THE INFLUENCES OF INITIALLY INDUCED INHOMOGENEITY OVER THE DYNAMICS OF MISMATCHED INTENSE CHARGED BEAMS*

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Abstract

Although undesired in many applications, the intrinsic spurious spatial inhomogeneity that permeates real systems is the forerunner instability that leads high-intensity charged particle beams to its equilibrium. In general, this equilibrium is reached in a particular way, by the development of a tenuous particle population around the original beam, conventionally known as the halo. In this direction, the purpose of this work is to analyze the influence of the magnitude of initial inhomogeneity over the dynamics of quasi-homogeneous mismatched beams. For that, all beam constituent particles, which are initially disposed in an equidistant form, suffer a progressive perturbation through a noise of a variable amplitude. Beam quantities are quantified as functions of the noise amplitude, which indirectly is assumed a consistent measure of the initial beam inhomogeneity. The results have been obtained by the means of full self-consistent N-particle beam numerical simulations and seem to be an important complement to the investigations already carried out in prior works.

INTRODUCTION

It is well-known that perfectly round and homogeneous beams do not suffer any thermalization effect under its excursion inside the magnetic focusing system. If the beam is assumed initially cold (all beam particles have initially a negligible velocity), during its linear path inside the magnetic focusing channel it remains cold. Beam particles cannot earn individually extra mechanical energy through the increasing of its own kinetic energy.

All of this is readily possible to show analytically by the calculus of the beam emittance. Emittance [1] is a macroscopic beam quantity that involves the velocity of each beam constituent. Emittance in average depends on a monotonic way of particle velocities: if the particle velocities in average increase during the confinement process, emittance also increases, and vice-versa. However, if the beam is initially cold, which implies emittance to be initially zero, after the beam confinement process the emittance conserves its initial value and thus remains zero. With this in mind, since emittance is a good indicator of the macroscopic increasing of beam kinetic energy, it is possible to say that for pretty homogeneous beams in average particles do not increase their velocities.

However, one will find a much different situation if, instead of considering an initial completely homogeneous beam, one considerer a quasi-homogeneous beam. By a quasi-homogeneous beam it should be understood the situation in which beam particles are not disposed equidistantly one from the other. The direct result of this is that in some region of the beam transversal section, particles are slightly closer while in other regions particles are slightly farther. The beam particles are not dispersed homogeneously anymore, giving rise to some kind of charge concentration/accumulation. This weak inhomogeneity introduced implies the forces inside the beam to be weakly nonlinear. Under the action of this kind of force, the beam particles pass to oscillate with a frequency that is dependant of its initial coordinate [2]. Associated to that, synchronization problems can potentially occur: during their dynamics, particles can eventually collapse their orbits, being expelled/ejected from the beam core. From the beam phase space picture, this ejection looks like a wave-breaking [3][4].

Once out of the core, the ejected particles become susceptible to the influences of the envelope mismatch. These particles are drive by the oscillation of the core and thus progressively acquire velocity. Since the velocity of some particles increase, the beam emittance also increases. In this way, due to the inhomogeneity, even initially cold beams suffer some heating and present an emittance growth that cannot be neglected. Note: beams particles just can be excited by the core movement because they are out of the beam [5]. While beam envelope mismatch scales its growth, the initial inhomogeneity acts as the forerunner mechanism that allows emittance increases [6].

Perfectly homogeneous beams are such an idealization pretty hard to achieve in the engineering sense. Always some inhomogeneity will be present and its influences over the beam dynamics are then something to be better investigated. Following this reasoning line, the purpose of this work is to evaluate how the time scale of the emittance growth depends of the magnitude of the initial inhomogeneity introduced in the beam density. In prior works [2][6], it has been found that the time scale of emittance growth has contributions from the initial inhomogeneity and mismatch. Also, it is of interest to verify qualitatively how beam quantities behave at equilibrium as a function of the initial inhomogeneity. The system considered here is an initially cold and azimuthally symmetric beam, which is focused by a constant magnetic field. The beam evolves in a linear channel surrounded by a conducting pipe. The z axis of the coordinate system adopted is aligned with the longitudinal direction. The radial coordinate $r = r(x, y)$ explores the beam transversal section. The quantity s is the time.

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THE APPROACH

The key point to achieve the goal is how to turn the beam density inhomogeneous in a controlled way. That is, how to progressively introduce inhomogeneity in the beam density in a way that its effects over the beam dynamics could be identified and then quantified.

One interesting approach is to suppose that initially all the beam particles are disposed in an equidistant form. However, each beam particle at radial coordinate r_i starts to be affected by a noise ξ_i. The generated noise ξ_i, comprised in the interval $[-1;1]$ with some probability distribution $P(\xi_i)$, has its intensity controlled by the parameter A, here defined as the noise amplitude, which is a constant. In this context, the coordinate r_i of each particle i of a beam composed by N constituents can be expressed as

$$r_i = (i + A\xi_i)\bar{r} \tag{1}$$

in which $\bar{r} = 1/N$ is a characteristic spatial scale associated with the distance between beam particles. For any particle, parameter A satisfies the condition $A\xi_i\bar{r} \ll r_0$, in which r_0 is the initial beam envelope. If $A = 0$, the particles are distributed homogeneously, composing what is called, for compactness, a crystalline beam. Nevertheless, if $A \neq 0$, the initial noise starts to perturb the spatial coordinate of each particle, imposing that fluctuates around $i\bar{r}$. Once probability distribution $P(\xi_i)$ of ξ_i is not uniform and N — although usually large — is finite, the noise introduced in the beam does not affect randomly each particle.

The noise breaks the equidistance characteristic between particles existent before, given rise to the concentration or accumulation of charge mentioned in the previous section. The beam loses its initial crystalline structure and starts to be inhomogeneous. Since parameter A controls the noise intensity, and the noise induces spatial fluctuations, indirectly A is a significant measure of the magnitude of inhomogeneity that exists in the beam density. Greater is A, more inhomogeneity permeates the beam. Although the beam cannot be considered homogeneous anymore because in fact $A \neq 0$, it is quasi-homogeneous, since the condition $A\xi_i\bar{r} \ll r_0$ is satisfied.

RESULTS

The influences of the inhomogeneity over the beam dynamics has been analyzed through full self-consistent N-particle beam numerical simulations. The method employed to simulate numerically the system is based on Gauss' Law: particles interact with each other just by the means of the generated electromagnetic fields. No collisions exist and thus just collective effects are accounted, such as desired. All the results have been obtained for $N = 10000$ particles, which has proportioned convergence to the results. The initial beam envelope for all numerical simulations is $r_0 = 1.5$. This means a 50% mismatch, once beam envelope of equilibrium has been set to $r_{\text{eq}} = 1$ by scaling. Fixing the beam envelope, it is expected that any influence of the envelope mismatch is suppressed and the effects of the inhomogeinity induced by the noise over the initial beam density can be better quantified. The only parameter that is varied between all the numerical simulations is the amplitude of noise A.

Figure 1 presents results that exemplify how the fluctuations introduced by the noise acts over the initial beam superficial density n_b. Panel (a) of this figure shows the result for the completely homogeneous case given by $A = 0$. As expected, no spatial fluctuations can be seen, and n_b is strictly constant. However, as noise amplitude A is increased from zero, spatial fluctuations in n_b arise. The beam becomes weakly inhomogeneous. The effects of the noise amplitude over the surface beam density are shown in Figure 1b for $A = 100$. The fluctuations along the radial coordinate r of the superficial density n_b can be clearly perceived.

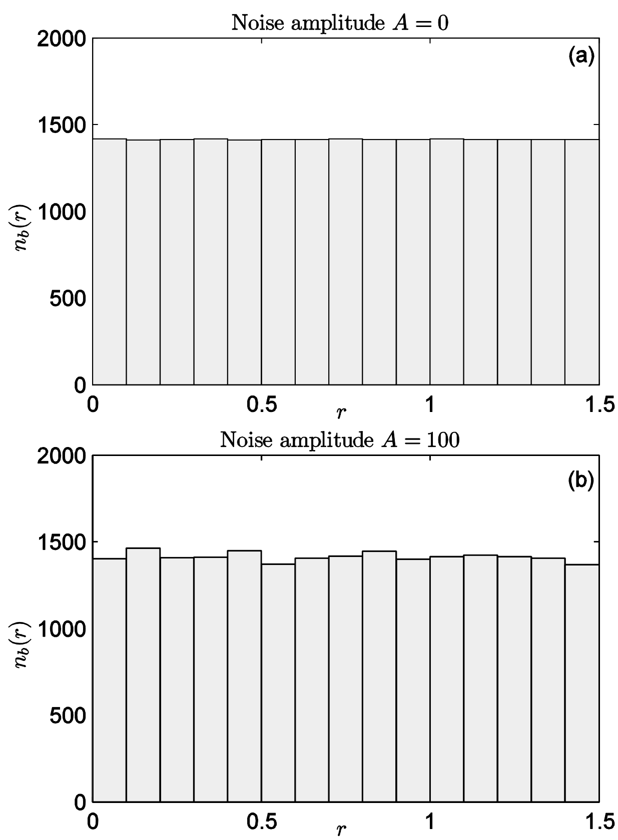

Figure 1: The initial density of beam particles $n_b(r, s = 0)$ for two distinct values of noise amplitude A. In panel (a) $A = 0$, the completely homogeneous beam case, and in panel (b) $A = 100$, a typical quasi-homogeneous beam. Note the spatial fluctuations in n_b due to the noise.

The influences of the noise amplitude in the density n_b can be also visualized in Figure 2, which presents the initial transversal beam section for $A = 100$. The noise perturbs the beam crystalline structure so particles accumulate in some regions, living gaps in its original unperturbed positions and consequently giving rise to the desired inhomogeneity in the superficial beam density n_b.

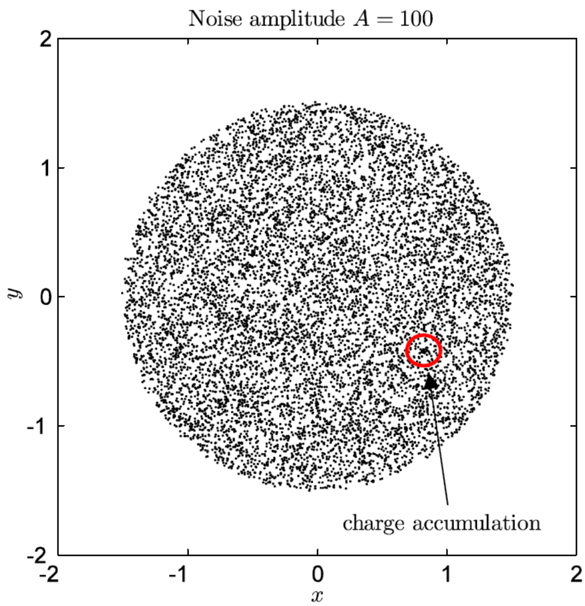

Figure 2: The spatial appearance of the initial beam density $n_b(r, s = 0)$. The amplitude of noise that perturbs the beam crystalline structure is $A = 100$. The process of charge accumulation induced by the noise is detached.

The envelope r_b and emittance ϵ as the beam evolves inside the focusing channel is calculated in respectively panels (a) and (b) of Figure 3. The value of the noise amplitude are $A = 0$ and $A = 100$ again.

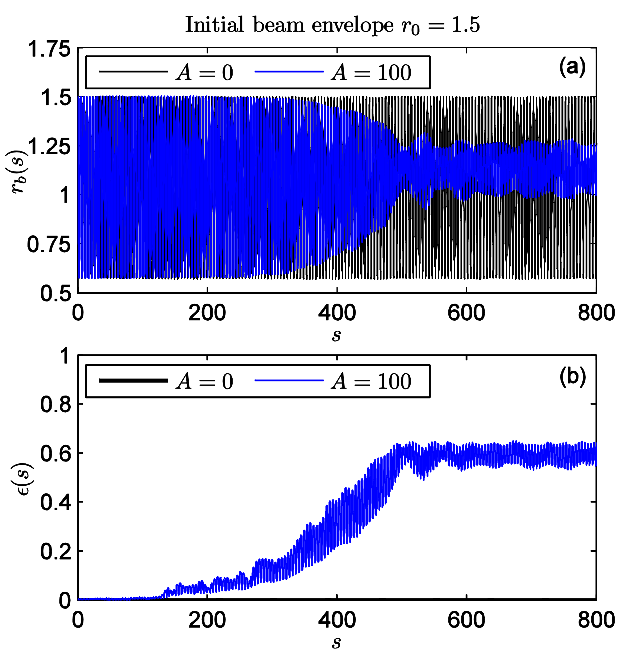

Figure 3: The beam (a) envelope r_b and beam (b) emittance ϵ as functions of the time s for $A = 0$ and $A = 100$. Results for $0 \leq s \leq 800$.

The noise determines that a small quantity of particles is ejected from the beam core and then drive by the envelope oscillations. Macroscopic potential energy from the envelope is transformed into kinetic energy for this small group of particles. The beam envelope decays as time evolves and beam emittance grows from zero to some equilibrium value. As expected, no emittance growth is observed for $A = 0$. The Figure 4 shows the emittance growth for different initial inhomogeneity magnitudes, specified by the noise amplitude $A = \{0,5,25,125,625\}$.

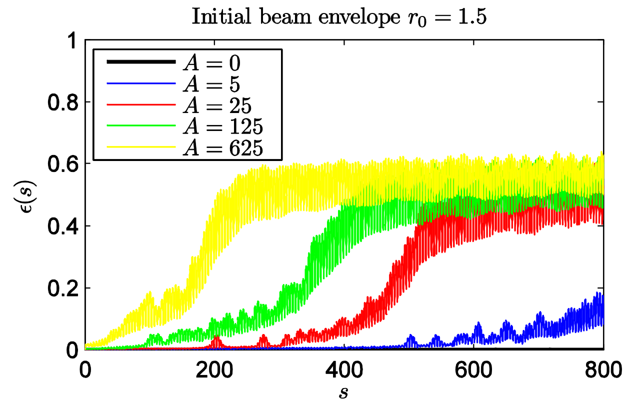

Figure 4: The emittance $\epsilon(s)$ for different values of noise amplitude A. Observe how the time scale of the emittance growth strongly depends of the noise amplitude A.

FINAL REMARKS

The initial inhomogeneity is determinant for the emittance growth observed in charged particle beams and has impact over its time scale. However, recent results show that no influences over macroscopic quantities such as envelope and emittance have been observed at the beam equilibrium. That is, r_b and ϵ at equilibrium do not depend of A. Future works will be deeper in this issue.

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