NONLINEAR SINGLE-PARTICLE EFFECTS IN MULTIPARTICLE TRACKING CODES FOR THE ANALYSIS OF COLLECTIVE INSTABILITIES

J.H. Rowland[1], R. Bartolini[1,2] , R. Fielder[1],
[1]Diamond Light Source Ltd, Oxfordshire, OX11 0QX, UK
and [2]John Adams Institute, University of Oxford, OX1 3RH, UK
R. Nagaoka, Synchrotron SOLEIL, St-Aubin, Gif-sur-Yvette, France

Abstract

Within the common programme on the analysis of collective instabilities at Diamond and SOLEIL, the numerical codes *mbtrack* and *sbtrack* [1,2] have been extended to include a full description of the nonlinearities in the storage rings by means of the nonlinear one-turn map. We present the details of the map implementation and the recent results on the analysis of the effects of the nonlinear terms of the map on the characteristics of the collective instabilities at the two machines.

INTRODUCTION

The onset of collective instabilities in storage rings can be influenced by the nonlinearities present in the equation of motion of a single particle. In fact, detuning with amplitude and detuning with momentum can prevent the build-up of coherent oscillation, by the so called Landau Damping mechanism. It becomes therefore important for a quantitative description of the collective effects to include in the numerical simulation the most possible accurate description of the storage ring nonlinearities, compatible with realistic computational times. To this aim we have extended the capabilities of the sbtrack and mbtrack codes [1-2] to be able to use the full one turn map of the storage ring which, in principle, can be computed at any order in the expansion with distance from the origin of the phase space and at any location inside the ring.

The code to simulate collective instabilities is split in two parts. In the first a low-order truncated Taylor map is computed to efficiently model the linear and nonlinear optics. The second code loads this and evaluates the map and the wakefield as well as the RF cavity to simulate one turn around the ring.

MAP TRACKING

Accurately modeling collective effects in the storage ring requires a model of the nonlinear optics. Full particle tracking using numerical integration is prohibitively expensive on a single machine so the 6D Taylor map truncated to a low order is used as an approximation of the one turn map. The map is produced by augmenting the symplectic integrator of the expanded Hamiltonian with an operator overloaded automatic differentiation library ADOL-C [3]. The output of tracking is a tensor of Taylor series coefficients. The coefficients are assembled into a tree describing the Taylor series expression as a collection of add and multiply nodes. The tree is then re-ordered to optimize the map application for numerical stability using Horner's rule and zero constant coefficients are folded. Finally code for a trivial stack machine is generated from the expression tree by a postorder traversal. Figure 1 shows the expression tree and stack program for the expression x+2x. The map code is evaluated by an interpreter in the collective effects program. The lattice describing the linear optics is set up in the code AT [4] and then the Hamiltonian parameters for the elements are saved in an interchange format used by the map tracking code.

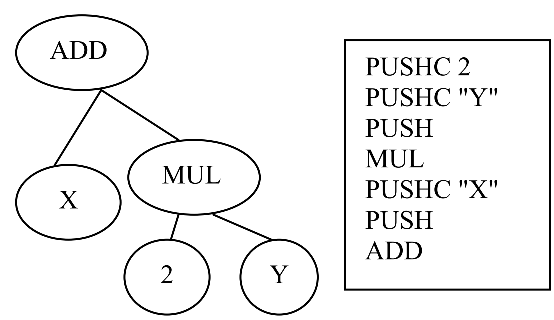

Figure 1. Expression Tree and Stack Program for x+2x

This separation into tree expression and stack machine code is typical of a compiler for a computer language. It allows for easy manipulation of the tree expression pointers for implementing optimizations before emitting code that is more efficiently evaluated.

HAMILTONIAN

The map tracking uses the expanded curvilinear Hamiltonian in dimensionless momentum co-ordinates, where V are the higher-order multipoles used in the straight elements.

$$H = \frac{px^2 + py^2}{2(1+\delta)} - \frac{x\delta}{\rho} + \frac{x^2}{2\rho^2} + V(x,y)$$

The implementation of this Hamiltonian in the code also takes into account the effect of fringe fields in dipoles using a thin quadrupole kick.

WAKEFIELD CONVOLUTION

By the convolution theorem, the wakefield convolution can be implemented with an FFT, giving O(n log n) computational complexity rather than O(n^2) for direct convolution. Here F is the Fourier transform operator and * denotes convolution.

$$a * b = F^{-1}(F(a) \cdot F(b))$$

The wakefield Green's functions are calculated using the broadband resonator formulae. The codes *sbtrack* and *mbtrack* can acquire the wakefields as computed by MAFIA [5] or Gdfidl [6] and build the total wakefield by a fitting procedure with a given number of broadband resonators.

COMPARISON OF MAP TRACKING WITH NUMERICAL INTEGRATION

The code AT was used to benchmark the map tracking, as it uses the same Hamiltonian and integrator. Several numerical quantities were compared. In Fig. 2 we report the vertical tune shift with momentum and it is clearly seen that increasing the order of the map from 3rd to 5th accounts correctly the higher order vertical chromaticity of the diamond storage ring.

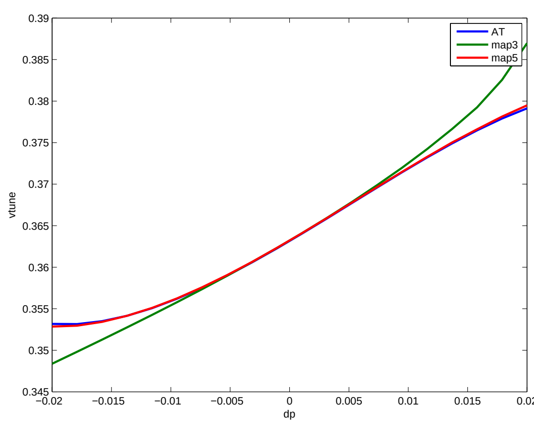

Figure 2. Convergence of the nonlinear chromaticity in the map to the result from AT as the map order is increased.

The map correctly describes also the higher order terms in the longitudinal motion as shown in Figure 3 where the longitudinal phase space plots for the diamond storage ring clearly show the RF bucket deformation due to the higher order momentum compaction factor.

NON-LINEAR MAPS

A preliminary investigation of the effects of including higher orders of non-linear maps was done using the diamond storage ring lattice, and results compared to the available experimental data. For the purposes of the simulation, the instability threshold was defined as the vertical beam size exceeding 1mm.

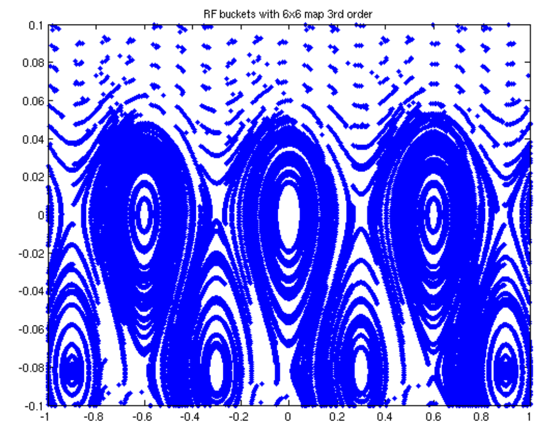

Figure 3: RF buckets from map tracking showing the presence of nonlinear momentum compaction

The current threshold as a function of the chromaticity have been investigated to check the convergence of these curves with the order of the map used. The results begin to converge after the 5th order (Figure 4a), with the curve at this point also being qualitatively closer to the measured instability threshold (Figure 4b). The simulated data show that mode 0 instability is dominating up to a normalized vertical chromaticity of 0.2. At higher chromaticity the instability is dominated by mode -1. The slight discrepancies found in the current threshold are under investigation and can probably be improved with a more accurate fit of the broad band impedance model used. Map to 6th order and higher reproduce similar results and it is not practical for most purposes due to the greatly increased processing time required for the tracking.

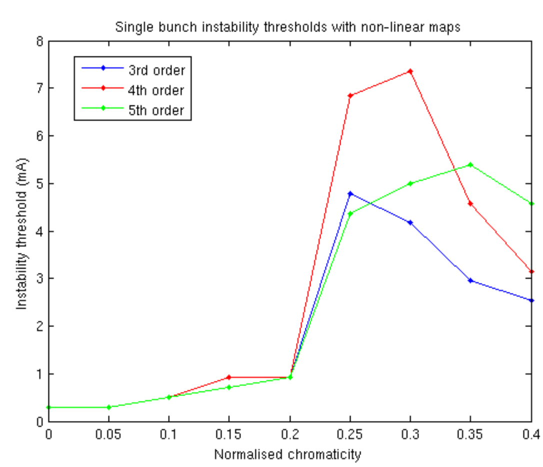

Figure 4a. Simulated single bunch instability current thresholds as a function of the normalised vertical chromaticity with various orders of non-linear maps.

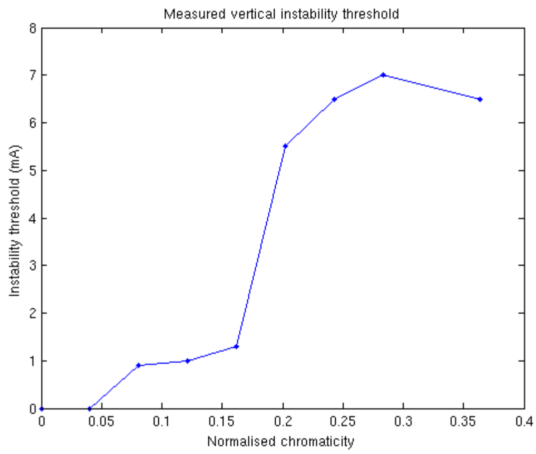

Figure 4b: Measured single bunch instability current threshold as a function of the normalised chromaticity. The machine is operated in single bunch with an RF voltage of 2.0 MV.

The nonlinear one turn map was also used to investigate longitudinal instabilities. No significant difference were found in the bunch lengthening and energy spread widening curves. Previous results [2] obtained with the linear version of *sbtrack* remains adequate to describe the single bunch longitudinal instability: an inductive impedance of of 149nH in addition to the broadband impedance could improve the match between model and measured data.

APPLICATION OF THE TRUNCATED ONE-TURN TAYLOR MAP TO SOLEIL

The developed routines that introduce nonlinear betatron motions of the tracked particles through a truncated Taylor series map have recently been implemented into both *sbtrack* and *mbtrack* at SOLEIL, along with making some preliminary test calculations. While for *sbtrack* that does the single bunch tracking, the map has been directly introduced in the loop over turns replacing the transfer matrix transformation, for *mbtrack* it has been introduced in the module *sbtransf* that communicates with *mbtrack* and takes care of the 1-turn map of each bunch. Prior to performing the tracking, nonlinear tune shifts generated by the map were followed as a function of the betatron amplitude for several specific sextupole configurations in the vertical plane (Figure 5). An expected trend has been obtained that the map gives larger tune shifts than those predicted from the analytical 2^{nd}-order formula at large amplitudes.

It has been observed that the implemented map does not bring about any increase in the computation time in both *sbtrack* and *mbtrack*, thanks to the map being a one-turn map as originally intended. The method therefore opens a wide possibility of treating the general case in which many insertion devices have their gaps closed, introducing strong complicated nonlinear fields, which may well counteract on the collective effect that gets enhanced simultaneously. Prior to extracting the instability thresholds from the tracking, however, a little more work was found necessary to develop treatment of the particles that exceeded the dynamic aperture, as the notion of the latter did not exist in the code so far and the diverged particles need be excluded from the evaluation of centre of mass motions. The work is in progress.

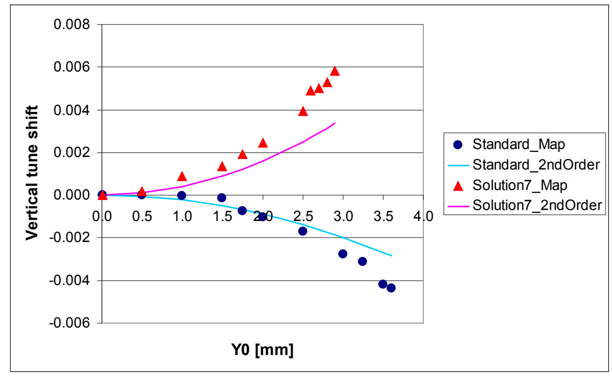

Figure 5: Vertical betatron tune shifts as a function of the initial amplitude for the standard sextupole setting (circles) and a setting with an increased 2^{nd}-order tune shift with amplitude (triangles), respectively in comparison with those predicted with the 2^{nd}-order formula.

CONCLUSION

We have described the work in progress in the collaboration Diamond-Soleil toward a more accurate description of the collective effects in the two machines and the improvement made on the sbtrack and mbtrack codes.

The match between model and machine is improved when nonlinear lattice terms are added to the collective effects simulation codes. Low-order taylor maps allow for fast approximate tracking and will allow these study to be extended to the computationally expensive multi-bunch case and to the inclusion of more complex nonlinearities coming from Insertion Devices.

REFERENCES

[1] R. Nagaoka, "Instability Studies Using Evaluated Wake Fields and Comparison with Observations at SOLEIL", EPAC'06.
[2] R. Nagaoka, R. Bartolini, J. Rowland, "Studies of Collective Effects in SOLEIL and DIAMOND Using the Multiparticle Tracking Codes *sbtrack* and *mbtrack*", PAC09, Vancouver, May 2009.
[3] Griewank, Juedes, Mitev, Utke, Vogel, Walther, "ADOL-C: A Package for the Automatic Differentiation of Algorithms Written in C/C++", 1999
[4] A.Terebilo, "Accelerator Toolbox for MATLAB", SLAC-PUB-8732
[5] CST MAFIA, CST UK LTD.
[6] W. Bruns, "The GdfidL Electromagnetic Field Simulator", http://www.gdfidl.de.