ALGORITHM FOR COMPUTATION OF ELECTROMAGNETIC FIELDS OF AN ACCELERATED SHORT BUNCH INSIDE A RECTANGULAR CHAMBER*

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Abstract

We discuss the feasibility of an application of an implicit finite-difference approximation to calculate the fields of a relativistic bunch moving with no restriction inside a vacuum chamber. We assume that a bunch trajectory is not straight but is inside a vacuum chamber or its branch. The bunch can be deflected by the fields of bending magnets. The bunch can be short enough to produce coherent synchrotron radiation (CSR).

INTRODUCTION

Accelerator physicists believe that electromagnetic phenomena of charged beams are governed by Maxwell's equations together with Newton's equations for particle dynamics. To understand the behavior of the beams and radiated fields we just need to find a solution to these equations for the case, which can fully describe the real accelerator environment. So, at first we make a model, which contains all the necessary components, but at the same time can be easily "inserts" into the equations. Sometimes, it is possible to find analytical solutions, but usually they are only work for one-dimensional cases and rarer for two-dimension cases. To find a solution in general we may transform the equations into a equivalent finite-difference form and solve them using computers. We can find a lot of finite-difference schemes, which approximate Maxwell's equations since the first one that was published in 1966 [1]. Most of them are so called explicit schemes. That means that the value of the field at the new time step is calculated only by the field values at the previous time step. Stability conditions for these schemes do not allow a time step to be greater than or equal to a space (mesh) step. This limitation brings an additional troublesome effect for short wavelengths compared a mesh step. We state that this effect works like a frequency dispersion media, which is "hidden" in the finite-difference equation.

DISPERSION OF THE EXPLICIT SCHEMES

Let's check the explicit scheme for the two-dimensional case. When the field components satisfy the equation:

$$\frac{\partial^2\Phi}{c^2\partial t^2}=\frac{\partial^2\Phi}{\partial z^2}+\frac{\partial^2\Phi}{\partial x^2}$$

Likewise the explicit scheme will be

$$\Phi^{n+1}-2\Phi^{n}+\Phi^{n-1}=\left(\frac{c\Delta t}{\Delta z}\right)^2\Delta_z^2\Phi^n+\left(\frac{c\Delta t}{\Delta x}\right)^2\Delta_x^2\Phi^n$$

The stability condition easily comes from the Fourier analyses $\Phi_k^n \sim e^{i\omega t+i\beta z+i\alpha x}$ and takes the form:

$$\sin^2\frac{\omega\Delta t}{2}=\left(\frac{c\Delta t}{\Delta z}\right)^2\times\sin^2\frac{\beta\Delta z}{2}+\left(\frac{c\Delta t}{\Delta x}\right)^2\times\sin^2\frac{\alpha\Delta x}{2}$$

For stability reasons we need the frequency ω to be real. This happens when the right part is less than one. To have stability for any longitudinal wave vector β and transverse wave vector α we need the following condition to be fulfilled

$$\left(\frac{c\Delta t}{\Delta z}\right)^2+\left(\frac{c\Delta t}{\Delta x}\right)^2\le 1 \quad \text{for } \Delta x=\Delta z \quad c\Delta t\le\frac{1}{\sqrt{2}}\Delta z$$

Now let's check the plane waves in free space. Without boundaries the plane waves must propagate at the speed of light. However the solution of the finite-difference equation shows that the propagation velocity depends upon the frequency.

$$V_{wave}=\frac{\omega}{\beta}=c\frac{2}{\beta c\Delta t}\times\arcsin\left(\frac{c\Delta t}{\Delta z}\sin\frac{\beta\Delta z}{2}\right)$$

This means that finite-difference equations contain something like a "hidden" dispersion media, which reveals itself at the wavelength comparable with the mesh size. A plot of propagation velocity as a function of frequency for different ratios of mesh steps to time steps is shown in Fig. 1.

This numerical dispersion disappears only when the time step becomes equal to the mesh step, but in this case the scheme is unstable.

The effect of the numerical dispersion may be very dangerous; it can greatly disturb the result. It may develop a strong diffusion of an initially smooth field distribution and reveal high frequency oscillations. Figure 2 shows snap shots of a short wave bucket propagating in free space. This is the result of using the explicit scheme for the case when the length of a bucket is equal to two mesh sizes.

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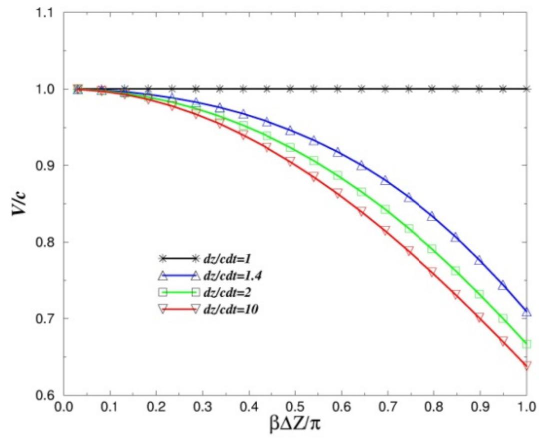

Figure 1: Propagation velocity of a plane wave in free space as a function of frequency for different ratios of mesh steps to time steps.

We can see that a wave bucket has distortion, modulation and diffusion: everything that comes from a dispersion media. The numerical dispersion really disturbs the bucket shape. If we want to have a better result using the explicit scheme we need to decrease the mesh size least five times

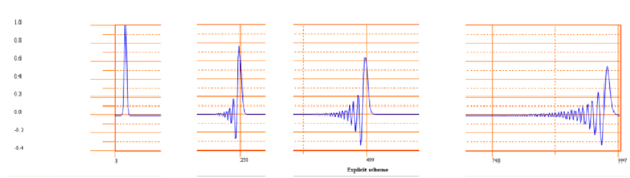

Figure 2: Snap shots of a short wave bucket, calculated by an explicit scheme.

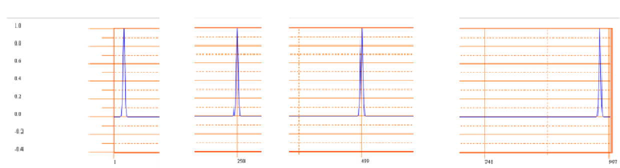

Figure 3: Snap shots of a short wave bucket, calculated by an implicit scheme.

In the wake field simulations this effect leads to an unphysical result like "self-acceleration" of a bunch head, which violates energy conservation. An explicit scheme is not very good at calculating the wake fields of very short bunches.

IMPLICIT SCHEME

As we stated before there will be no numerical dispersion if the time is equal to a mesh step. We can fulfil this condition by using the stable implicit scheme. In the implicit scheme for the calculation of the space derivatives we assume that the field at some time can be approximated by the average value of the field at a previous and a new time step. Of course the implicit algorithm is more complicated as it requires the solution of a system of equations. However as we have interested in the waves that are propagating in the longitudinal direction we really only need a "partially" implicit scheme, which for our equation is the following:

$$\Phi^{n+1} - 2\Phi^{n} + \Phi^{n-1} = \left(\frac{c\Delta t}{\Delta z}\right)^2 \Delta_z^2 \Phi^n + \left(\frac{c\Delta t}{\Delta x}\right)^2 \Delta_x^2 \frac{1}{2}\left(\Phi^{n+1} + \Phi^{n-1}\right)$$

The dispersion relation for this scheme is:

$$\sin^2\frac{\omega\Delta t}{2} = \frac{\left(\frac{c\Delta t}{\Delta z}\right)^2 \sin^2\beta\frac{\Delta z}{2} + \left(\frac{c\Delta t}{\Delta x}\right)^2 \sin^2\alpha\frac{\Delta x}{2}}{1 + 2\left(\frac{c\Delta t}{\Delta x}\right)^2 \sin^2\alpha\frac{\Delta x}{2}}$$

This equation shows that this scheme is stable in the case of equal mesh and time step $c\Delta t = \Delta z$. Figure 3 demonstrates the effectiveness of this scheme. Now a wave bucket keeps the same shape for a long time.

WAKE FIELD SIMULATIONS

The described implicit algorithm has been used in computer code designed in 1976 for wake field dynamics studies at the Novosibirsk Electron-Positron Linear Collider VLEPP [2]. Later the code was used for calculating wake fields of very short bunches at the TESLA Linear Collider and TTF FEL [3]. All of these calculations were important for the feasibility of these projects. Recently we got an opportunity to make a comparison with wake field measurements at LCLS. Results of a computation of the fields of a several micron long bunch showed very good agreement [4]. Figure 4 shows how very complicated electromagnetic fields can be when they are generated by a short bunch in the LCLS bellows.

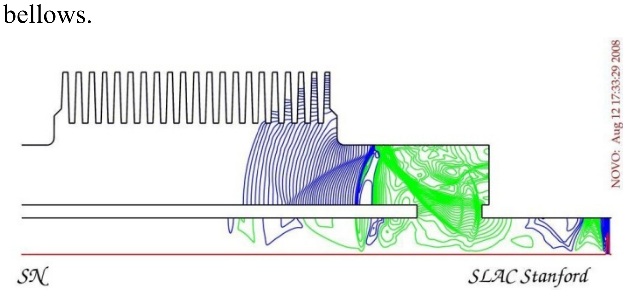

Figure 4: Electric force lines of the wake field, excited by a short bunch in LCLS bellows.

TRAVELLING MESH

To decrease the amount of needed memory we can use a travelling mesh. This is very important for bunch compressor simulations at higher beam energies where the bunch length is a micron but the distance between bends is tens of meters. The mesh will move with the speed of light and we can definitely assume that the electromagnetic field in front of the bunch is zero; even if the bunch motion is not straight. Because time delay due to the bending magnet in the chicane is very small, we do not need more space for the bunch. A travelling mesh does not change the accuracy of the scheme or any conditions of stability.

BUNCH PARTICLES

To simulate the real shape of a non-monochromatic bunch moving, for example, in a bending magnet we will use an ensemble of particles. We will track each particle and average the current (particle velocities) over the mesh. The charge density distribution will be integrated using the continuity equation for charge and current. Because we are using a travelling mesh particles will stay in same cells for many time steps. This will help to smooth out errors of particle transitions from one cell to another. Figure 5 shows the charge distribution of a bunch rotated by the vertical magnetic field. Initially this bunch was travelling with a speed very close to the speed of light and had a Gaussian distribution in all directions.

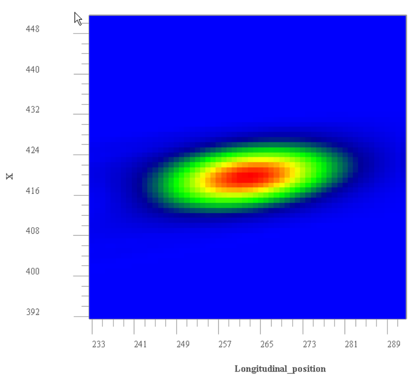

Figure 5: Charge distribution of a bunch rotated by the vertical magnetic field.

MASSIVE PARALLELIZATION

The algorithms for particle tracking and for the field calculations can be adapted to massively parallel computers. Using a Fourier transform for the vertical coordinate we can associate each mode (it can be 100 or more modes) with six-eight (field components and/or space charge density and current) elementary processors. We will need to communicate between processors only if we want to include radiation reaction on the beam motion. Parallelization for the particle motion can be done in the same way as for "particle in cells" technique [5].

CSR FIELD DYNAMICS

To check the accuracy of this method we can make a comparison with results of other CSR codes and with the one-dimensional analytical approach. We can also verify the method from the physical point of view. For this purpose we have developed a code, which contains only the core part of the method and which can show the results in the form of electric force line distributions. First snapshots of electric force line distributions for a short bunch moving in a bending chamber are shown in Fig. 6. It will be nice to see the radiation of the waves from a bending bunch, but instead we have found that the radiation fields and the bunch fields are moving together. They are staying together because the time delay due to the bend is small. Radiation fields can be "cut out" by the crotch if the bunch goes to a branch of the chamber, for example when it goes to the dump. We also can see from the plot that some part of the bunch field will propagate straight and separate from the other part of the bunch field. We may assume that this is an important fraction of the CSR field – edge radiation. This part is correlated with the position of the bunch at the entrance of the deflecting magnetic field. We have seen such a correlation at the LCLS dump.

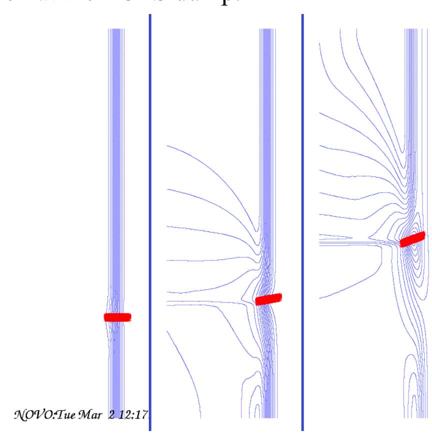

Figure 6: Snapshots of electric force lines of the field of a short bunch in a bending chamber.

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