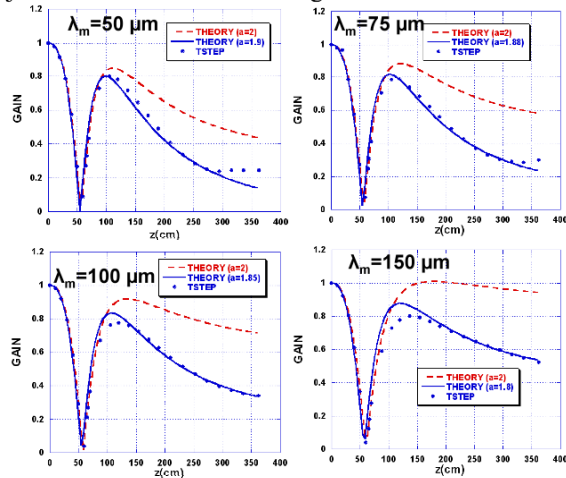

Figure 2: Comparison between TSTEP simulations and the linear theory for $\psi_0 = -82°$ rf phase at four different initial modulation wavelengths. The dashed line and solid lines were obtained by setting $a = 2$ and $a = 1.95 - 0.001\lambda[\mu m]$ respectively in the linear theory.

Further simulation-vs-theory comparisons are reported in fig. 3 showing the linear gain at the exit of RF compressor over a range of perturbation wavelengths for a uncompressed ($\psi_0 = 0°$) and compressed ($\psi_0 = -82°$) beam.
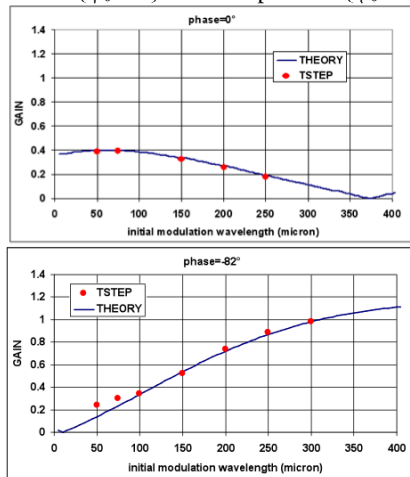

Figure 3 Comparison between TSTEP and linear theory for the linear gain at the RFC output over a spectrum of perturbation wavelengths; (top) C=1, (bottom) C=2.

Finally we report in fig. 4 evolution of the energy modulation amplitude for an initial modulation wavelength of 75 μm. It was retrieved from the simulation data by first removing the correlation phase-energy in a window selected around the beam core in the longitudinal phase space and then carrying out a sinusoidal fit.
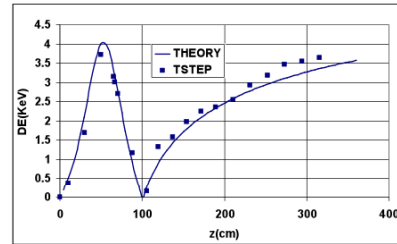

Figure 4: Evolution of the energy modulation amplitude.

CONCLUSIONS

The model presented here indicates that for parameters of interest in FEL applications, the amplification of small density perturbations through an RF compressor tends to be quite modest (relative to the peak current) if not outright smaller than unity (i.e. implying damping of the initial perturbation) even in the absence of any uncorrelated energy-spread induced mixing. This does not imply that the dynamics of small-amplitude density perturbations in the rf compressor should be neglected altogether as these perturbations can seed an instability downstream, if further compression by magnetic chicanes is applied. Indeed, a scenario in which rf compression is supplemented by magnetic compression represents the most likely mode of operation envisioned for the accelerator drivers of FEL-based 4th generation light sources. We expect that the theory elaborated in this paper will represent a useful tool in the evaluation of the compression schemes for these FEL sources.

ACKNOWLEDGMENTS

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