# STRONG-STRONG SIMULATIONS FOR SUPER B FACTORIES

K. Ohmi, KEK, Tsukuba, Japan

### *Abstract*

Super B factories are designed with very low emittance and very low beta function at the interaction point. The two beams collide with a large crossing angle, thus the overlap area of the beams is limited at a small part of their length. Simulation of the beam-beam effects is hard because of the longitudinal slice of the beam is the order of 100. We discuss three methods for the simulation. One is the soft Gaussian approximation. Second is a simplified method, which is mixture of the particle in cell and Gaussian approximation. Third is fully strong-strong simulation using the particle in cell. The shifted Green function is used to calculate the beam-beam force for less overlap of the beam distribution. Luminosity and its degradation due to IP optics errors in Super B factories are discussed.

## INTRODUCTION

Super B factories are designed so as to collide the two beam with a large Piwinski angle (LPA). The overlap area of the colliding beam is small, thus the beam-beam parameters due to hourglass effect is suppressed even the beta function of the collision point is very small $\beta_y \ll \sigma_z$. The ratio of the bunch length and typical collision overlap area is $\sigma_z\phi/\sigma_x \sim 20$ for Super B factories, where $\phi$ is half crossing angle. Table 1 shows the parameters of SuperKEKB and SuperB.

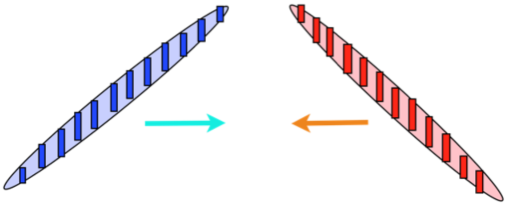

Figure 1: Schematic view of beam-beam collision with a large Piwinski angle

The beam-beam performance is evaluated using weak-strong simulations for this collision scheme. In the weak-strong simulations, one (strong) beam is represented by a fixed charge distribution, and motion of another (weak) beam, which is represented by macro-particles, are studied. The strong beam is sliced into many pieces along longitudinal. The region $\Delta s \sim \beta_y$ or $\sigma_x/\phi$ should be sliced into 5 pieces or more typically. The number of sliced is 100 or more for $\sigma_z\phi/\sigma_x \sim 20$. The simulation is based on single particle dynamics, therefore the number of macro-particle is related to the statistics, but is not essential for the dynamics. The simulation can be performed with less number of particles than the strong-strong simulations.

In the strong-strong simulation, the number of macro-particles affects the beam-beam dynamics, thus enormous number of macro-particles are necessary to avoid dynamical effects of the numerical noise. For electron storage rings, beam particles are fluctuated by the radiation excitation. Square of the fluctuation in a unit of the beam size is roughly equal to the radiation damping rate $T_0/\tau$ per turn, where $T_0$ and $\tau$ are revolution and the radiation damping time, respectively. The number of macro-particles should be more than the radiation damping rate, otherwise the numerical noise dominates the radiation excitation. For the strong-strong simulation,

Table 1: Parameter list for SuperKEKB and SuperB. These parameters used in simulations are not latest one.

| | SuperKEKB | | SuperB | |
|---|---|---|---|---|
| | HER | LER | HER | LER |
| Circumference (m) | 3016 | - | | |
| Energy (GeV) | 7 | 4 | 7 | 4 |
| Emittance x (nm) | 1.7 | 3.2 | 1.6 | 2.8 |
| Coupling (%) | 0.48 | 0.4 | 0.25 | 0.25 |
| Beta x (mm) | 25 | 32 | 20 | 35 |
| Beta y (mm) | 0.42 | 0.27 | 0.39 | 0.22 |
| Bunch population($10^{10}$) | 6.53 | 9.04 | 5.52 | 5.52 |
| Bunch length (mm) | 5 | 6 | 6 | 6 |
| Half crossing angle (mrad) | 41.5 | - | 24 | - |
| Collision repetition (ns) | 4 | 4 | 4 | 4 |
| Beam-beam parameter | 0.09 | | 0.15 | |
| Piwinski angle $\sigma_z\phi/\sigma_x$ | 32 | 21 | 25 | 15 |

we require 10,000 macro-particles in a slice, thus 2,000,000 for 200 slices. Since the collision is evaluated slice by slice, 200x200=40,000 interactions have to be evaluated per collision. This is enormous number. To complete the simulation in a realistic time, some devices are necessary. We present three trials for the strong-strong beam-beam simulations for Super B factories.

## SOFT GAUSSIAN MODEL

The potential calculation is heavy duty for computers. If the beam distribution is approximated by a Gaussian distribution, the beam-beam force is given by analytic formula [1] as follows,

$$F_y + iF_x = \frac{2Nr_e}{\gamma}\sqrt{\frac{\pi}{2(\sigma_x^2-\sigma_y^2)}}\left[w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - \exp\left(-\frac{x^2}{\sigma_x^2}-\frac{y^2}{\sigma_y^2}\right)w\left(\frac{\sigma_y x/\sigma_x + i\sigma_x y/\sigma_y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right] \qquad (1)$$

where w() is the complex error function. Using the formula reduces the simulation time. Single beam-beam collision is evaluated by 200x200=40,000 sub-collisions using Eq.(1). $\sigma_x$ and $\sigma_y$ are calculated for each slice pair in each sub-collision. Figure 2 shows the luminosity evolution for SuperKEKB. The legend CW and NoCW indicate the collision with and without crab waist scheme, respectively. The luminosity is $9\times10^{35}$ cm$^{-2}$s$^{-1}$ and $<6\times10^{35}$ cm$^{-2}$s$^{-1}$ with and without the crab waist, respectively, in this simulation.

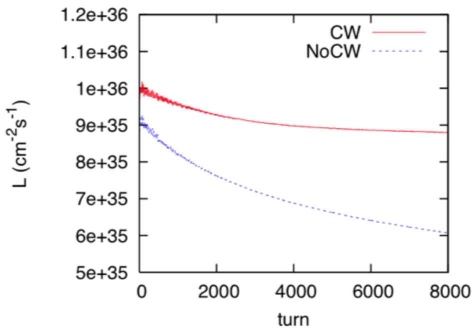

Figure 2: Evolution of luminosity with soft Gaussian model for SuperKEKB.

## SIMPLIFIED STRONG-STRONG SIMULATION

Normally the strong-strong simulation is performed using the particle in cell (PIC) method [1]. The beam particle distribution $\rho(\mathbf{r})$ is mapped on 2D grid space (($N_x$,$N_y$) for the area of ($\Delta$x,$\Delta$y)), and then the potential is calculated by solving Poisson equation in the space. Green function is used for the potential calculation in an isolated system. The potential in transverse plane **r**=(x,y) is given by

$$\phi(\mathbf{r}) = -\frac{1}{2\pi\varepsilon_0}\int d\mathbf{r}' G(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}') \qquad (2)$$

where

$$G(\mathbf{r}) = \ln|\mathbf{r}| \qquad (3)$$

The integration is performed by the convolution of their Fourier Transform [2] to save the CPU time. The Green function should be defined in twice large area $(-\Delta x,-\Delta y)<\mathbf{r}-\mathbf{r}'<(\Delta x,\Delta y)$ for the convolution method, while the density and potential are mapped in $(-\Delta x/2,-\Delta y/2)<\mathbf{r}<(\Delta x/2,\Delta y/2)$. The calculation cost of this method is 8 times for 128x256 grid space (actually 256x512 for Green function) compare than the soft Gaussian method. Therefore a simplified method has been tried to reduce the number of the potential calculation. When two slices collide at s, the separation of the slices is $2s\phi$. For $s\gg\sigma_x/\phi$, the sub-collision between two slices is approximated to be interaction between Gaussian distribution because the separation is larger than the size $\sigma_x$. Figure 3 shows the evolution of the luminosity.

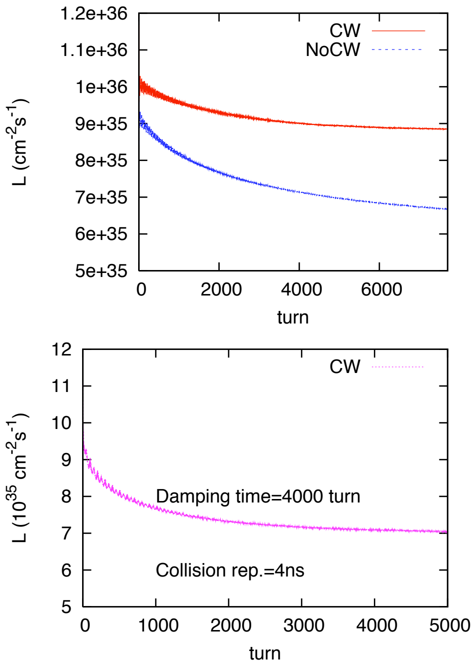

Figure 3: Evolutions of luminosity with a simplified strong-strong simulation for SuperKEKB (top) and SuperB (bottom).

The separation of two beams is closer than $5\sigma_x$, the beam-beam force is calculated by PIC method, otherwise by Gaussian approximation. Simulations are performed with 150 slices. In the total number of sub-collisions of 22,500, PIC and Gaussian calculations are 1,670 and 20830, respectively for SuperKEKB. They are 2,510 and 19,990 for SuperB.

The luminosity for SuperKEKB, which is $6.7\times10^{35}$ cm$^{-2}$s$^{-1}$ without crab waist, is somewhat less than the design $8\times10^{35}$ cm$^{-2}$s$^{-1}$. The luminosity with and without crab waist is almost consistent with that of Gaussian model. The luminosity for SuperB is less than the design $1\times10^{36}$ cm$^{-2}$s$^{-1}$. This design of SuperB, which is not newest, orients beam-beam parameter larger than 0.1. The simulation shows the beam-beam parameter is degraded around 0.08. While weak-strong simulations show no degradation. Simulations with high beam-beam parameter are complex, because noises and errors immediately degrade the luminosity. There may be difficulty in the boundary of Gaussian approximation and PIC calculation.

## FULL STRONG-STRONG SIMULATION

In the collision of LPA, two beam interact with a large separation for a large s~$\sigma_z$. Grid space cover all area is wide and ineffective. The vertical beam size is a large due to the hourglass effect. Shifted Green function [3] is helpful for the separation of the source and force areas. Potential for the case is calculated by Green function with a constant offset as follows,

$$\phi(\mathbf{r}) = -\frac{1}{2\pi\varepsilon_0}\int d\mathbf{r}' G(\mathbf{r}-\mathbf{r}'-\mathbf{r}_0)\rho(\mathbf{r}'+\mathbf{r}_0) \tag{4}$$

The Green function should be defined in the area ($-\Delta x - x_0, -\Delta y - y_0) < \mathbf{r}-\mathbf{r}'-\mathbf{r}_0 < (\Delta x - x_0, \Delta y - y_0$). The Green function table $G_{ij}$ is assigned as follows,

$$f(x_{i+}-x_0,y_{i+}-y_0)-f(x_{i+}-x_0,y_{i-}-y_0)-f(x_{i-}-x_0,y_{i+}-y_0)+f(x_{i-}-x_0,y_{i-}-y_0) \tag{5}$$

$$f(x_{i+}-2\Delta x-x_0,y_{i+}-y_0)-f(x_{i+}-2\Delta x-x_0,y_{i-}-y_0)-f(x_0-2\Delta x+x_{i-},y_{i+}-y_0)+f(x_{i-}-2\Delta x-x_0,y_{i-}-y_0) \tag{6}$$

$$f(x_0+x_{i+},y_0-2\Delta y+y_{i+})-f(x_0+x_{i+},y_0-2\Delta y+y_{i-})-f(x_0+x_{i-},y_0-2\Delta y+y_{i+})+f(x_0+x_{i-},y_0-2\Delta y+y_{i-}) \tag{7}$$

$$f(x_0-2\Delta x+x_{i+},y_0-2\Delta y+y_{i+})-f(x_0-2\Delta x+x_{i+},y_0-2\Delta y+y_{i-})-f(x_0-2\Delta x+x_{i-},y_0-2\Delta y+y_{i+})+f(x_0-2\Delta x+x_{i-},y_0-2\Delta y+y_{i-}) \tag{8}$$

$$\text{where } f(x,y)=\int dxdyG(\mathbf{r}) = -3xy + x^2\tan^{-1}(y/x) + y^2\tan^{-1}(y/x) + xy\log(x^2+y^2) \tag{9}$$

$$x_{i\pm} = (i-1\pm 0.5)\Delta x/N_x \qquad y_{j\pm} = (j-1\pm 0.5)\Delta y/N_y$$

Eq.(5)-(8) show $G_{ij}$ for ($i=1, N_x, j=1, N_y$), ($i=N_x+1, 2N_x, j=1, N_y$), ($i=1, N_x, j=N_y+1, 2N_y$), ($i=N_x+1, 2N_x, j=N_y+1, 2N_y$), respectively, so as to satisfy the periodic condition for $2\Delta x$ and $2\Delta y$.

The number of the PIC calculation including the potential solving is huge; square of the number of the beam slices, ~40,000 per collision. The calculation is too heavy for the present KEK super computer.

KEK super computer is replaced by a new system in 2011. Perhaps two types of computers are installed to succeed the present system. One is SMP type of computer with rather small number of nodes, and the other is the parallel computer with enormous number of nodes. For these beam-beam simulations, SMP type of computer, which has a high performance per node, is suitable, because frequent communications for $\rho(\mathbf{r})$ and $\phi(\mathbf{r})$ can be a bottleneck for parallel computers connected by network.

The full strong-strong simulation will be started with the new supercomputer.

## CONCLUSIONS

Overlap of two beams during collision is very small for super B factories with a large Piwinski angle scheme. The strong-strong simulation is very hard to complete because of many longitudinal slicing of a bunch. Several trials of strong-strong simulations were performed, soft Gaussian model, simplified PIC method and full strong-strong simulation. The luminosity is somewhat lower than the design but is not so disaster. There may be problem in the strong-strong simulations, for example, numerical noise, potential smoothness along longitudinal axis and others. Full strong-strong simulation has not performed yet due to too heavy. We waist for new KEK super computer.

## REFERENCES

[1] Many references exist for the strong-strong simulations. The simulation using Green function is written in K. Ohmi, Phys. Rev. E 62 (2000) 7287.

[2] R.W. Hockney and J.W. Eastwood, "Computer simulation using particles". IOP Publishing Ltd (1988).

[3] J. Qiang et al., Phys. Rev. ST-AB 5, 104402 (2002).