

# LATTICE DESIGN AND STUDY TOOLS REGARDING THE SUPER-B PROJECT

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## Abstract

Lattice design and study tools are being developed, and related beam and spin dynamics simulations are being performed, in the framework of the international collaboration regarding the super-B project. The present contribution reports on this work.

## INTRODUCTION

The assessment and qualification of the super-B lattice parameters as produced by the on-going design studies [1] require high precision dynamic simulations and *ad hoc* tools. Preliminary investigations have been undertaken using the stepwise ray-tracing code Zgoubi [2]. That code is based on a numerical resolution of  $\vec{F} = q\vec{v} \times \vec{B}$ , thus allowing (i) high precision magnetic fields modelling, (ii) high precision particle and spin tracking; in addition, it provides most of the tools relevant to detailed numerical simulation of beam and spin dynamics in super-B high-energy (HER) and low-energy (LER) rings, as spin tracking [3, 4], synchrotron radiation [5], together with a number of optical elements and other dynamics numerical tools.

This paper reports on series of preliminary numerical experiments performed in the recent months, of which the aim is twofold, (i) evaluating the capabilities of the code as applied to super-B HER and LER lattices, (ii) starting delivering precision analysis of lattice behavior.

Table 1: Super-B Parameters

	MAD8	Ray-tracing
<b>HER</b>		
Energy /GeV		6.7
Orbit length /m	1258.3581	1258.3582
$Q_x, Q_y$	40.5750, 17.5950	[40].5750, [17].5950
$Q'_x, Q'_y$	0.042, -0.0038	0.062, -0.0019
$\alpha, \sqrt{1/\alpha}$	$4.361 \cdot 10^{-4}, 47.88$	$4.371 \cdot 10^{-4}, 47.83$
Max $\beta_x, \beta_y$ /m	388.57, 1126.40	388.55, 1126.35
Max Dx /m	0.6346	0.6346
<b>LER</b>		
Energy /GeV		4.18
Orbit length /m	1258.3581	1258.3582
$Q_x, Q_y$	42.5744, 18.6019	[42].5749, [18].5949
$Q'_x, Q'_y$	-0.620, -0.678	-0.624, -0.676
$\alpha, \sqrt{1/\alpha}$	$4.049 \cdot 10^{-4}, 49.69$	$4.053 \cdot 10^{-4}, 49.67$
Max $\beta_x, \beta_y$ /m	396.54, 1013.17	387.25, 1146.77
Max Dx /m	0.5118	0.5118

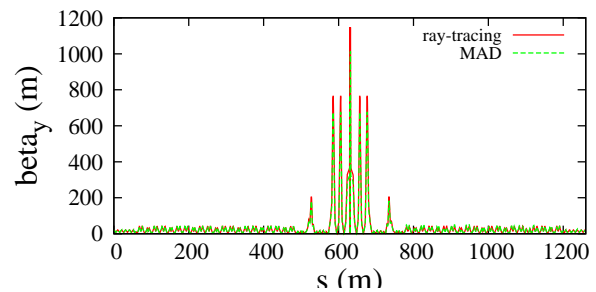
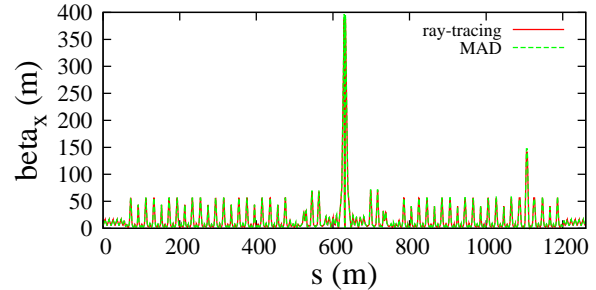


Figure 1: Optical functions in LER. Ray-tracing and MAD8 curves are superimposed.

## PARAXIAL MOTION

The input data files to Zgoubi are obtained from translation from MAD8 survey output. A preliminary step prior to further ray-tracing simulations is to get agreement between the two methods as to first and main second order parameters. This is illustrated in Tab. 1 and Fig. 1. All optical elements in the ray-tracing methods are hard-edge but for the eight spin rotator solenoids in LER. Field in bends and multipoles is derived from the scalar potential

$$V_n(s, x, y) = (n!)^2 \left\{ \sum_{q=0}^{\infty} (-)^q \frac{\alpha_{n,0}^{(2q)}(s)}{4^q q!(n+q)!} (x^2 + y^2)^q \right\} \left\{ \sum_{m=0}^n \frac{\sin(m\frac{\pi}{2}) x^{n-m} y^m}{m!(n-m)!} \right\}$$

with  $n$  the multipole order,  $\alpha(s)$  the longitudinal form factor, a step function in the present study, however liable to simulate extended fringe fields [2]. Solenoids (in LER) are simulated using for the on-axis field

$$s/\sqrt{s^2 + r^2} + (L - s)/\sqrt{(L - s)^2 + r^2}$$

followed by Taylor expansion off-axis, with  $L, r$  respectively length and radius.

## LARGE EXCURSION MOTION, DAS

Ray-tracing allows computing large amplitude motion with great precision on magnetic fields and, inherent to the numerical integration, including effects in relation with non-linearities of kinematical origin. Sample results of maximum stable amplitudes tracking are given in Fig. 2.

**01 Circular Colliders**

**A02 Lepton Colliders**

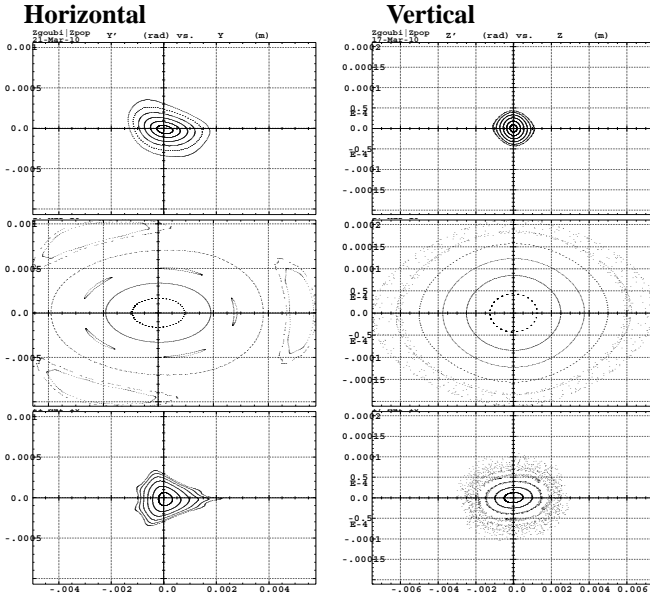


Figure 2: Maximum stable motion in HER, from ray-tracing,  $dp/p = +1\%, 0, -1\%$  from top to bottom row.

Comparisons with MAD8 have been undertaken [6] as it is often taken as a reference in the literature, it has been observed that strong discrepancies arise; given this, comparisons with other numerical methods are in order.

Dynamic apertures are obtained by repeating maximum stable H and V amplitude search above, that yields DAs in HER and LER are shown in Fig.3, as observed at center of QDI, RF section, over 1000 turns - the damping time is  $\approx 5000$  turns, see next Section.

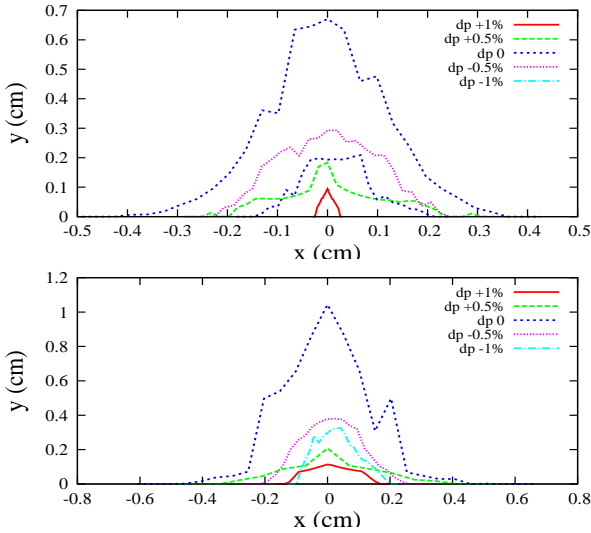


Figure 3: 1000-turn DA in HER (top) and LER (bottom).

## EQUILIBRIUM EMITTANCES

Stochastic synchrotron radiation and its effects on particle dynamics are simulated in a very regular manner, as follows [2, 5]. The number of photons,  $k$ , emitted in a step  $\Delta s$  is given by

### 01 Circular Colliders

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Table 2: Theoretical Parameters Entering SR Simulations in LER

Synchronous energy $E_s$	GeV	4.18
Orbit length	m	1258
Revolution time	$\mu s$	4.20
Average energy loss /particle/turn, $E_{loss}$	keV	865
Equivalent $\rho$ , $88.5E_s^4[GeV]/E_{loss}[keV]$	m	31.2
$\hat{V} \sin(\phi_s)$	kV	865.13

$p(k) = \lambda^{-k} \exp(-k) / k!$  with  $\lambda = 20e_0 \beta^2 B \rho \Delta s / 8\bar{h}\sqrt{3}$  the mean number of photons per radian, and  $r_0 = e^2/4\pi\epsilon_0 m_0 c^2$  the classical radius of the particle,  $e$  the elementary charge,  $\bar{h} = h/2\pi$ ,  $h$  the Planck constant,  $\beta = v/c$ ,  $B\rho$  the particle stiffness.  $\lambda$  is evaluated at each integration step from the current values  $\beta$ ,  $B\rho$  and  $\Delta s$ , then a value of  $k$  is drawn by a rejection method. These  $k$  photons are assigned energies  $\epsilon = h\nu$  at random, in the following way. The cumulative distribution of the energy probability law  $p(\epsilon/\epsilon_c)d\epsilon/\epsilon_c$  writes

$$\mathcal{P}(\epsilon/\epsilon_c) = \frac{3}{5\pi} \int_0^{\epsilon/\epsilon_c} \int_{\epsilon'/\epsilon_c}^{\infty} K_{5/3}(x) dx$$

with  $K_{5/3}$  the modified Bessel function,  $\epsilon_c = \bar{h}\omega_c$ ,  $\omega_c = 2\pi 3\gamma^3 c/2\rho$  the critical frequency of the radiation in constant field with bending radius  $\rho$ ;  $\omega_c$  is evaluated at each integration step from the current values  $\gamma$  and  $\rho$  (assumes constant magnetic field over  $\Delta s$ ). Quantities in the ‘‘theoretical’’ column in Tab. 3 derive from the following regular expressions :

Let  $e_i[kV] = 88.5E^4[GeV] \alpha_i/2\pi$  be the energy loss in dipole  $i$  (of  $N$  in the ring) characterized by its deviation  $\alpha_i$  with local radius  $\rho_i$ , and  $\epsilon_i = 0.683E^3[GeV]/\rho_i[m]$  the average photon energy, then, energy loss per particle  $E_{loss}[keV] = \sum_i^N e_i$ , a sum over the  $N$  dipoles; number of photons per particle,  $N_{phot} = \sum_i^N e_i/\epsilon_i$ ; beam  $\sigma_E/E = \gamma\sqrt{C_q/2\rho}$  with  $C_q = 55\hbar/32\sqrt{3}m_0c \approx 3.84 \cdot 10^{-13}$ ; damping time  $\tau_E[turns] = E_s/E_{loss}$ ,  $\tau_E[ms] = 4.2 \cdot 10^{-3} E_s/E_{loss}$ .

The stepwise SR simulation method yields equilibrium emittances and other parameters as damping times,  $\bar{H}$ , etc. Simulations have been realized with the hypotheses gathered in Tab. 2. SR in the sole dipoles is assumed here, whereas the code allows switching SR in other magnets. Note that the quantities in Tab. 2,  $E_{loss} = 865.13$  keV,  $N_{phot} = 541$ ,  $E_{phot} = 3.45$  keV as obtained from the

Table 3: SR in LER, Comparing Theoretical and Monte Carlo Data

		theoretical	Zgoubi
$E_{loss}$	keV	865.1	878.4
$N_{phot}$		541.4	541.3
$\epsilon_x/\pi$		$1.8 \cdot 10^{-9}$	$1.4 \cdot 10^{-9}$ (a)
$\tau_x$	ms		$\approx 28$ (b,a)
$\sigma_E/E$		$0.67 \cdot 10^{-3}$	$0.68 \cdot 10^{-3}$ (c)
$\tau_E$	ms	20.3	$\approx 15$ (b,c)

(a) After Fig. 4-top.

(b) Taken at 1/e of initial amplitude.

(c) After Fig. 4-bottom.

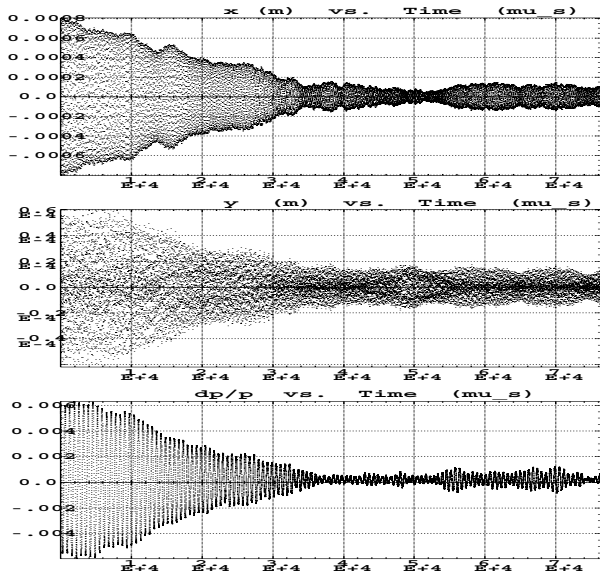


Figure 4: 20000-turn tracking, reaching equilibrium in presence of synchrotron radiation. Top and middle : respectively  $x$ -motion and  $y$ -motion, starting with pure  $x$ ; bottom: starting with pure  $dp/p$ .

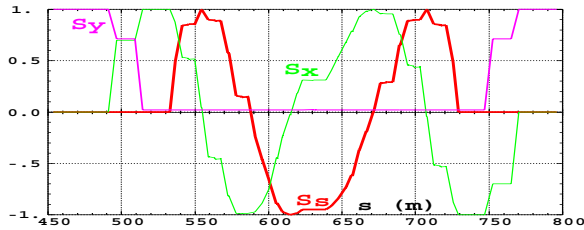


Figure 5: Spin components in IR region (spin is vertical,  $S_y \equiv 1$ , elsewhere).

oretical radiation formulas are identical to those obtained from the stochastic method. Sample behavior is displayed in Fig. 4 : a particle is launched with either  $x$  motion or non-zero  $dp/p$ , equilibrium of respectively  $\epsilon_x/\pi \approx 1.5$  nm,  $\sigma_{dE/E} \approx 6.5 \cdot 10^{-4}$  are reached.

## SPIN MOTION

Thomas-BMT equation for spin motion is solved simultaneously with particle motion, if requested [3]. In LER, the spin vector of the electron beam has to be longitudinal at the interaction point. This is ensured by means a rotator comprised of two sets of four solenoids, located on both sides of the IR. One way to find the  $\vec{n}_0(s)$  spin vector is to use the built-in fitting procedure in Zgoubi, with constraint of identical spin coordinates after one turn (Fig 6).  $\vec{n}_0(s)$  is vertical in the arcs at appropriate momentum; it is then clear that the method can be used for adjusting the rotator and IR bends so to reach longitudinal polarization at IP.

Typical spin behavior for the present tuning of V12 optics is displayed in Fig. 5; improvement is being carried on, in particular it has to be noted that the conditions of vertical  $\vec{n}_0(s)$  in the arcs as displayed in Fig. 5 are for

$\delta p/p = +1.252\%$  chromatic closed orbit.

Another way to find the  $\vec{n}_0(s)$  is the *stroboscopic averaging* [7] method, This second method permit to get rid of closed orbit, and allow full phase space study, especially with synchrotron radiation allowing spin diffusion, work is being carried on.

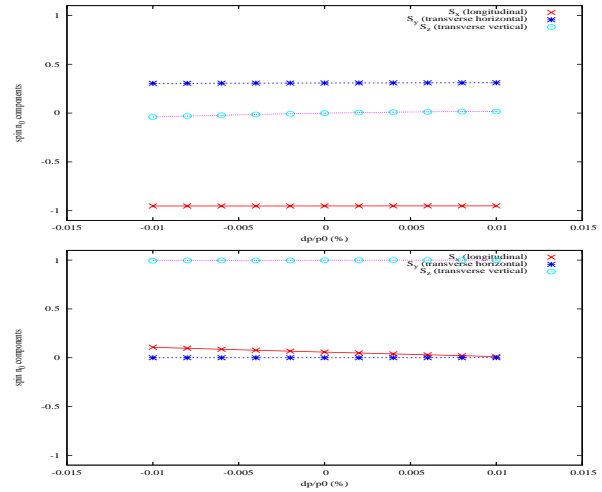


Figure 6: Spin components at the IP (up) and RF (down) section in function of momentum.

## COMMENTS

Given the acute insight in the physics which the ray-tracing tool allows, checks as well as tight comparisons with theory and other codes are considered worth being carried on. It is foreseen to explore further analysis methods as frequency map techniques, based on Zgoubi outputs. Sokholov-Ternov effect will be implemented - however the time scale in super-B lattice is *a priori* well beyond storage time scale. Implementation of beam-beam effects is being contemplated.

## REFERENCES

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