

EFFECTS OF DIRECT SPACE CHARGE ON THE TRANSVERSE MODE COUPLING INSTABILITY

D. Quatraro, G. Rumolo

European Organization for Nuclear Research (CERN), CH-1211 Genève 23, Switzerland

Abstract

The effects of direct space charge forces on the Transverse Mode Coupling Instability (TMCI) are studied using numerical techniques. We have implemented a third order symplectic integrator for the equation of motion, taking into account non linear space charge forces coming from a Gaussian shaped bunch. We performed numerical simulation for the Super Proton Synchrotron (SPS) bunch at 26 GeV of kinetic energy, using either resistive wall or broad band transverse wake fields. In both cases the result of applying direct space charge, leads to an intensity threshold increase up to 100% for the resistive wall wake.

INTRODUCTION

The Transverse Mode Coupling Instability (TMCI) was first observed in the Super Proton Synchrotron (SPS) few years ago [1].

The role of the direct space charge has been pointed out in many papers (e. g. [2, 3]) as a possible source able to increase the bunch intensity threshold N_{th} [ppb]. One goal of this paper is also to study the SPS case with a more accurate space charge model: in the past, HEADTAIL simulations have been carried out applying incoherent space charge tune shift estimating an increase of the beam intensity [4]. In the following analysis we scan over the transverse beam size (which does not play any role in the TMCI) thus scanning over space charge force strength.

THE MODEL

We limit our analysis to a 4D phase space $\mathbf{x} = (x, \dot{x}, z, \delta)$ and the equations of motion in the horizontal and longitudinal plane [m/s⁻²] read

$$\begin{cases} \ddot{x} + \omega_x^2 x = \kappa f_x^{s.c.} \\ \ddot{z} + \omega_s^2 z = 0 \end{cases} \quad (1)$$

with $\kappa = 2r_0 \frac{Nc^2\lambda}{\gamma^3}$ and defining the linear frequencies

$$\begin{cases} \omega_x = \omega_0 Q_x (1 - \xi \dot{z} / \beta c \eta) \\ \omega_s = \omega_0 Q_s \end{cases} \quad (2)$$

where $C = 2\pi R$ is the ring circumference, $T_0 = C/\beta c = 2\pi/\omega_0$ is the revolution period, and η the slippage factor. The initial distribution in the horizontal and vertical plane are matched to the quadratic potentials.

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Assuming the bunch shape remains Gaussian and the beam round, the transverse space charge force $f_x^{s.c.}$ is given by [5]

$$f_x^{s.c.} = e^{-z^2/(2\sigma_z^2)} \frac{1}{x} \left[1 - e^{-r^2/(2\sigma_x^2)} \right] \quad (3)$$

where $\sigma_{x,z}$ stands for the rms bunch size in horizontal/longitudinal plane. No longitudinal space charge forces have been taken into account in this work.

In order to integrate the equation of motion we used 3-th order splitting symplectic algorithm which ensures the preservation of the energy and does not create an artificial emittance growth. In the transverse plane the Hamiltonian of the system reads

$$H_T(x, \dot{x}, z) = \underbrace{\frac{\dot{x}^2}{2} + \omega_x^2 \frac{x^2}{2}}_{H_0} + \varphi(x - \bar{x}, z) = H_0 + V \quad (4)$$

with $\bar{x} = \mu_{1,0}, \mu_{i,j} = \sum_{n=0}^M x_n^i \dot{x}_n^j / M$, M is the number of macroparticles and $\varphi(x)$ being the space charge potential: the space charge forces which are given by $\kappa f_x^{s.c.}(x) = -\partial\varphi/\partial x$ and the analytical expressions are in Eq. (3).

The splitting algorithm reads

$$\mathbf{x}(\Delta t) = e^{\frac{\Delta t}{2} D_\varphi} \circ e^{\Delta t D_{H_0}} \circ e^{\frac{\Delta t}{2} D_\varphi} \mathbf{x}(0) + \mathcal{O}(\Delta t^3) \quad (5)$$

The operator $e^{\Delta t D_H}$ in Eq. (5) formally reads

$$e^{\Delta t D_H} = \sum_{i=0}^{\infty} \frac{t^i D_H^i}{i!}, \quad (6)$$

where $D_H^i = \overbrace{[\cdot, [\cdot, \dots [\cdot, H]]]}^{i \text{ times}}$, and $[\cdot, H]$ is the usual Poisson bracket. The flow generated by H_0 is a simple rotation in the phase-space after the scaling $x \rightarrow x/\sqrt{\omega_x}, \dot{x} \rightarrow \sqrt{\omega_x} \dot{x}$. The N particles bunch is thought as made of n slices populated by an ensemble of M macroparticles. Each slice has its own rms quantities which are updated during the simulation as well as during the integration of the equation of motion.

Assuming one wake field interaction per turn each particle feels the action of the wake field excited by the other particles traveling in front of it. If we slice the beam in n slices, then j -th particle, belonging to the k -th slice of the beam will change its momentum \dot{x} [m/s] according to

$$\Delta \dot{x}_j = \frac{\beta c e^2 N}{E_0 M} \sum_{l=k+1}^n n_l \bar{x}_l W_1(- (k-l) dz) \quad (7)$$

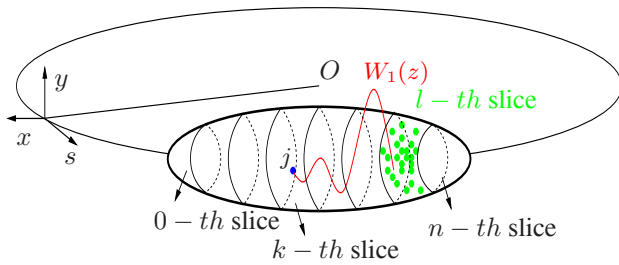


Figure 1: The j -th particle (\bullet), located in the k -th slice of the bunch, feels the wake field force W_1 excited by the l -th slice. The force is proportional to the number of particles N_l and the centroid position \bar{x}_l of the l -th slice.

where $E_0 = \gamma m_0 c^2$ and \bar{x}_l is the offset of the l -th slice. In Fig. 1 we can find a scheme explaining the mechanism.

In the following section we show the result obtained simulating the SPS 26 GeV bunch applying resistive wall and broad band wake fields. We will find the bunch intensity threshold N_{th} with or without applying the space charge forces.

SIMULATIONS

In this section we will show the results obtained for different wake field forces. We will compare the mode shifting analysis vs. the bunch population. For each wake force we show the mode analysis without and with space charge. In general the mode analysis is much more noisy when space charge forces are considered. For the resistive wall wake we also scanned over the horizontal size σ_x in order to observe the dependence of N_{th} on the space charge forces.

Resistive Wall Wake Fields

We applied the following horizontal resistive wall wake field

$$W_{\perp}(z) = cZ_0 \frac{1}{\pi^2 b^3} \sqrt{\frac{\pi}{c^2 Z_0 \sigma |z|}} \frac{\text{sgn}z - 1}{2}, \quad (8)$$

where $Z_0 = \mu_0 c$ and $b = 3.6$ cm is the pipe radius. In Table 1 we list the parameters used for the simulations.

Table 1: SPS parameters used for the simulations.

Q_x	Horizontal tune	26.18
Q_s	Synchrotron tune	$3 \cdot 10^{-3}$
β	Relativistic factor	0.99935
R	Machine radius [m]	1100
α	Momentum compaction factor	$1.88 \cdot 10^{-3}$
ξ_x	Chromaticity	0.0

We used a bunch length of $\sigma_z = 10$ cm. In Fig. 2 we can observe the mode shifts vs. the bunch population.

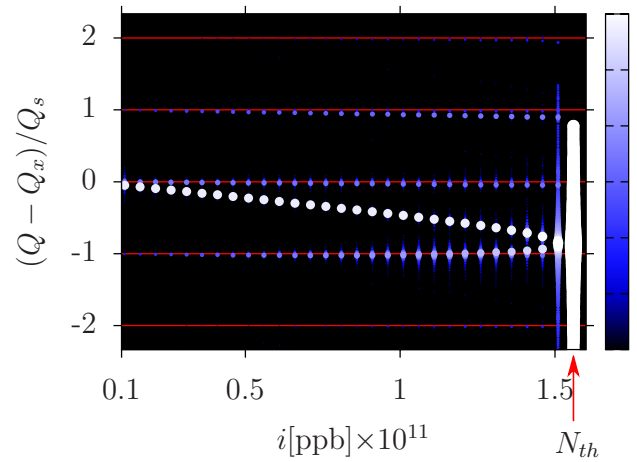


Figure 2: Mode shifting for resistive wall and no space charge.

It can be clearly seen that the instability starts when the mode “-1” and “0” merge while increasing the bunch intensity N .

On the other side if we take into account the space charge forces we can observe in Fig. 3 that the TMCI threshold increased up to almost 100% by setting $\sigma_x = 4$ mm.

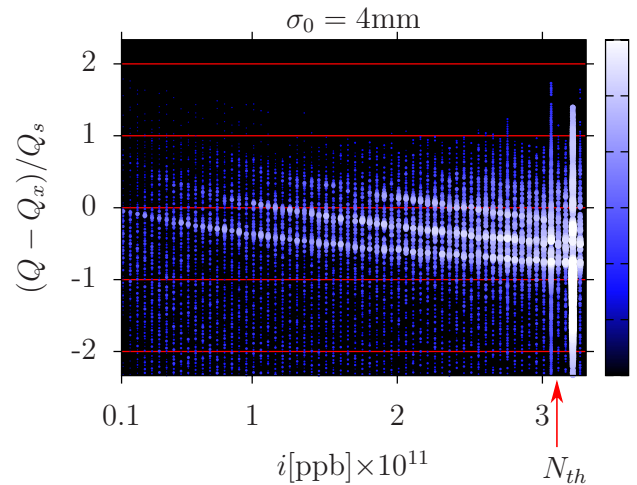


Figure 3: Mode shifting for resistive wall and space charge.

In Fig. 4 we show the behavior of the intensity threshold as a function of the horizontal beam size and thus of the space charge forces.

Broad Band Wake Fields

Here we show the same mode analysis using a broad-band impedance

$$W(z) = Z_t \frac{\omega_r^2}{\hat{\omega} Q} \sin\left(\frac{\hat{\omega} z}{\beta c}\right) e^{\omega_r z / 2Q\beta c} \quad (9)$$

with $\hat{\omega} = \sqrt{\omega_R^2 - \alpha^2}$, $\alpha = \omega_r / 2Q$: $Z_t = 5$ M Ω /m, $\omega_R = 1.3 \cdot 2\pi$ GHz and $Q = 1$. For these simulations we used

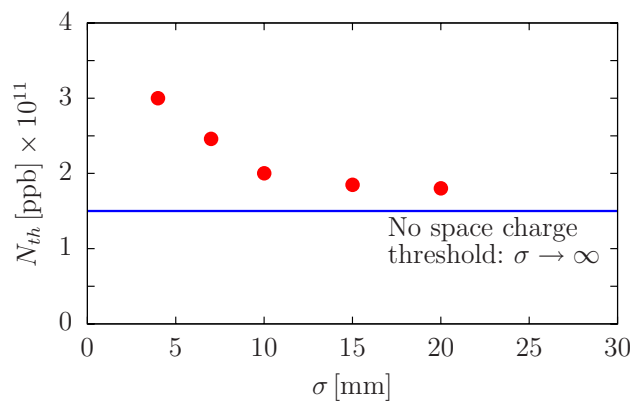


Figure 4: TMCI intensity threshold as a function of the horizontal beam size.

the value $\sigma_z = 30$ cm, in order to excite a higher mode merging. It is also interesting to study the impact of the space charge when higher modes are involved. In Fig. 5, 6 we find the result for the broad band wake fields with and without space charge forces.

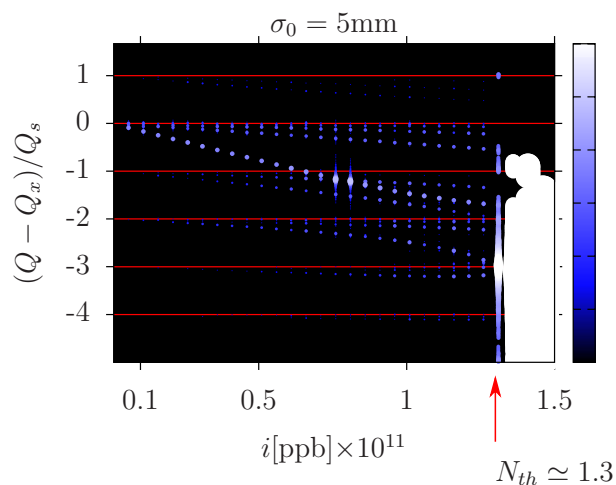


Figure 5: Mode shifting for broad band wake fields without space charge.

From Fig. 5 it is clear that the TMCI instability comes from the merging between the modes “-3” and “-2”, as expected [6]. In Fig. 6 we can see the mode analysis when space charge forces are applied. We can observe that the gain in term of bunch population is around 10%.

Despite the fact that the mode analysis is complicated and noisy the TMCI instability seems to be also driven by the “-3” and “-2” mode coupling. According to Ref. [2, 3] one important parameter to quantify the space-charge effects is the ratio $\Delta Q_{sc}/Q_s$. From Eq. (3) (linearizing the space charge forces, and for $z = 0$) we get

$$\Delta Q_{sc} = \frac{1}{2\omega_0^2 Q_x} \frac{r_0 N c^2}{\gamma^3 \sqrt{2\pi} \sigma_x^2 \sigma_z}. \quad (10)$$

For the resistive wall simulations we have $\Delta Q_{sc}/Q_s \approx 14$, whereas for the broad band ones we have $\Delta Q_{sc}/Q_s \approx 50$.

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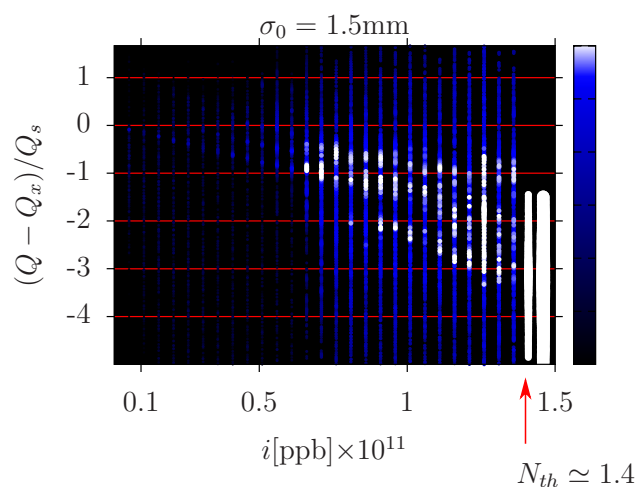


Figure 6: Mode shifting for broad band wake fields with space charge.

This consideration might suggest that the effect of the space charge on the TMCI might be much less important when high order modes merge, as already anticipated[7].

CONCLUSIONS

We numerically studied the effect of the direct space charge forces on the TMCI via a 2-D model, and assuming non linear forces for Gaussian beams. We found that space charge can increase the TMCI intensity threshold. The simulations suggest that the smaller the beam (transversally) the higher the TMCI intensity threshold. For the simulated SPS parameters this phenomenon is enhanced for low resistive wall driven TMCI.

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