HORIZONTAL IMPEDANCE OF THE KICKER MAGNET OF RCS AT J-PARC

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Abstract

There is the famous formula of the horizontal impedance for the matched-traveling wave kicker. However, the real and the imaginary parts of the impedance do not satisfy the Hilbert transformations. On the other hand, the impedance measured by a loop method does not directly give the beam impedance. In this paper we theoretically derive the beam impedance and reproduce the impedance by using the estimated inductances of the kicker by the loop measurement.

INTRODUCTION

There is the famous horizontal impedance formula of the matched-traveling wave kicker that was obtained by Nassibian and Sacherer [1]. While the formula is simple and easy to use, the derivation of it is too intuitive and the results do not satisfy the Hilbert transformation [2], which means that the formula cannot satisfy the causality. In order to satisfy the condition, the imaginary part of the impedance should be capacitive, otherwise the constant part of the impedance, that corresponds to the δ-function in the wake function, should be deleted from the formula. The previous measurement results of the horizontal kicker impedance strongly suggested that the constant part of the imaginary part of the impedance did not exist [3].

In view of the ratio of the signal to the noise, it is generally easier to precisely measure the longitudinal impedance than to do the transverse impedance. Then, the horizontal impedance at the measurement was derived by the position-dependence of the longitudinal impedance in the reference [3]. Though the physical meaning of the impedance measured by the method should be the horizontal impedance, there was a doubt whether this impedance was quantitatively equal to the rigorously defined transverse impedance[1].

In the present experiment, we measure the horizontal impedance by stretching twin wires with shorted end and detecting the reflection coefficient. It has been considered that the measured impedance is identical to the beam impedance. But, this method can only detect the time- dependence of the magnetic field, and it is necessary to study whether the measured impedance is the beam impedance.

In this report, we develop a new theory to describe the kicker impedance, which satisfies the causality condition. First, we review the previous formula of the horizontal kicker impedance, and explain that it becomes capacitive if the wake function satisfied the causality condition, though it should be inductive intuitively. After we derive the formula of the horizontal impedance due to the coil of the kicker, we obtain the beam impedance by using the inductances of the kicker estimated by the loop method.

A QUESTION ABOUT THE PREVIOUS FORMULA OF THE HORIZONTAL IMPEDANCE

Let us explain the questions about the famous formula of the transverse impedance for the matched-traveling wave kicker [1]. The corresponding wake function $W(\zeta)$ for the kicker magnet is given by [1]

$$W(\zeta) = W_0\left[\Theta(\zeta) - \Theta\left(\zeta - \frac{Lc}{v_p}\right) - \frac{Lc}{v_p}\delta(\zeta)\right], \quad (1)$$

where c is the velocity of light, L is the longitudinal length of the kicker, $\Theta(x)$ is the step function, $\delta(x)$ is the δ-function, $v_p = 1/\sqrt{L_k C_k}$, L_k and C_k are the inductance and the capacitance of the kicker, respectively. In Eq.(1), the horizontal wake function $W(\zeta)$ has an amount for the positive value of ζ ($\zeta > 0$), while it is completely zero for the negative value of ζ ($\zeta < 0$). This expresses that the wake function satisfies the causality. Since a beam should lose the energy passing through the kicker, $W'(\zeta)$ becomes positive as ζ approaches to zero. Then, the amplitude of wake function W_0 should be positive.

Since the horizontal impedance Z_x for the relativistic particle is defined as Fourier transformation of the wake function $W(\zeta)$, it is express as

$$Z_x(\omega) = -\frac{1}{jcx_0}\int_{-\infty}^{\infty} d\zeta W(\zeta) e^{-j\frac{\omega}{c}\zeta}, \quad (2)$$

where x_0 is the displacement of the source particle. By substituting Eq.(1) into Eq.(2), the horizontal impedance Z_x is obtained as

$$Z_x = \frac{W_0 L}{v_p x_0}\left[\frac{1-\cos\omega\frac{L}{v_p}}{\omega\frac{L}{v_p}} - j\left(1 - \frac{\sin\omega\frac{L}{v_p}}{\omega\frac{L}{v_p}}\right)\right]. \quad (3)$$

Equation (3) looks similar to the well-known formula [1]:

$$Z_x = \frac{W_0 L}{v_p x_0}\left[\frac{1-\cos\omega\frac{L}{v_p}}{\omega\frac{L}{v_p}} + j\left(1 - \frac{\sin\omega\frac{L}{v_p}}{\omega\frac{L}{v_p}}\right)\right]. \quad (4)$$

However, the sign of the imaginary part of Eq.(3) is negative, which means the impedance is capacitive.

This result also contradicts the previous measurement results [3]. The measurement results suggested that the horizontal impedance Z_x be described as

$$Z_x = \frac{W_0 L}{v_p x_0}\left(\frac{1-\cos\omega\frac{L}{v_p}}{\omega\frac{L}{v_p}} + j\frac{\sin\omega\frac{L}{v_p}}{\omega\frac{L}{v_p}}\right), \quad (5)$$

which is rewritten to the wake function:

$$W(\zeta) = W_0\left[\Theta(\zeta) - \Theta\left(\zeta - \frac{Lc}{v_p}\right)\right], \quad (6)$$

where there was no instantaneous force (δ-function).

In the next section, we explain a new theory and derive the formula of the kicker impedance.

THE HORIZONTAL IMPEDANCE

In this section, the kicker that is combined with right(the positive side of x) and left (the negative side of x) coils with self-inductance $\tilde{L}_k$, such as J-PARC, is considered [4]. The horizontal impedance due to the kicker coils is found by calculating the induced current via the transmission line model. When all terminal impedances of the kicker are the same, the equations are expressed as

$$\frac{dV_k}{dz} = -j\omega L_k I_k - j\omega M(x_0) I_0 e^{-jkz}, \quad (7)$$

$$\frac{dI_k}{dz} = -j\omega C_k V_k, \quad (8)$$

where $k = \omega/\beta c$, V_k and I_k are the induced voltage and the current in the right coil, respectively, those for the left coil is obtained by $-V_k$ and $-I_k$, respectively, M_b is the mutual inductances between the right and the left coils, L_k is given by $\tilde{L}_k - M_b$, $M(x_0)$ is equal to $(M^r(x_0) - M^l(x_0))/2)$, $M^r(x_0)$ and $M^l(x_0)$ are the mutual inductances between the beam and the right coil and between the beam and the left coil, respectively.

The boundary conditions are given by the condition that the terminal impedances should be equal to R at $z = 0$ and $z = L$. The solution is described as

$$I_k = I_1^{(0)}(x_0, R) e^{-j\theta^{(0)} z} + I_1^{(1)}(x_0, R) e^{-j\theta^{(1)} z} + I_1^{(2)}(x_0, R) e^{-j\theta^{(2)} z}, \quad (9)$$

where

$$I_1^{(0)}(x_0, R) = \frac{M(x_0) c\theta^{(1)}\theta^{(0)}}{Z_c(\theta^{(0)2} - \theta^{(1)2})} I_0, \quad (10)$$

$$I_1^{(1)}(x_0, R) = M(x_0) A(R) I_0, \quad (11)$$

$$I_1^{(2)}(x_0, R) = M(x_0) B(R) I_0, \quad (12)$$

where

$$A(R) = \frac{c\theta^{(0)}(R\theta^{(1)} + Z_c\theta^{(0)})}{Z_c(\theta^{(1)2} - \theta^{(0)2})} \times \frac{\left[(R+Z_c) - e^{-j(\theta^{(1)}+\theta^{(0)})L}(R-Z_c)\frac{(R\theta^{(1)}-Z_c\theta^{(0)})}{(R\theta^{(1)}+Z_c\theta^{(0)})}\right]}{\left[(R+Z_c)^2 - e^{-j2\theta^{(1)}L}(R-Z_c)^2\right]}, \quad (13)$$

$$B(R) = \frac{c\theta^{(0)}(R\theta^{(1)} + Z_c\theta^{(0)})}{Z_c(\theta^{(1)2} - \theta^{(0)2})} \times \frac{\left[(R-Z_c) - e^{j(\theta^{(1)}-\theta^{(0)})L}(R+Z_c)\frac{(R\theta^{(1)}-Z_c\theta^{(0)})}{(R\theta^{(1)}+Z_c\theta^{(0)})}\right]}{\left[(R-Z_c)^2 - e^{j2\theta^{(1)}L}(R+Z_c)^2\right]}, \quad (14)$$

$\theta^{(0)} = k$, $\theta^{(1)} = \omega\sqrt{L_k C_k}$, $\theta^{(2)} = -\theta^{(1)}$, $k = \omega/c\beta$ and $Z_c = \sqrt{L_k/C_k}$.

By solving the Maxwell equations, the magnetic and the electric fields for j-th mode in vacuum are obtained as

$$H_x^{(j)} = \frac{y}{|y|}\frac{I_i^{(j)} e^{-j\theta^{(j)} z}}{a}\sum_{m=1}^{\infty}\cos\frac{m\pi(x+a)}{2a} \times \frac{\cos\frac{m\pi(x_i+a)}{2a}\sinh\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2\pi^2}{4a^2}}(|y|-b)}{2\sinh\sqrt{\theta^{(j)} - k^2\beta^2 + \frac{m^2\pi^2}{4a^2}}b}, \quad (15)$$

$$H_y^{(j)} = \frac{I_i^{(j)} e^{-j\theta^{(j)} z}}{a}\sum_{m=1}^{\infty}\sin\frac{m\pi(x+a)}{2a} \times \frac{m\pi\cos\frac{m\pi(x_i+a)}{2a}\cosh\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2}{4a^2}}(|y|-b)}{4a\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2}{4a^2}}\sinh\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2}{4a^2}}b}, \quad (16)$$

$$H_z^{(j)} = 0, \quad (17)$$

$$E_x^{(j)} = \frac{k\beta Z_0 I_i^{(j)} e^{-j\theta^{(j)} z}}{a\theta^{(j)}}\sum_{m=1}^{\infty}\sin\frac{m\pi(x+a)}{2a} \times \frac{m\pi\cos\frac{m\pi(x_i+a)}{2a}\cosh\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2}{4a^2}}(|y|-b)}{4a\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2}{4a^2}}\sinh\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2}{4a^2}}b}, \quad (18)$$

$$E_y^{(j)} = -\frac{y}{|y|}\frac{k\beta Z_0 I_i^{(j)} e^{-j\theta^{(j)} z}}{a\theta^{(j)}}\sum_{m=1}^{\infty}\cos\frac{m\pi(x+a)}{2a} \times \frac{\cos\frac{m\pi(x_i+a)}{2a}\sinh\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2\pi^2}{4a^2}}(|y|-b)}{2\sinh\sqrt{\theta^{(j)} - k^2\beta^2 + \frac{m^2\pi^2}{4a^2}}b}, \quad (19)$$

$$E_z^{(j)} = 0, \quad (20)$$

where the aperture size of the kicker is specified by $2a \times 2b$, $Z_0 = 120\pi$, $I_i^{(j)}$ is the intensity of i-th(that specifies the right($i = 1$) or the left($i = 2$) coil's) current with the wave number $\theta^{(j)}$ and x_i is the position of i-th coil ($x_1 = a$ and $x_2 = -a$, respectively).

Finally, the horizontal impedance Z_x (excited by TEM mode) is expressed as

$$Z_x = \sum_{i=1}^{2}\sum_{j} \frac{dI_i^{(j)}}{dx_0} \frac{\beta^2 Z_0 (e^{j(k-\theta^{(j)})L} - 1)}{2I_0 a\theta^{(j)}} \sum_{m=1}^{\infty} \sin\frac{m\pi}{2}$$
$$\times \cos\frac{m\pi(x_i + a)}{2a}\left(\frac{m\pi}{2a\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2\pi^2}{4a^2}}tn} - 1\right)$$
$$+ \sum_{i=1}^{2}\sum_{j} \frac{dI_i^{(j)}}{dx_0} \frac{\beta^2 Z_0 (e^{j(k-\theta^{(j)})L} - 1)}{8I_0 a\theta^{(j)}}$$
$$\times \left(\frac{\sin\frac{\pi(x_i+2a)}{2a}}{1 - \cos\frac{\pi(x_i+2a)}{2a}} - \frac{\sin\frac{\pi x_i}{2a}}{1 - \cos\frac{\pi x_i}{2a}}\right), \qquad (21)$$

where

$$tn = \tanh\sqrt{\theta^{(j)2} - k^2\beta^2 + \frac{m^2\pi^2}{4a^2}}b. \qquad (22)$$

MEASUREMENT OF THE IMPEDANCE

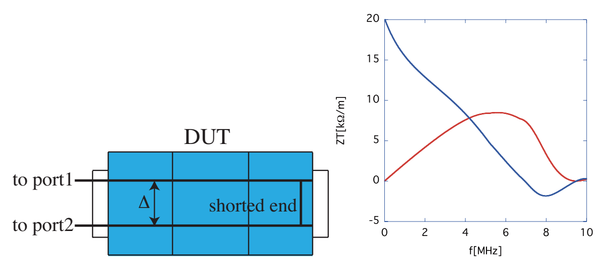

Figure 1: The left figure is the schematic picture of the setup of the twin-wire measurement. The right figure is the horizontal beam impedance reproduced by using estimated L_k and M by the measurement. The red and the blue lines represent the real and the imaginary parts of the impedance, respectively.

In order to measure the transverse impedance, following Nassibian and Sacherer [5], we made a loop in the kicker, which is sandwiched by aluminum chambers, by two copper wires where either end of the loop was shorted. The two wires are directly connected to Network analyzer with coaxial cables, because we focus on the low frequency region of the impedance. The horizontal impedance Z_x^{loop} of the loop is obtained as

$$Z_x^{loop} = \frac{cZ}{\omega\Delta^2}, \qquad (23)$$

where $\Delta(= 34)$mm is the distance between wires,

$$Z = Z_{c,diff}\frac{1 + S_{11}^{(kick)}}{1 - S_{11}^{(kick)}} - Z_{c,diff}\frac{1 + S_{11}^{(ref)}}{1 - S_{11}^{(ref)}}, \qquad (24)$$

$Z_{c,diff}(\simeq 722\Omega$ in our case) is the characteristic impedance for the differential mode, $S_{11}^{(kick)}$ and $S_{11}^{(ref)}$ are the reflection coefficients in the case that DUT is the kicker and that DUT is replaced by the reference pipe (aluminum chamber), respectively.

Theoretically, Z is calculated by using the transmission line mode. The voltage V_0 and the current I_0 of the Lecher line inside the aluminum chambers is described as

$$V_0 = \sqrt{\frac{L'}{C'}}(C_0 e^{-j\sqrt{L'C'}\omega z} - D_0 e^{j\sqrt{L'C'}\omega z}), \qquad (25)$$
$$I_0 = C_0 e^{-j\sqrt{L'C'}\omega z} + D_0 e^{j\sqrt{L'C'}\omega z}, \qquad (26)$$

where C_0 and D_0 are the arbitrary constants, L' and C' are the inductance and the capacitance of the Lecher line inside the pipe, respectively. Further, the induced voltage and the current of the coil and those of the wire in the kicker part are found by solving

$$\frac{d\vec{u}}{dz} = \mathcal{M}\vec{u}, \qquad (27)$$

where

$$\vec{u} = \begin{pmatrix} V_0 \\ I_0 \\ V_k \\ I_k \end{pmatrix}, \mathcal{M} = \begin{pmatrix} 0 & -j\omega L_w & 0 & -j\omega 2M \\ -j\omega C_w & 0 & 0 & 0 \\ 0 & -j\omega M & 0 & -j\omega(\tilde{L}_k - M_b) \\ 0 & 0 & -j\omega C_k & 0 \end{pmatrix}, \qquad (28)$$

L_w and C_w are the inductance and the capacitance of the Lecher line inside the kicker, respectively, and M is the mutual inductance between the right coil and the loop. The matching condition between the kicker and the aluminum chambers determines the solution. Substituting the obtained V_0/I_0 for the case that DUT is the kicker, and that for the case that DUT is replaced by the reference pipe into Eq.(24), Z_x^{loop} is theoretically obtained.

Here it is noticeable that L_w and L' are generally different due to the ferrite in the kicker. However, if there was no ferrite, and if both values are equal like strip line monitor, Z_x^{loop} becomes capacitive. This is because the loop method can only detect the magnetic field effect of the beam impedance.

Therefore, we only use the measured data to estimate L_w, L_k and M. The beam impedance is reproduced by substituting them into Eq.(21). The result are shown in the right figure of Fig.1. Since the new impedance formula includes the effect of the electric effect as well as the magnetic field in the impedance, the reproduced kicker impedance becomes inductive, and δ-function does not exist in the wake function.

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