

STABILITY ANALYSIS OF FREE-ELECTRON LASER RESONATORS

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Abstract

The stability of free-electron laser (FEL) resonators differs from that of resonators of conventional lasers, because of the nature of the FEL interaction. We study the stability of FEL resonators (especially for g_1 not equal to g_2) using simulations, as well as using a simple thin-lens model, and show that the near-concentric configuration is preferable, while the confocal configuration becomes unstable. Thus, longer cavities are preferred, which could be an advantage for high average-power FELs. Resonator stability imposes constraints on the wavelength tuning range of the FEL, with operation becoming unstable at shorter wavelengths. A systematic study of the dependence on cavity length shows that as the wavelength is decreased, mode competition first sets in, and the FEL takes longer to reach saturation. At even shorter wavelengths the FEL simply does not lase.

INTRODUCTION

For conventional lasers, the preferred cavity configuration is generally the confocal one and is well studied in standard texts [1] using an empty-cavity analysis. In the case of free-electron lasers (FELs), the presence of an undulator within the resonator introduces a strong, nonlinear, interaction, that must necessarily affect the stability of the system. Hence the empty cavity analysis is not expected to be sufficient. For FELs it is typically found that the near-concentric configuration is experimentally preferred. Stability of modes for symmetric FEL resonators (i.e. $g_1 = g_2$) has been studied in Ref.[2]. It was found that changing the geometry of the resonator affects its stability, and that near-concentric resonators are indeed the most stable. In next section of this paper we extend the previous work and study this effect for asymmetric resonators (i.e. $g_1 \neq g_2$), both analytically (using a simple thin lens model), and with the simulation code TDAOSC [2,3] (that uses the full, nonlinear, FEL equations of motion). In Section 3 we study the stability of the resonator as a function of the FEL wavelength, and in Section 4 we investigate the dependence of stability of an FEL resonator on the length of the cavity.

THIN LENS ANALYSIS

Consider a resonator comprising two concave mirrors with radii of curvature R_1 and R_2 , respectively, placed at a distance D apart. The stability condition for such an empty

cavity is given by [1]

$$0 \leq g_1 g_2 \leq 1, \quad (1)$$

where, $g_1 = 1 - D/R_1$, and $g_2 = 1 - D/R_2$. The FEL interaction will tend to focus the laser beam, and can therefore be modeled as a lens. In the simplest case, it can be modeled as a thin lens, of focal length f , placed at the centre of the resonator. For this cavity configuration the stability condition can be calculated, using a matrix analysis, to be,

$$0 \leq 2g_1 g_2 - g_1 - g_2 + h[2g_1 g_2 + g_1 + g_2] \leq 4, \quad (2)$$

where $h = 1 - D/2f$. Therefore only those cavity configurations which satisfy Eq.(2) will be stable. Clearly, $h = 1$ ($f = \infty$) corresponds to the empty cavity case. It is optimal to choose the focal length of the FEL thin lens to be equal to $D/2$, so that the radiation is focused onto the outcoupling hole, and maximum power can be outcoupled.

Figure 1 shows the g_2 vs g_1 curves for these two values of h , $h = 1$ ($f = \infty$) and $h = 0$ ($f = D/2$) obtained using Eq.(2) (circles and squares, respectively). The effect of the thin lens with $h = 0$ ($f = D/2$) is to widen the stability region in the first quadrant (i.e. g_1 and g_2 both positive), relative to the empty cavity case. Whereas, in the third quadrant (i.e. g_1 and g_2 both negative) the stability region reduces at higher values of g_1 and is nearly same as empty cavity case for low values of g_1 . Overall, therefore, it can be seen that stability is compromised near the confocal region, whereas for near-concentric configurations, longer cavities remain stable. This is an advantage, because with longer cavities one can reduce the power densities on the mirrors - an issue that could be particularly significant for high average-power FELs.

Simulation Results

In order to investigate the validity of the above, rather simple, analysis, we performed computer simulations using the time-independent, axisymmetric code TDAOSC. We chose a radiation wavelength $\lambda_R = 25 \mu\text{m}$, undulator parameter $a_w = 0.637$, undulator length $L = 2$ m, and undulator period $\lambda_U = 5$ cm. We chose a cavity of length $D = 12.3$ m with concave mirrors (i.e. the stability parameter g is negative for both the mirrors). One of the mirrors has a hole to outcouple the cavity power. In the simulation we varied R_1 and R_2 in such a way that it always satisfied the empty cavity stability condition with $g_1 g_2 = 1$. Now, if the FEL oscillator is stable then it should lase and a high output power should be obtained. But we observed that only certain configurations which are very close to thin lens model

stability criteria could lase and others don't. Figure 1 shows the comparison between empty cavity, thin lens model and TDAOSC simulations. From Fig. 1 one can see that our simple thin lens model, where the FEL is modeled as a thin lens at the centre of the cavity, with focal length half the cavity length, agrees well with the simulations. We have confirmed that for values of f different from $D/2$, the results do not agree well with the simulations.

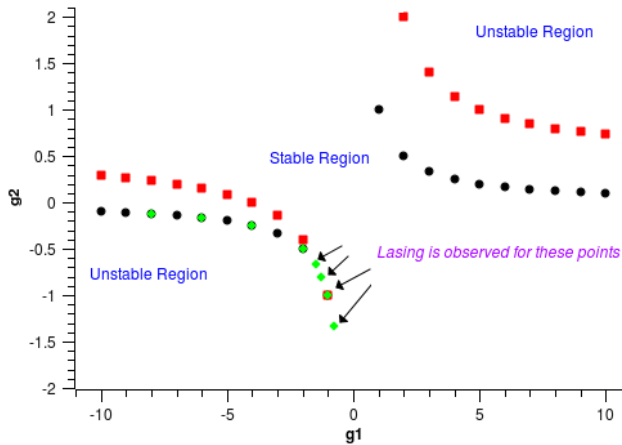


Figure 1: Comparison between empty cavity (black circles), thin lens model (red squares) and TDAOSC simulation (green diamonds).

WAVELENGTH DEPENDENCE

One of the major advantages of the FEL over conventional lasers is that FELs are widely tunable in wavelength. In actual operation, it is not possible to change the mirrors or the cavity length on the fly. It is therefore important to explore the stability of FEL resonators as a function of wavelength. To this end we performed TDAOSC simulations for both, a concentric cavity ($D = 12.3$ m and $R_1 = R_2 = 6.15$ m) as well as a confocal cavity ($D = 6.15$ m and $R_1 = R_2 = 6.15$ m), at various wavelengths. All other parameters, including the hole size (4 mm), were fixed. Figure 2 shows the output power as a function of wavelength, for both concentric and confocal cases. It can be seen that at long wavelengths, there isn't much difference between the confocal and the concentric resonators. This is because the size of the optical beam is much larger than the hole size, the outcoupled fraction is small and so the mode is not strongly perturbed. However, at shorter wavelengths, resonator configuration has a strong effect on the out-coupled power. The concentric resonator delivers much higher power, and is therefore clearly preferable. The reason for this is evident from Fig. 3, which shows the optical beam profile (mode) on the out-coupling mirror, for both cases, at a wavelength of $25 \mu\text{m}$. The concentric resonator supports a near-Gaussian mode, and so the out-coupled power is high. For the confocal resonator, how-

ever, the dominant mode is completely non-Gaussian - it has a minimum at the centre, and therefore the outcoupled power is lower. The situation worsens as you go to shorter wavelengths. Hence, resonator stability imposes important constraints on the wavelength tuning range of an FEL.

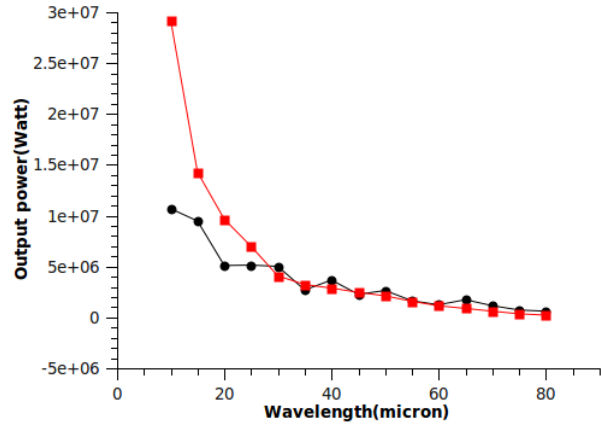


Figure 2: Output power as a function of wavelength for the concentric (red squares) and confocal (black circles) configurations.

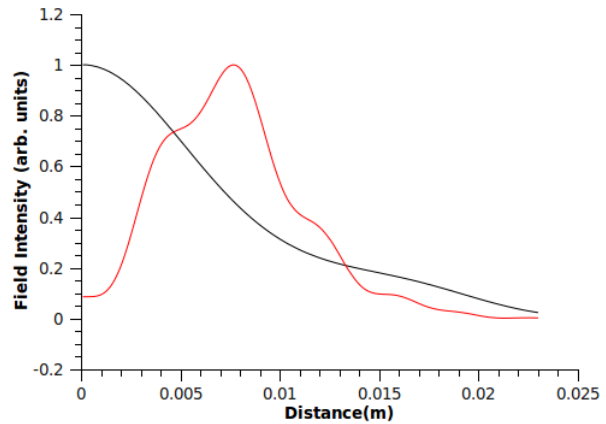


Figure 3: Mode profile for concentric (black) and confocal (red) configurations on out-coupling mirror for $25 \mu\text{m}$ case for hole radius of 4 mm.

CAVITY LENGTH DEPENDENCE

Another, and more practical, way to change the stability parameter g , is to keep the mirror radii fixed, and vary the length D of the cavity. We studied this for a range of wavelengths. We found that at longer wavelengths the out-coupled power changes little with cavity length, but at shorter wavelengths the length of the cavity plays a significant role in determining the stability of the resonator. Figure 4 show the output power as a function of cavity

length, from which it is clear that for a radiation wavelength of $10\ \mu\text{m}$, only configurations very close to confocal or very close to concentric could lase. For longer wavelengths, $25\ \mu\text{m}$ to $80\ \mu\text{m}$, lasing was observed at all cavity lengths starting from confocal to concentric, but the output power obtained was low due to greater diffraction at longer wavelengths. Figure 5 shows the cavity power as a function of pass number for a cavity length of $10.3\ \text{m}$, which indicate nearly smooth saturation of cavity power for $50\ \mu\text{m}$ and $80\ \mu\text{m}$. But for $25\ \mu\text{m}$ the intra-cavity power oscillates from pass to pass, indicating mode competition, and it takes much longer to reach the saturation. Further decrease in wavelength causes the instability to grow so much that at $10\ \mu\text{m}$ no lasing is observed.

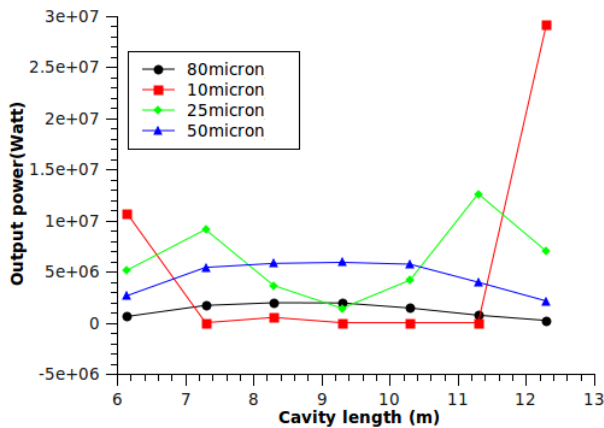


Figure 4: Output power as a function of cavity length.

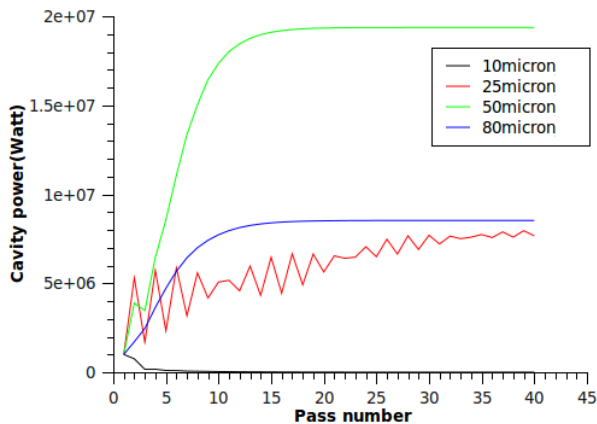


Figure 5: Cavity power as a function of pass number for $D = 10.3\ \text{m}$.

CONCLUSION

In section 2 we discussed the stability of an FEL oscillator using simple thin lens model, and were able to investigate, analytically, the stability of an asymmetric resonator

($g_1 \neq g_2$) with an internal FEL undulator. We found that the region of stability is different from that of an empty resonator. In particular, that the stability regime can extend beyond that of the empty cavity, and favours near-concentric configurations. Also with our analytical model, we could explain the TDAOSC simulation results.

Since for all practical cases a single cavity should give good output power for all wavelengths, we performed simulations and showed that at shorter wavelengths the output power obtained is larger than at longer wavelengths. This is due to greater diffraction at longer wavelengths. We also found that at shorter wavelengths (less than $25\ \mu\text{m}$) substantial output power can be obtained only when the cavity configuration is close to confocal or concentric but for longer wavelengths (greater than $25\ \mu\text{m}$) a stable but low output power is obtained at all cavity lengths starting from confocal to concentric.

In conclusion, for practical FEL oscillator applications the near-concentric resonator configuration is preferable over confocal configuration because concentric case gives more output power and a good mode profile on the out-coupling mirror. The range of wavelengths over which an FEL can be usefully operated, is constrained by the resonator stability.

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