THE TRANSVERSE LINAC OPTICS DESIGN IN MULTI-PASS ERL[*]

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Abstract

In this paper, we analyzed the linac optics design requirement for a multi-pass energy recovery linac (ERL) for arbitrary number of linacs. A set of general formula of constrains for the 2-D transverse matrix is derived to ensure design optics acceptance matching throughout the entire accelerating and decelerating process. Meanwhile, the rest free parameters can be adjusted for fulfilling other requirements or optimization purpose. As an example, we design the linac optics for the future MeRHIC (Medium Energy eRHIC) project and show the optimization for small β function.

INTRODUCTION

The uniqueness of Energy Recovery Linac (ERL) attracts the attention of many accelerator physicists in the past decades, because of its ability to provide high current beam with low beam emittance. It is proved to be suitable in many applications such as synchrotron light sources, free electron lasers and electron ion colliders. In contrary to the ring-type accelerators, the electron beam is used once; only the energy of the beam is recovered before the beam dumping. This feature avoids many serious beam dynamics issues in ERL that are common in ring-type accelerators. However, there do exist significant challenges in designing the ERLs, such as the beam break-up instability (BBU) due to the higher order modes in RF cavities and ion trapping in linacs.

In the MeRHIC[1], (Medium Energy eRHIC) project (shown in Figure 1), a 3-pass ERL is proposed to accelerate electron beam to 4 GeV for collision with the ion beam from the existing RHIC ring. In each pass, the electron beam is accelerated by two linacs. Each linac contains 6 cryomodule. And 6 superconducting RF cavities are included in each cryomodule. Between cryomodules, there is 1m-drift space for a quadrupole set to be installed to control the transverse optics function. We present our optics design of the linacs and discuss the optimization to mitigate the beam dynamics challenges.

OPTICAL FUNCTIONS IN LINACS

Without the knowledge of the detail information of the cavity, we only consider the acceleration effect. In high

gradient SRF cavity, the energy change of the electron beam is significant and the adiabatic damping effect cannot be neglected. Consider a linac with length L and the energy gain is δE, an electron beam enters the linac with energy E_0. The transverse transport map of it for the coordinate $(x, p_x = P_x/P_0)$ is not symplectic any more, since the reference particle's momentum P_0 is not constant. Here, we assume the electron velocity is very close to speed of light, which implies $dP/P = dE/E$. The determinant of the map should equal to $E_0/(E_0 + \delta E)$.

A simplest map for the cavity can be built as the energy jump between two L/2 drift space, written as:

$$M = \left(\begin{array}{cc} 1 & L/2 \\ 0 & 1 \end{array} \right) \left(\begin{array}{cc} 1 & 0 \\ 0 & \frac{E_0}{E_0+\delta E} \end{array} \right) \left(\begin{array}{cc} 1 & L/2 \\ 0 & 1 \end{array} \right) \quad (1)$$

A more accurate map can be obtained if we observe the energy change is continuous. The map is directly derived from the solution of the differential equation:

$$\frac{dp_x}{ds} = \frac{k_E}{E_0 + k_E s} \quad (2)$$

where $k_E = dE/ds$ is the energy change slope in Linac. If we define $L_E = E_0/k_E$, the map has a simple form as:

$$M = \left(\begin{array}{cc} 1 & L_E \log \frac{L_E+L}{L_E} \\ 0 & \frac{L}{L+L_E} \end{array} \right) \quad (3)$$

When the L_E approaches infinity, Eq 3 reduces to a map of drift space with length L, which corresponds to the ultra relativistic case so that the energy gain in linac is negligible comparing with the beam energy. Also the equation 3 is the special case of Ref when all details of RF cavity are ignored.

With these non-symplectic maps, we need to determine optical function (β and α functions) inside the linac and match to those in the arcs. By defining a reference momentum P_r, we can transform to a new coordinate $(x, p_{xr} = P_x/P_r)$, in which the map returns to a symplectic one.

$$\left(\begin{array}{c} x \\ p_x \end{array} \right)_{E=E_i} = \left(\begin{array}{cc} 1 & 0 \\ 0 & \frac{(\gamma\beta)_r}{(\gamma\beta)_i} \end{array} \right) \left(\begin{array}{c} x \\ p_{xr} \end{array} \right) \quad (4)$$

the subscript i refers to a specific energy in linac.

In the above equation, the beam velocity $\beta = v/c$ and Lorentz factor γ are grouped together to avoid confusion with optics functions. And the map changes to:

$$\tilde{M} = \left(\begin{array}{cc} 1 & 0 \\ 0 & \frac{(\gamma\beta)_2}{(\gamma\beta)_r} \end{array} \right) \cdot M \cdot \left(\begin{array}{cc} 1 & 0 \\ 0 & \frac{(\gamma\beta)_r}{(\gamma\beta)_1} \end{array} \right) \quad (5)$$

01 Circular Colliders
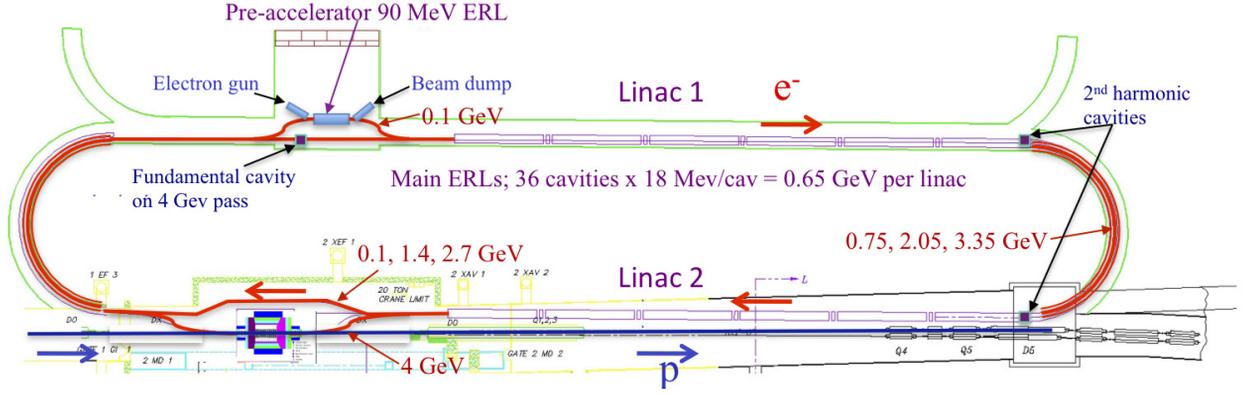
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Figure 1: Layout of MeRHIC. The red lines represent the energy recovery paths. There are 3 passes that vertically overlay with each other for different energies.

where the subscript 1, 2 and r represent the momentum at specific position 1 and 2 and the reference momentum, which can be chosen arbitrarily. Obviously, the determinant of $\tilde{M}$ is unity.

We can rewrite $\tilde{M}$ in norm form which transform the betatron oscillation to a pure rotation R with the phase advance as the rotation angle:

$$\tilde{M} = \begin{pmatrix} \sqrt{\tilde{\beta}_2} & 0 \\ -\frac{\tilde{\alpha}_2}{\sqrt{\tilde{\beta}_2}} & \frac{1}{\sqrt{\tilde{\beta}_2}} \end{pmatrix} R \begin{pmatrix} \frac{1}{\sqrt{\tilde{\beta}_1}} & 0 \\ \frac{\tilde{\alpha}_1}{\sqrt{\tilde{\beta}_1}} & \sqrt{\tilde{\beta}_1} \end{pmatrix} \quad (6)$$

Here we defined the pseudo beta and alpha functions with tilde above the symbols. It is starghtforward to prove that they link with the normal ones at energy E_i as:

$$\begin{aligned} \tilde{\beta}_{E=E_i} &= \beta \frac{(\gamma\beta)_r}{(\gamma\beta)_i} \\ \tilde{\alpha}_{E=E_i} &= \alpha \end{aligned} \quad (7)$$

It is worthwhile to note that the phase advance does not change due to this transformation. With the norm form we can easily calculate the optics functions and phase advance at any position if we know the knowledge of the following: map between the initial point to this position, the optics functions at initial point and the energies of both positions.

PHILOSOPHY OF LINAC DESIGN

There is larger amount of parameters that can be optimized to achieve the desired optics functions in linac. An uncompleted list includes the transfer map of the energy recovery paths, the configuration of the focusing magnets between linacs and the strength of focusing quadrupoles. In MeRHIC, over 30 parameters can be changed if necessary. To optimize them in one time needs extensive computation power and the result usually does not have clear realistic meanings.

To simplify the process, we impose additional limitations on the parameters. First we choose doublet as the focusing magnets between linacs and fix the length of each

quadrupole because here only the integrated field is important. Second, only two configurations of the doublet strength are considered. One is the constant gradient case, in which all quadrupoles have same strength, although the beam experiences weaker focusing when the energy increases in linac; the other is alternate gradient case, in which the strength is proportional to the beam energy at the quadrupole position in the lowest energy pass. In two-linac layout as MeRHIC, the doublet strength increases in linac 1 and decreases in linac 2. This gradient configuration has more natural meaning than other alternate gradient because the beam experiences same focusing force in its lowest energy stage.

We use a transfer maps to represent the energy recovery paths that connect one linac exit to the next linac entrance. For one transverse direction, a symplectic map has 3 independent variables. A simplification method is to make the map as simple focusing map with focal length f.

$$M_A = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \quad (8)$$

This map doesnot change the β function but only the α function by β/f. Therefore the focal length f, or the α function at the entrance of the next linac can be adjusted to minimize the average β function as:

$$\frac{\partial \bar{\beta}(\beta_i = \beta_0, \alpha_i)}{\partial \alpha_i} = 0 \quad (9)$$

Here we minimize the average β function because the small values is favorable to many beam dynamics issues including beam break-up and ion trapping effects. Depending on different optimization goal, we can use other functions in above equation if the they are functions of α.

In this discussion, the whole pass is symmetric between the acceleration and deceleration stage, as shown in Figure 2. Starting from the highest energy, we can calculate each focal length to optimize the beta function in the next linac.

Not breaking the symmetry of acceleraton and deceleration stage, we can choose a more general form for the arc. The transfer map of a symmetric arc gives,
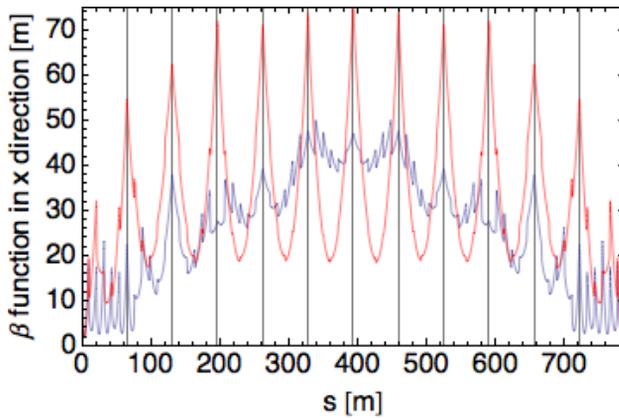
Figure 2: The β function of horizontal direction in all 12 linac passes. All arcs are considered as map and represented by vertical grid lines. The red line correponds to the constant gradient doublet and the average β function $(\bar{\beta})$ is minimized. The blue one is for alternate gradient doublet and the $\bar{\beta} + \sigma_\beta$ is minimize.

$$M_A = \begin{pmatrix} d & (d^2 - 1)\,b \\ 1/b & d \end{pmatrix} \qquad (10)$$

or its transpose. The parameter d and b can be choosen arbitrarily. The benefit we can get from expanding the one variable matrix to M_A is that both β and α functions can be changed simultaneously. Therefore, each energy pass of both linacs can be optimized seperately. If the optics function at one linac exit is (β_1, α_1), the values at next linac entrance is (β_2, α_2), M_A has the well known form as in Eq. (6) with the phase advance as

$$\psi = \arctan \frac{\beta_1 - \beta_2}{\beta_1 \alpha_2 + \beta_2 \alpha_1} \qquad (11)$$

For each energy pass in linac, the initial optics function (β_i, α_i) is given by solving

$$\frac{\partial f(\beta_i, \alpha_i)}{\partial \beta_i} = 0; \ \frac{\partial f(\beta_i, \alpha_i)}{\partial \alpha_i} = 0 \qquad (12)$$

The above equations only need to be solved at acceleration stage. The deceleration part is just the mirror symmetry with respect to highest energy recovery path, because the same matrix transform $(\beta_2, -\alpha_2)$ to $(\beta_1, -\alpha_1)$.

In figure 3, we show the individually optimized linac optics in the accelerating stage (first 6 linacs). At grid lines, the arcs' transform matrix are calculated by the optics functions at nearby linac ends. Comparing figure 2 and 3, we can not see much improvement, which means the single variable method already give good lattice. However, the 2-variable method isolates the optimization of one linac with others. No matter how many linacs is used in ERL, the procedure will be the same.

A special case, where our interest is paid, is quadrupole-free linac. Using the same example, we keep the current gap between SRFs, but set all quadrupole as zero strength.
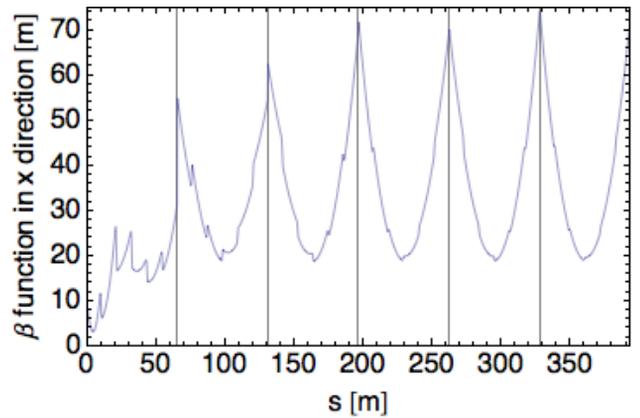

Figure 3: The β function of horizontal direction in 6 linac passes (The acceleration stage). All linacs are optimized individually. Other parameters are same as the red curve of figure 2.
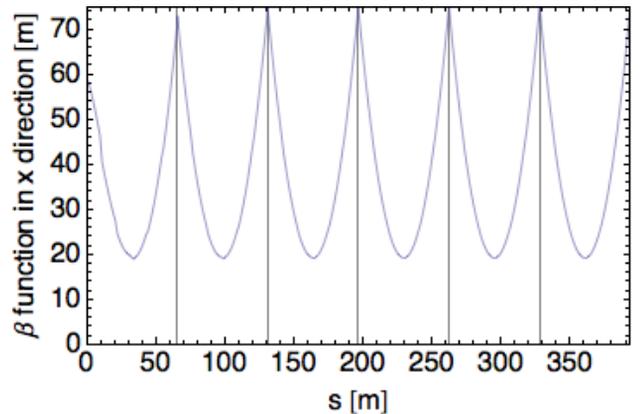

Figure 4: The β function of horizontal direction in 6 linac passes (The acceleration stage). All linacs are optimized individually. All quadrupole strengths are set to zero.

In figure 4, the beta function is only slight higher than non-zero quadrupole layout, except for the lowest energy case. The advantage of quadrupole-free layout is obvious. No cool-warm transition is needed and linac can be more compact by reducing the space between SRFs. However, the feasibility of this scheme needs to be confirmed by BBU simulations.

In this paper, we present our methods to design the linac optics with optimization for different purpose. As an example, the results for minimizing the beta function for MeR-HIC are provided. The methods can be expanded to any ERL linac design with arbitrary number of linacs.

REFERENCES

[1] V. Ptitsyn et al, "MeRHIC–Staging Approach to eRHIC", PAC09, Vancouver, Canada, 2009