COHERENT SYNCHROTRON RADIATION SIMULATIONS FOR THE CORNELL ENERGY RECOVERY LINAC*

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Abstract

Coherent Synchrotron Radiation (CSR) can be a detrimental effect on particle bunches with high charge and short bunch lengths. CSR can contribute to an increase in emittance and energy spread, and can limit the process of bunch compression. It is especially important in Energy Recovery Linacs (ERLs), because any relative energy spread induced at high energy is magnified after deceleration, and any energy lost by the particles is energy that cannot be recovered. Here we present CSR simulation results using the particle tracking code BMAD for the main operation modes in the proposed Cornell ERL, including an additional bunch compression mode. These simulations consider the effect of CSR shielding, as well as CSR propagation between bends.

INTRODUCTION

When a charged particle is transversely accelerated in a bending magnet, it produces radiation according to the well-known synchrotron radiation spectrum. When N such particles are bunched on a scale of length σ, the power spectrum per particle at frequencies less than c/σ in this spectrum is enhanced by roughly a factor N, resulting in increased radiation, and likewise increased energy losses from the individual particles. This Coherent Synchrotron Radiation (CSR) was first calculated in a seminal paper by Schwinger [1].

Here we are concerned with the effect of this radiation on the particle distribution. We refer to the CSR wake $W_{\text{csr}}(z)$ as the energy change per unit length of a particle with longitudinal position z in a bunch, and it can be shown that for ultra-relativistic particles this W_{csr} scales with the factor

$$W_0 \equiv N r_c m c^2 \left(\frac{\kappa}{\sigma^2}\right)^{2/3}, \qquad (1)$$

in which m is the mass of a single particle, r_c is its classical electromagnetic radius, and κ is the curvature of its trajectory (see, for example, Ref. [2]).

Additionally, the low frequencies of the radiation spectrum are "shielded" when the particles travel inside a conducting structure. Also in Ref. [1], Schwinger calculates the radiation spectrum for particles traveling on a circle between two infinite conducting parallel plates, and concludes that the lowest frequencies are suppressed depending on the plate separation height h, as well as κ and σ.

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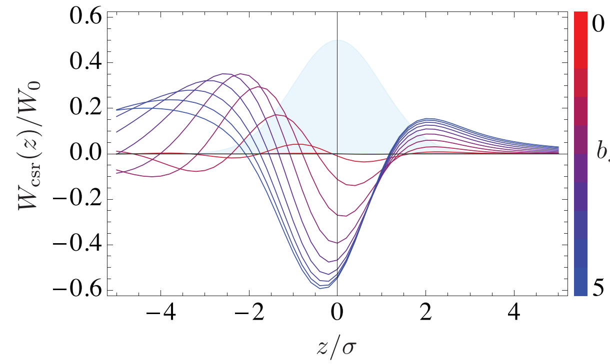
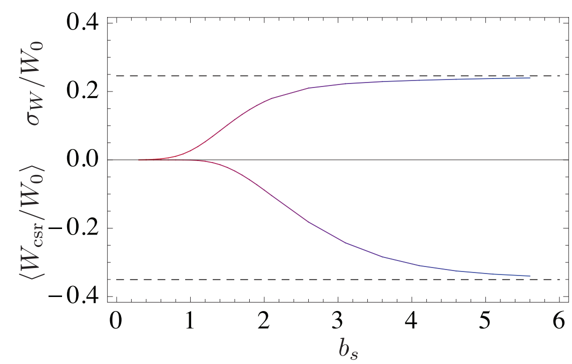


(a) Shielded CSR wakes for various b_s



(b) Average and standard deviation of the wakes in Fig. 1(a)

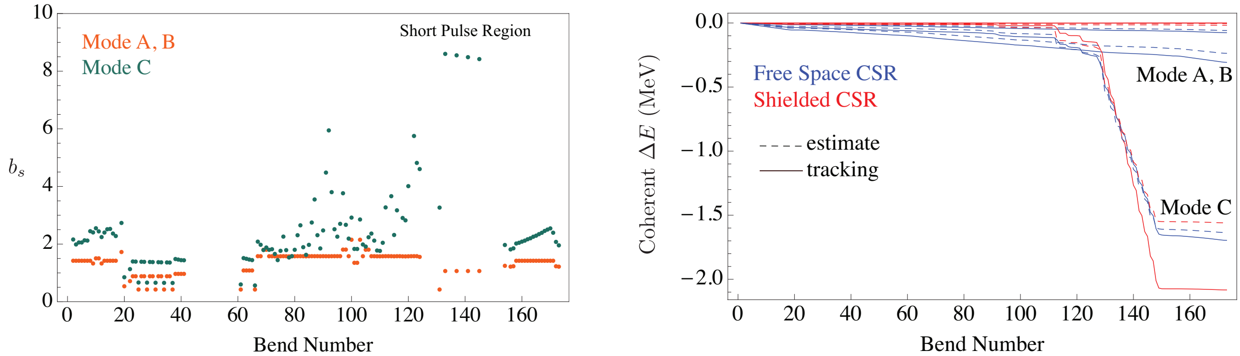
Figure 1: Normalized shielded steady-state CSR wakes W_{csr} for a Gaussian bunch of length σ, along with average energy loss and energy spread from these curves for various shielding parameters b_s.

His formulas, however, are difficult to evaluate numerically due to the presence of very high order Bessel functions, but fortunately an excellent approximation for ultra-relativistic particles can be found in the appendix of Ref. [3]. Some manipulation reveals that this suppression depends entirely on the shielding factor

$$b_s \equiv h \left(\frac{\kappa}{\sigma^2}\right)^{1/3}, \qquad (2)$$

and the longitudinal bunch density (see section 5.2.2 in Ref. [4] and a heuristic argument in Ref. [5]).

To visualize this suppression, Fig. 1(a) shows the CSR wakes for a bunch with Gaussian longitudinal density for various b_s. The averages and standard deviations of these wakes over the distribution are shown in Fig. 1(b), revealing that shielding plays a significant role when $b_s \lesssim 3$. The dashed lines are drawing at the limiting values of these curves, and are approximately -0.35 and 0.25. We refer to this as the "free-space" regime.



(a) b_s for all bends in the Cornell ERL

(b) Cumulative energy loss estimates and tracking results

Figure 2: The shielding parameter b_s at all bends in the Cornell ERL for the different bunch operation modes is shown in Fig. 2(a). Modes A and B use the same optics, and bunches in this mode have durations of $\sigma_z/c \simeq 2\text{ps}$. An optional Mode C compresses the bunches to 25 fs. Figure 2(b) shows an estimate for the cumulative coherent energy losses through these bends using the bottom curve in Fig. 1(b), along with particle tracking results from *Bmad* that take into account all entrance and exit transients for the CSR wake.

SIMULATIONS

The proposed Cornell ERL has operating modes A and B, which are detailed in Ref. [6]. For the purposes here, we only need to know that Mode A uses bunches of charge 77 pC and a bunch duration of $\sigma_z/c \simeq 2\text{ps}$ and Mode B has this same length but with 19 pC of charge. Here we also show calculations for an optional Mode C that uses 19 pC bunches but starting at $\sigma_z/c \simeq 1\text{ps}$. All modes operate at a 1.3 GHz repetition rate. Additionally, this Mode C compresses the bunch at high energy to approximately 25 fs. With the design bunch lengths, the b_s for these modes in all of the bends in this machine are shown in Fig. 2(a). This plot suggests that CSR in modes A and B will be strongly shielded, but that Mode C will not be, especially in the short pulse region. Simulations for a mode with special high charge (1 nC) non-energy recovered bunches are presented in Ref. [7].

To more accurately calculate CSR in this machine, we use the particle tracking code *Bmad* [8], which simulates shielding using the image charge method, and is able to calculate all of the entrance and exit transient effects that are not seen in curves like Fig. 1(a). This code has been benchmarked against other codes as well as analytical formulas in Ref. [5]. For comparison, we run three types of tracking simulation: one with no CSR effects, one with free space CSR, and one that includes shielding by the conducting vacuum chamber.

Figure 2(b) shows the cumulative energy losses due to CSR by tracking 10^5 macroparticles through the 2.8 km ERL lattice, along with estimates from the values in Fig. 2(a). As expected, Modes A and B are well shielded, but Mode C suffers in the short pulse region. Interestingly, the calculation with shielding loses the most energy. This is because the calculation with free space CSR suffers so much in the compression process that its final compressed bunch duration is 31 fs, whereas the shielded bunch is able to be compressed to achieve a duration of 23 fs. At this shorter length, the shielded bunch (now in fact not shielded at all, with $b_s \approx 8$) radiates energy at a rate of a factor $(31/23)^{4/3} \approx 1.5$ more than the free space bunch.

Figure 3 illustrates this limiting of the bunch length in Mode C as the charge is further increased. With higher charge, the longitudinal phase spaces at the first short pulse undulators are increasingly warped and have increasing durations. There are five short pulse undulators which are joined by isochronous achromatic sections, and in the last such undulator one can roughly see the application of a curve similar to the $b \to \infty$ CSR wake in Fig. 1(a). Also notice that the horizontal emittance is greatly degraded as the charge is increased. The 19 pC bunch, however, is relatively undamaged.

Finally, Fig. 4 shows the emittances and transverse beam sizes in Mode C at all 14 undulators from particle tracking. Undulators 1–9 all receive the non-compressed 1 ps bunches and suffer very little CSR damage. The bunches in short pulse undulators 10–14, however, experience significant increases in horizontal emittance. The CSR shielding effect here marginally limits this damage compared to free space CSR, but more critically allows these bunches to be compressed to shorter lengths.

SUMMARY

In short, these simulations show that the performance of the proposed Cornell ERL in modes A and B is not limited by CSR. Bunches in an optional short pulse Mode C are able to achieve a duration of 25 fs, but suffer significant emittance degradation. For an accurate simulation of this mode, the effect of CSR shielding must be included.

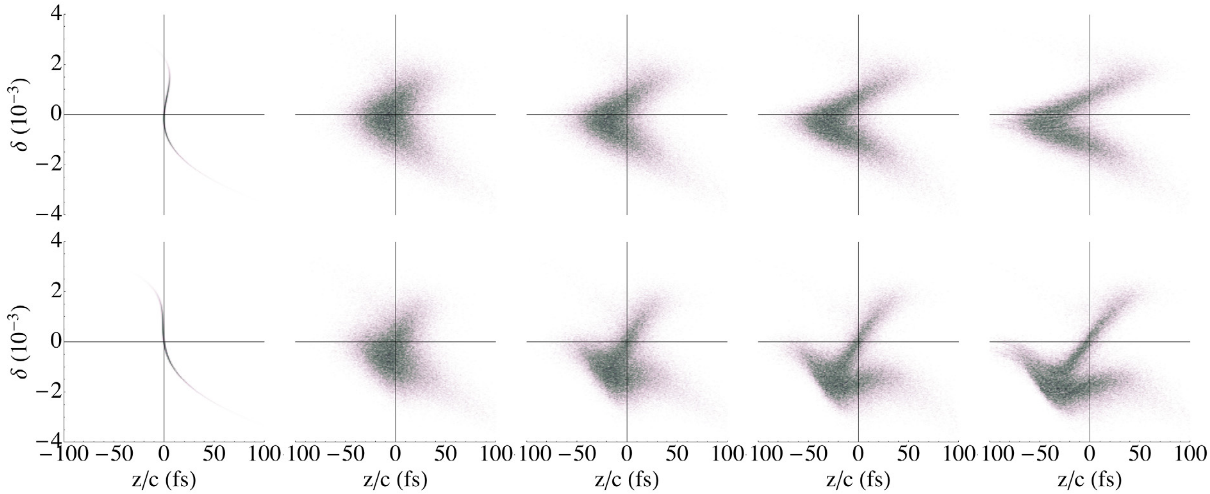
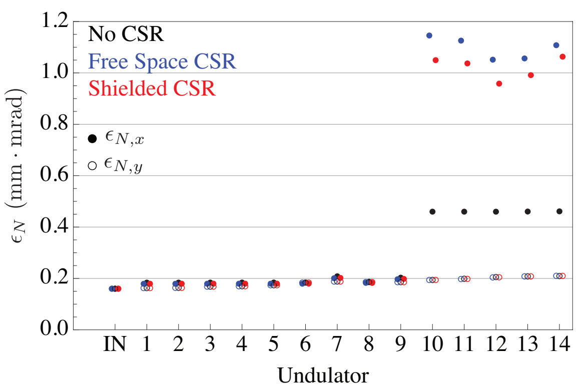
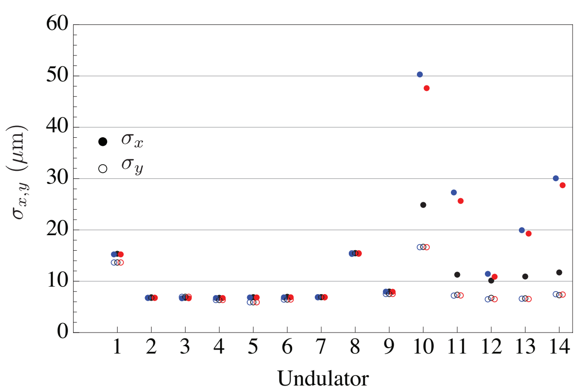


Figure 3: Longitudinal phase space for various bunches in the first short-pulse undulator (top row) and last short-pulse undulator (bottom row) from *Bmad* simulations. From the left, the bunch charges are: $(0, 19, 38, 57, 76)$ pC. The blurring of the distribution is mainly due to incoherent radiation. The leftmost plot which shows the designed bunch, which has all incoherent and coherent radiation effects turned off. The bunch durations in the first row are approximately $(10, 23, 27, 29, 33)$ fs, and the corresponding normalized horizontal emittannces are approximately $(0.2, 1.0, 1.9, 3.0, 4.4)$ mm-mrad.



(a) Normalized emittances $\epsilon_{N,x}$, $\epsilon_{N,y}$



(b) Transverse beam sizes

Figure 4: Normalized emittance and transverse beam sizes in all of the undulators in the Cornell ERL from tracking simulations in *Bmad*.

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