BEAM DYNAMICS IN COMPTON STORAGE RINGS WITH LASER COOLING*

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Abstract

Compton sources are capable to produce intense beams of gamma-rays necessary for numerous applications, e.g. production of polarized positrons for ILC/CLIC projects, nuclear waste monitoring. These sources need high current of electron beams of GeV energy. Storage rings are able to accumulate a high average current and keep it circulating for a long time. The dynamics of circulating bunches is affected by large recoils due to emission of energetic photons. We report results of both an analytical study and a simulation on the dynamics of electron bunches circulating in storage rings and interacting with the laser pulses. The steady-state transverse emittances and energy spread, and dependence of these parameters on the laser pulse power and dimensions at the collision point were derived analytically and simulated. It is shown that the transverse and longitudinal dimensions of bunches are dependent on the power of laser pulses and on their dimensions as well. Conditions of the laser cooling were found, under which the electron bunches shrink due to scattering off the laser pulses.

INTRODUCTION - COMPTON RINGS

Compton rings are perspective bright sources of electromagnetic radiation ranged from hard x rays to gamma rays due to their ability to store high current beams and keep them circulating for a long time. With developing of the lasers with high average power and optical resonators capable to accumulate high power and dense pulses, the intense Compton sources are coming into play (see e.g. [1]).

The dynamics of circulating bunches is affected by large recoils due to emission of energetic photons. With developing of the optical resonators storing the powerful high density pulses, the beam behavior will be changed. The report is aimed to present a survey of the results of theoretical study on the beam dynamics in Compton storage rings. The results are verified by the simulations.

Steady Parameters

Conservative Hamilton system (ensemble of oscillators) holds occupied by particles phase volume conservative. Thus, initial emittance preserved in time (saying nothing of its magnitude). To get the equilibrium specific emittance, the system should open to externals: become nonconservative, [2, 3].

Perturbations of the conservative system in general can be decomposed into (a) perturbation of the potential function, (b) excitation, and (c) damping. The equilibrium state of the system (distribution in phase space) is settled due to balance of excitation and damping.

Perturbed canonical equations have a form

$$\begin{aligned} \dot{x} &= p\ ; \\ \dot{p} &= -U'_x(x) + F(p,t)\ . \end{aligned} \tag{1}$$

Here $F(p,t)$ is a random function representing the recoils due to scattering off laser photons and emission of the synchrotron radiation.

Both the longitudinal and transversal dynamics of circulating particles may be reduced to the form (1). With the random function dependence on the variables of the system (multiplicative noise), the steady solutions to (1) may exist. This solutions govern with the momenta of the random function $F(p,t)$:

$$\begin{aligned} \lambda &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left\langle \int_t^{t+\Delta t} F(p,t)\,\mathrm{d}t \right\rangle\ ; \\ S &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left\langle \left[\int_t^{t+\Delta t} F(p,t)\ \mathrm{d}t\right]^2 \right\rangle\ . \end{aligned} \tag{2}$$

The stationary solution to (1) is given by:

$$\rho_{\mathrm{st}}(x,p) = N\exp\left(-\frac{2\alpha}{S}\tilde{\mathcal{H}}\right)\ , \tag{3}$$

where N is the normalizing factor; $\alpha = \left.\mathrm{d}\lambda_2/\mathrm{d}p\right|_{p=0}$;

$$\tilde{\mathcal{H}} = \frac{p^2}{2} + U(x) + \lambda\phi \tag{4}$$

is the 'averaged stochastic Hamiltonian.' The last term in (4) appears in the longitudinal dynamics only. It represents the fact that in the Compton scattering the electron lost a portion of its energy and never gain.

RESULTS OF ANALYTICAL STUDY

'Linear' Approach. Steady State

We considered a model of Compton ring with the collision point (CP) set in the dispersion–free section with minimum of the betatron functions. For a simple model of seldom interaction of the electrons with a 'wide' laser pulse

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(uniform distribution of photons within the bunch volume), and making use of the Fokker–Plank–Kolmogorov method, the results are as follows (see [2]). Also we neglected the last term in (4): the energy losses during the synchrotron oscillation are negligible as compared with the separatrix height (energy acceptance).

Partial energy spread – induced by Compton interactions alone – reads (see [4]):

$$\sigma_p^2 = \frac{7}{10}\gamma\gamma_{\rm las} \ , \tag{5}$$

where $\sigma_p \equiv \sqrt{\langle(\gamma_i-\gamma)^2\rangle}\Big/\gamma$ is a relative r.m.s. spread; γ_i, γ are Lorentz–factors for individual electrons and for the synchronous particle, resp.; $\gamma_{\rm las} = E_{\rm las}\,/m_0c^2$ the ratio of the energy of the laser photon to electron's rest energy (the equivalent photon Lorentz factor).

Partial transverse emittances read as

$$\epsilon_{x,z} = \frac{3}{10}\beta_{x,z}^{(CP)}\frac{\gamma_{\rm las}}{\gamma} \ , \tag{6}$$

with $\beta_{x,z}^{(CP)}$ being magnitudes of the betatron functions at CP.

The transverse rms dimensions of the bunch at CP corresponding to emittances (6) are

$$\sigma_{x,z}^2 = \frac{\beta_{x,z}^2\gamma_{\rm las}}{3\gamma} \ . \tag{7}$$

As it can be seen from the derived expressions, the longitudinal size of the bunches (squared) is proportional to the energy of electrons, while the transverse sizes inversely proportional to it. Also the transversal dimensions of the bunch in CP are proportional to magnitude of the betatron function in this point. It should be emphasized that Compton partial steady–state emittances are independent on the intensity of Compton interactions.

'Quantum' Lifetime

The derived above density distribution (3) enables us to determine the beam losses caused by fluctuational nature of the Compton interactions. (The inverted loss rate is referred to as 'quantum' lifetime.)

Restricting the consideration to the case of a small average energy of scattered–off photons as compared with the energy acceptance of the ring:

$$\gamma^2\gamma_{\rm las}\sqrt{\frac{2\pi\eta h}{\gamma\gamma_{\rm rf}}} \ll 1 \ .$$

Here η is the linear momentum compaction factor; h the harmonic number; $\gamma_{\rm rf} = eV_{\rm rf}\,/m_0c^2$ the reduced rf voltage.

Relative 'quantum losses' rate is equal to:

$$\tau_{\rm qf}^{-1} = \frac{\omega_0 n_{\rm x}}{2\pi}\sqrt{\frac{9\gamma_{\rm rf}}{2\pi\gamma\eta h}}\exp\left(-\frac{3\gamma_{\rm rf}}{2\pi\gamma_{\rm las}\gamma^2\eta h}\right) \ . \tag{8}$$

Here $n_{\rm x}$ is the average number of scattered–off photons by each electron per turn, ω_0 the frequency of bunches circulation along the ring orbit.

Compton Interactions + Synchrotron Radiation

For a realistic case of the electron bunches which undergo two independent perturbations – from the Compton scattering and due to emission of the synchrotron radiation – the emittances (and the squared energy spread as well) are the weighted sum of the partial emittances, the Compton one (see above) and the synchrotron (i.e. natural) one:

$$\epsilon_{x,z} = \frac{\epsilon_{\rm C}w_{\rm C} + \epsilon_{\rm s}w_{\rm s}}{w_{\rm C} + w_{\rm s}} \ , \tag{9}$$

where $w_{\rm C,s}$ are the average losses of energy by the circulating electron due to specific process (partial energy losses).

If several perturbations act upon the bunch, the total emittances become dependent on the excitation and damping caused by the specific processes.

Narrow Laser Pulse

For the laser pulses with nonuniform density distribution, the bunch steady–state emittances become dependent on the laser pulse dimensions at CP. The gaussian density distribution allows one to estimate reduction of the emittances. It results from the implicit relation:

$$\epsilon_{x,z}(\sigma_{\rm las}) = \epsilon_{\rm x,z}^{\rm unif}\frac{{\rm I}_0(t^2/4)}{[{\rm I}_0(t^2/4) + {\rm I}_1(t^2/4)]} \ , \tag{10}$$

where $\epsilon_{\rm x,z}^{\rm unif}$ is the transversal emittance (6); ${\rm I}_{0,1}$ are the Bessel functions, $t \equiv \sigma_{x,z}/\sigma_{\rm las}$.

As it can be seen from (10), for the uniform laser pulse ($t \ll 1$) emittance is equal to the linear one. For the 'narrow' laser pulses ($t \gg 1$) the emittance decreased twofold. Equally, this result is applicable to the longitudinal emittance (the spread or the bunch length) in the case of a short laser pulse and non–head–on collision.

Intensive Laser Pulse

The presented above the 'linear model' represents seldom electron–to–laser photons interactions: number of interactions for the period of synchrotron oscillations $\ll 1$. In this limiting case, the momenta of the random function (2) are reduced to the momenta of the single interaction multiplied by the average frequency of interactions. This is not valid for the powerful laser pulses as in the gamma–ray sources for positron production [5].

The beam dynamics changes due to the high power laser pulses, two effects come into play.

First, the 'losses' term (last term in the stochastic hamiltonian becomes sufficient). It reduces the acceptance of the ring and sufficiently increases the quantum losses exponentially dependent on the acceptance. In the Fig. 1, there are presented the phase plane of non–perturbing system and the 'stochastic' one. The 'stochastic phase trajectories' correspond to the loops of equal density of electrons in the phase space.

Second, the statistical properties of perturbations due to Compton interactions are changed. These changes are

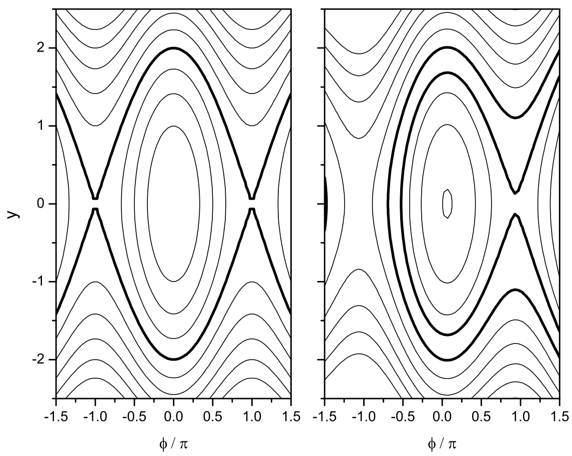

Figure 1: Loops of equal particle density. The left corresponds to non perturbing system, right subjected to interactions with photons

rather interesting: Up to unity efficiency of interactions per the synchrotron period, the dispersion of recoils rises faster then the average losses. With higher efficiency (we look in the future with desire of such a powerful lasers and resonators to become available) the dispersion reduces $\sim n_x^{-1/2}$. Since emittances are proportional to ratio of the dispersion to the average losses, they will have maximum at the efficiency of about unity per the synchrotron period.

CONCLUSION

The analytical model of the electron storage ring with the laser cooling was elaborated. In the model, the collision point where circulating electrons collide with the laser photons is set in the dispersion–free section of the orbit. The betatron functions in this point have extremal (minimal) values.

Extensive simulations verified the main results of the analytical study. In particular, the energy spread from simulations is equal to the predicted one, with the predicted deviations both due to the power and the dimensions of the laser pulse, see Fig.2 and Fig.3.

It is noteworthy that considering the laser–induced kinetics of the bunches, we get Robinson's sum rule for the laser cooling: the longitudinal decrement is twofold as much as the transverse ones. Also, the longitudinal decrement obeys the known relation: it equals to ratio of the losses to the energy of circulating electrons. Robinson's sum rule for Compton cooling from the simulations coincides with the predicted fairly well.

To meet the conditions for effective laser cooling, the power in the laser pulse should be limited to amount which produces the scattering rate $n_x \lesssim 0.1$. Also the smallest attainable laser pulse dimensions at CP (nonlinear cooling) are desired.

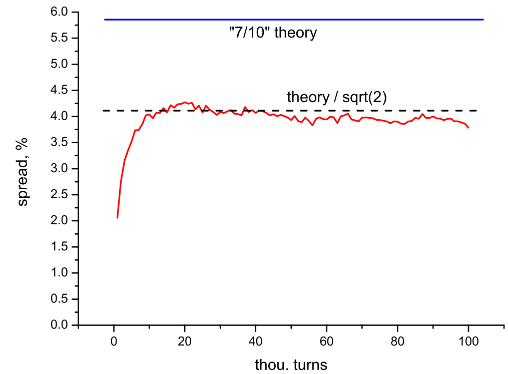

Figure 2: Simulated energy spread in Compton gamma–ray ring vs. time.

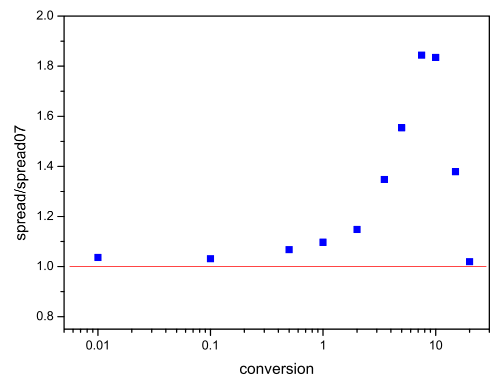

Figure 3: Ratio of the steady-state squared mean spread to the theoretical one vs. the conversion factor (number of scattered off laser photons per pass through CP).

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