A PARTICLE-CORE MODEL FOR MISMATCHED AND INHOMOGENEOUS INTENSE CHARGED PARTICLE BEAMS*

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Abstract

Beams of charged particles usually reach their stationary state by the development of a halo. Halo formation is in fact a macroscopic transcription of microscopic instabilities acting inside the beam over its constituent particles. Previous works have investigated individually the role of the initial envelope mismatch and of the magnitude of inhomogeneity in halo formation. However, it is clear that in real implemented beams both act together. In this sense, the main purpose of this work is to consider now concomitantly both effects. As a final product of the investigation, a particle-core model is presented. The agreement with full self-consistent N-particle beam numerical simulations is satisfactory and the results provided by the model seem to be more compatible with that would be expected experimentally.

INTRODUCTION

Completely homogeneous beams evolve inside the magnetic focusing structures with no emittance growth. The beam constituents cannot individually couple and then be excited by macroscopic oscillations such as that from the envelope. Particles orbits stay limited to the beam boundary established by the envelope and halo is not formed. Emittance is a macroscopic quantity associated with the beam heating [1].

However, this is not the case of initially inhomogeneous beams. In such systems, particles can be expelled from the beam core due to the breaking of density waves [2][3][4]. If the beam envelope is mismatched, these ejected particles can be continuously driven by its oscillations and the system directs to its equilibrium with a progressive halo formation [5][6].

Although for cold and regular inhomogeneous beams the time scale of halo formation can be adequately predicted by the time with which the first density wave breaks, for quasi-homogeneous or thermal beams this is not possible. Particle jets are pretty less prominent and particles, instead of in groups, leave the beam individually. In this situation, it is interesting to describe the interaction between particles and the whole beam [7].

The purpose of this work is to develop a particle-core approach for an initially mismatched and inhomogeneous beam. The beam evolves in a linear channel and is surrounded by a conducting pipe.

THE MODEL

As mentioned before, for practical purposes, the beam has been considered azimuthally symmetric. In this situation, the beam spatial behavior can be enough described by the transversal radial coordinate R. Also, for convenience, the initial beam density has been supposed parabolic in the form

$$n_b(R,s) = \begin{cases} \dfrac{N_b}{\pi r_b^2} + \eta\left[\dfrac{N_b}{\pi r_b^2}\left(\dfrac{2R^2}{r_b^2} - 1\right)\right], 0 \leq R \leq r_b \\ 0, r_b < R \leq r_w \end{cases} \quad (1)$$

in which envelope r_b and the magnitude of inhomogeneity η can be a function of time/axial coordinate s.

Since the interest resides on the oscillations of r_b and η, one primary step to achieve the goal is to determine the transversal Lagrangian L of the beam. In this way

$$L = \int_0^{r_b(s)} \mathcal{L}\, n_b(\mathbf{R}_{\perp o}) d\mathbf{R}_{\perp o} \quad (2)$$

where $\mathbf{R}_{\perp o}$ is the radial coordinate that formally explores the beam transversal section and

$$\mathcal{L} = \frac{1}{2}V^2 - \frac{1}{2}\kappa_{zo}R^2 + Q(R_o)\ln(R) \quad (3)$$

is the transversal Lagrangian of a beam ring initially at coordinate $R(s=0) = R_o$. k_{zo} is the coefficient of magnetic focusing, a constant for the purposes of this work. Important to emphasize: the transversal Lagrangian $\mathcal{L}$ pertains to a ring with radius R_o while L refers to the whole beam.

The equation (3) contains two quantities that have to be determined. One is the beam velocity profile V, which should be a function of radial coordinate R and accounts how each beam ring evolves with the time s. The other is the dimensionless fraction of charge $Q(R_o)$, which specifies the charge trapped by a Gauss' surface at $R = R_o$.

By the use of the continuity equation, one can determine the beam velocity profile V, which can be formally written as

$$\mathbf{V}_\perp = -\frac{\mathbf{e}_r}{2\pi R n_b}\int \frac{\partial}{\partial s} n_b d\mathbf{R}_\perp. \quad (4)$$

The dimensionless fraction of charge $Q(R_o)$ is defined in the form

$$Q(R_o) = -\frac{K_b}{N_b}\int_0^{\mathbf{R}_{\perp o}} n_b(\mathbf{R}'_{\perp o}) d\mathbf{R}'_{\perp o} \quad (5)$$

in which $\mathbf{R}'_{\perp o}$ is just an auxiliary variable to perform the integration. Inserting the initial beam density of equation (1) into equation (4) and (5), one respectively obtains for the beam velocity profile

$$V = \frac{1}{2}\frac{(r_b\dot{\eta} - 4\eta\dot{r}_b)R^3 + (-2r_b^2\dot{r}_b - r_b^3\dot{\eta} + 2r_b^2\eta\dot{r}_b)R}{-r_b^3 - 2\eta r_b R^2 + r_b^3\eta} \quad (6)$$

and for the fraction of charge

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$$Q(R_o) = \eta \frac{R_o^4}{r_b^4} + (1-\eta)\frac{R_o^2}{r_b^2} \tag{7}$$

Observe that $V = V(R)$.

With equations (4) e (5), the transversal Lagrangian $\mathcal{L}$ of equation (3) for the beam rings is now completely determined. As consequence, equation (2) can be readily integrated. Proceeding in this way, it is found that

$$L = L(\eta, \dot{\eta}, r_b, \dot{r}_b). \tag{8}$$

The expression for the transversal Lagrangian L is omitted for compactness. Observe that L is a function of the magnitude of inhomogeneity η and beam envelope r_b. With the aid of the corresponding Euler-Lagrange equations, it is possible to obtain the following second-order ODEs for envelope r_b e inhomogeneity η

$$\begin{aligned} \ddot{\eta} &= F_\eta(\eta, \dot{\eta}, r_b, \dot{r}_b) \\ \ddot{r}_b &= F_{r_b}(\eta, \dot{\eta}, r_b, \dot{r}_b) \end{aligned} \tag{9}$$

in which F_η and F_{r_b} are also omitted but known functions. In this way, equation (9) can be readily integrated by the usual numerical methods, and $\eta(s)$ as well as $r_b(s)$ can be determined. The results obtained for both quantities will be shown in the next section.

As stated by Equation (1), the superficial beam density n_b depends of just r_b and η. To know how n_b evolves with the time s, it is possible to suppose that all beam dynamics is related with r_b and η. Beam density n_b is governed by both r_b and η, which are functions obtained by the numerical integration of Equation (9).

Since a model for the dynamical behaviour of the density n_b has been developed, many macroscopic beam quantities can be promptly calculated. And between the many possible quantities, one of them is the emittance.

Emittance has great interest in beam physics once it contains information about its kinetic energy. Essentially, the emittance ϵ is defined in the form

$$\epsilon = \sqrt{4(\langle \mathbf{V}_\perp^2 \rangle \langle \mathbf{R}_\perp^2 \rangle - \langle \mathbf{R}_\perp \cdot \mathbf{V}_\perp \rangle^2)}, \tag{10}$$

in which the angle brackets $\langle \quad \rangle$ denotes phase space average.

While in breathing homogeneous beams emittance is a constant, in the current case it is not. Because r_b and η are functions of s, then the beam emittance ϵ is also expected to be dependant of the time s. Through straightforward algebra, the following ODE is obtained for ϵ

$$\frac{d}{ds}\epsilon^2 = -\frac{1}{18} r_b^2 \eta(\eta+1)\dot{\eta} \tag{11}$$

which is a function of the free parameters of the proposed model, that are the magnitude η of inhomogeneity and the beam envelope r_b.

All the equations presented before, from (1) to (11), are associated with the collective and synchronized movement of the beam. In this way, the equations are adequate to describe just the particles that have a fluid-like behaviour. However, many other particles desynchronize during the beam evolution inside the magnetic focusing channel.

To describe these desynchronized particles, it is necessary to develop a model. The first population of particles, that which have a very organized macroscopic movement, can be identified as the beam core. Thus an adequate model is the one based on particle-core interactions. The core is adequately represented by n_b and the particles can be supposed as test-particles. This particle-core model is described by the following equations

$$\frac{d^2}{ds^2} R + \kappa_{zo} R = F, \tag{12}$$

in which $k_{zo} R$ the magnetic focusing force and

$$F = \begin{cases} K_b\left[\eta \dfrac{R^3}{r_b^4} + (1-\eta)\dfrac{R}{r_b^2}\right], 0 \le R \le r_b \\ K_b/R\,, r_b < R \le r_w \end{cases}, \tag{13}$$

is the force applied by the core in each test-particle. Observe that $F = F(\eta, r_b, R)$ is nonlinear both inside and outside the beam. Then test-particles are nonlinear excited by the core and can, with great potential, exhibit a chaotic dynamics.

RESULTS

The figure 1 presents the solutions for $\eta(s)$ e $r_b(s)$ obtained with numerical integration for long times of equation (9) with specific initial conditions. Note that the dynamics of both quantities are coupled.

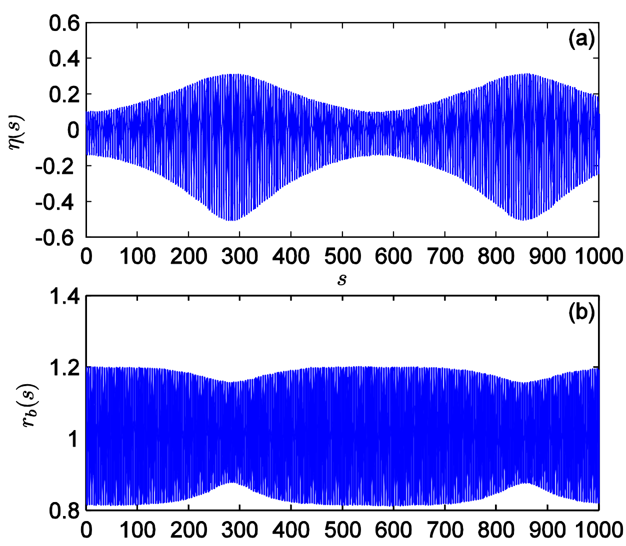

Figure 1: Numerical solutions for (a) inhomogeneity $\eta(s)$ and (b) envelope $r_b(s)$. Initial conditions are $\eta(s=0) = 0.1$, $\dot{\eta}(s=0) = 0$, $r_b(s=0) = 1.2$, and $\dot{r}_b(s=0) = 0$.

Figure 2 shows the comparison between the model and self-consistent N-particle beam numerical simulations for short times. Numerical simulations are based on Gauss' Law: particles at R suffer a resultant force that is due to the action of all particles comprised in the interval $(0, R)$. The beam macroscopic quantities compared are the envelope r_b and emittance ϵ.

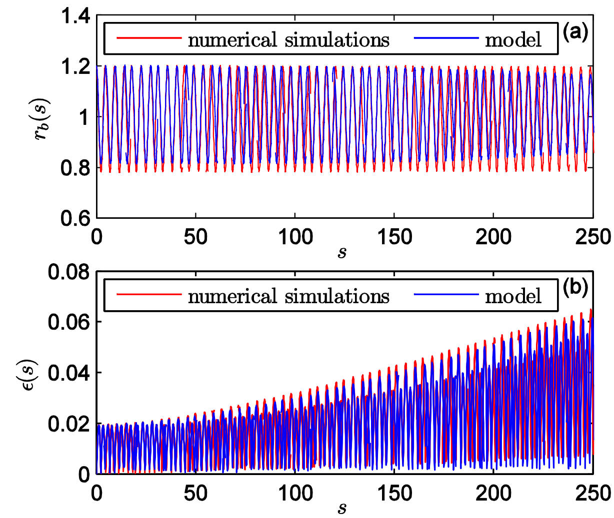

Figure 2: Comparison between the model and numerical simulations for short times. Beam initial characteristics are $\eta = 0.1$ and $r_b = 1.2$. The results are for (a) envelope r_b and (b) emittance ϵ. Numerical simulations employ $N = 10000$ particles.

Figure 3 presents the dynamics of a particle initially positioned inside the beam. The result in panel (a) is obtained with numerical simulations. In panel (b), the dynamics of a test-particle calculated with the particle-core model is shown. The beam particle in Figure 3a has the same initial coordinates of the test-particle of Figure 3b. Note that the model predicts reasonably the way with which the particles are ejected from the beam core.

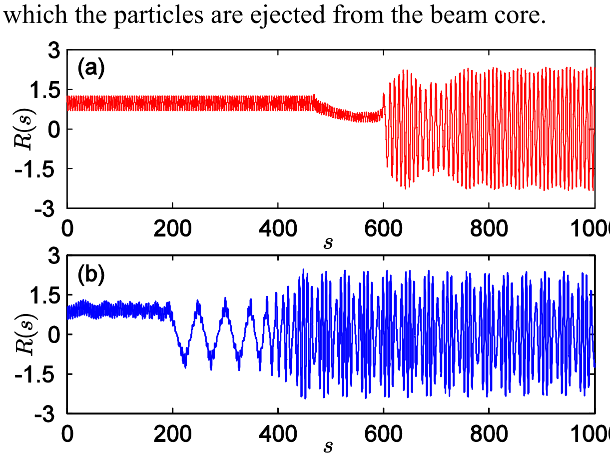

Figure 3: Radial coordinate $R(s)$ of a particle initially disposed at $R(s = 0) = 1$ (inside the beam) obtained with (a) numerical simulations and (b) the particle-core model. The results are for $\eta = 0.1$ and $r_b(s = 0)= 1.3$.

Finally, in Figure 4 the orbit of the beam particle and the test-particle under the action of r_b and η are shown. Both particles are disposed inside the beam with same initial conditions such as the case of Figure 3. The orbits of Figure 4 have been obtained with cumulative Poincaré sections of the original $(R, \dot{R})$ particle orbit. Results in panel (a) are provided by numerical simulations and in panel (b) by the particle-core model. Very similar pattern is observed in both panels.

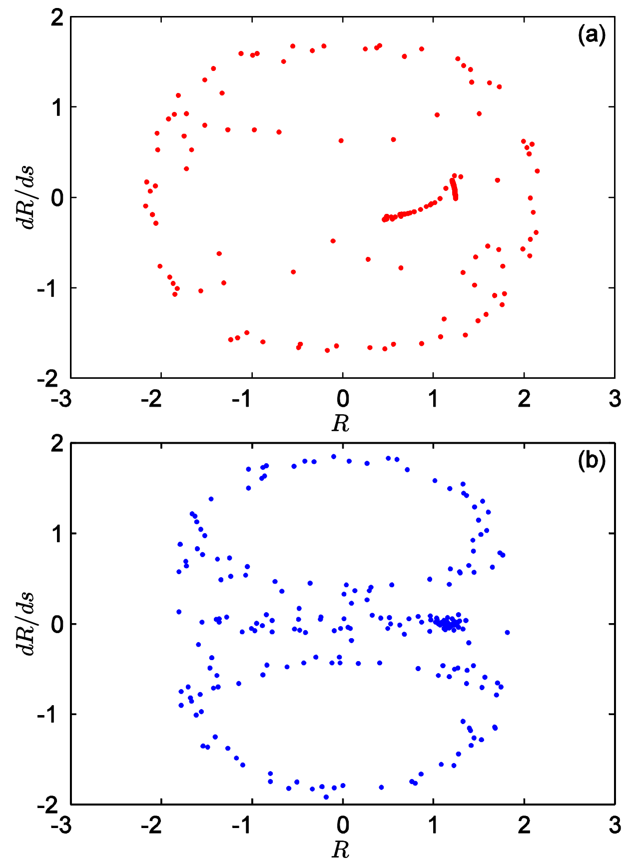

Figure 4: Phase space orbit of a particle initially disposed at $R(s = 0) = 1$ (inside the beam). In (a) the results of numerical simulations and in (b) the results of the particle-core model. $\eta = 0.1$ and $r_b(s = 0) = 1.3$.

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