THEORETICAL STUDIES OF TE-WAVE PROPAGATION AS A DIAGNOSTIC FOR ELECTRON CLOUD*

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Abstract

The propagation of TE waves is sensitive to the presence of an electron cloud primarily through phase shifts generated by the altered dielectric function, but can also lead to polarization changes and other effects, especially in the presence of magnetic fields. These effects are studied theoretically and also through simulations using WARP. Examples are shown related to CesrTA parameters, and used to observe different regimes of operation as well as to validate estimates of the phase shift.

THE USE OF TE WAVES AS A DIAGNOSTIC FOR ELECTRON CLOUD

Electron-cloud instabilities are a significant concern for accelerators with high current and repetition rate [1, 2]. One method to detect and measure the electron cloud is by propagating electromagnetic (EM) waves through the beam pipe, which acts as a waveguide [3, 4]. As this wave passes through regions of large electron-cloud buildup, the reduced permittivity inside the plasma causes a phase shift in the detected signal. As the electron cloud is generated by the bunch structure of the accelerated beam, it is highly modulated and even small (<1 mrad) shifts in phase can be detected as a modulated signal. Here, we focus on the use of TE (transverse electric) waveguide modes as a diagnostic.

ELECTRON CLOUD TRANSVERSE PROFILE EFFECTS

The effect of a uniform electron cloud density at low temperature has been studied in Ref. [5] using the solution for the longitudinal wave number $k_z^2 = (\omega^2 - \omega_c^2 - \omega_p^2)/c^2$, where ω is the frequency of the TE wave, ω_c is the usual cutoff frequency for the waveguide, and ω_p is the plasma frequency corresponding to the electron cloud density, $\omega_p^2 = n_e e^2/\epsilon_0 m_e$. The effect of the plasma is to change the permittivity inside the waveguide. As a result, in the limit of small plasma frequency it is found that the relative phase shift in comparing transmission with or without the electron cloud is proportional to the electron density. The shift for a distance Δz of propagation is

$$\Delta\Phi \simeq \frac{\omega_p^2}{2c\sqrt{\omega^2-\omega_c^2}}\,\Delta z. \qquad (1)$$

More generally, if the electron cloud density is weak enough so that $\omega_p^2 \ll \omega^2 - \omega_c^2$. and is slowly varying longitudinally compared to the longitudinal wavelength, then the interaction between the EM wave and the plasma can be treated as a small perturbation. This allows the phase shift to be approximated as

$$\Delta\Phi \simeq \frac{1}{2c\sqrt{\omega^2-\omega_c^2}} \int dz\, \frac{\int dxdy\, \omega_p^2 |\vec{E}|^2}{\int dxdy\, |\vec{E}|^2}, \qquad (2)$$

where $\vec{E}$ is the electric field of the EM wave defined by the guided mode. Because ω_p^2 is proportional to n_e, this defines a weighted average of the electron density, favoring regions where the guided mode is more intense. This result is analogous to the Slater perturbation technique for the wave equation of a deformed resonant cavity [6]; however, instead of a small volume that completely excludes the nominal EM fields, here we have an extensive volume of plasma that only slightly reduces the electric fields.

For the lowest mode in a rectangular waveguide, the weighted density is then

$$n_{\text{eff}} = \frac{2}{ab} \int dxdy\, n_e(x,y) \cos^2(\pi x/a), \qquad (3)$$

where a and b are taken to be the longer and shorter dimensions of the waveguide, respectively. Note that the weighting only modifies horizontal variations. Simulations of the EM waves and plasma in the beam pipe were performed using WARP [7]. Parameters roughly corresponded to experiments performed on CesrTA [4, 8]. TE waves of 2.05 GHz are launched into the beam pipe, and propagated at low power through 2 m of electron cloud. The simulation duration is 20 ns, which is sufficient for the entire volume to approach its equilibrium. Typical electron cloud densities are up to 10^{12} m^{-3}, but to enhance the visibility of the effects the simulations use an average density of 10^{15} m^{-3}. The corresponding plasma frequency of 0.284 GHz is still well below the other relevant frequencies. The beam pipe is chosen to be rectangular with horizontal and vertical dimensions of 0.09 m and 0.05 m, respectively. The lowest mode has vertical polarization and its cutoff is 1.67 GHz. Without electron cloud, $k_z \simeq 8\pi$ m^{-1}, for a wavelength of 0.25 m. Thus, there are 8 periods in the simulated region. A uniform plasma reduces k_z so that there are roughly 7.75 periods. When the plasma is concentrated at smaller $|y|$ there is no change, but when concentrated at smaller $|x|$ the number of periods is further reduced to 7.40. The vertical field on axis for these cases is shown in Figure 1.

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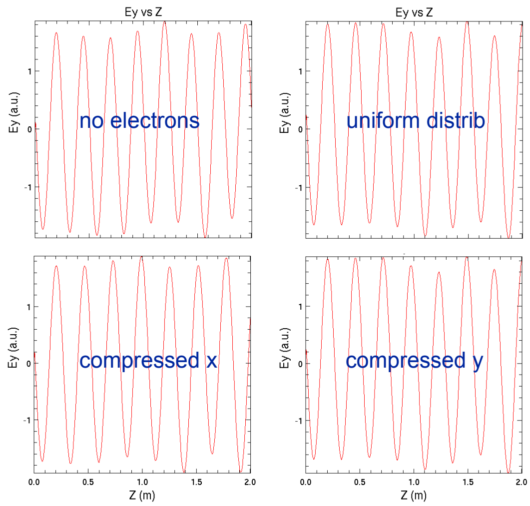

Figure 1: A comparison of TE wave propagation for the cases of no electron cloud and distributions that are uniform, compressed horizontally, or compressed vertically. The average electron density is 10^{15} m^{-3} in all cases with electrons.

MAGNETIZED PLASMA EFFECTS

Note that the previous results are only for the limit of a cold electron distribution, and for an unmagnetized system. The presence of magnetic fields can significantly alter this behavior even at low plasma densities. One reason is that additional cutoffs and resonances occur which may be close to the EM wave frequency. Moreover, the presence of the magnetic field alters the symmetry of the dielectric tensor. As a result, polarization directions can shift as the mode propagates, opposite circular polarizations can have significantly different properties, and TE and TM waves can be coupled to each other. Some analysis of waveguide modes in magnetic fields and warm plasma can be found in Refs. [9, 10, 11].

The general wave equation in an external magnetic field $\vec{B}_{\rm ext}$ is given in Ref. [12] and depends on the angle between $\vec{k}$ and $\vec{B}_{\rm ext}$, as well as the quantities $P = 1-\omega_p^2/\omega^2$, $R = 1-\omega_p^2/[\omega(\omega-\Omega_c)]$, and $L = 1-\omega_p^2/[\omega(\omega+\Omega_c)]$. The electron cyclotron frequency, $\Omega_c = eB_{\rm ext}/m_e$, is here taken to be positive. The wave properties become particularly complex for propagation at oblique angles to the magnetic field. Note that the R term exhibits strong enhancement near the cyclotron resonance, $\Omega_c \simeq \omega$, while for $\Omega_c \gg \omega$ the effect of the electron density is suppressed except for the P term, which corresponds to either electrostatic waves or "O" waves where the electric field is parallel to $\vec{B}_{\rm ext}$.

For a waveguide, the boundary condition introduces coupling between modes, some of which may not have solutions at the transmitted frequency. When $\vec{k}$ is perpendicular to $\vec{B}_{\rm ext}$ and the electric field polarization is parallel to $\vec{B}_{\rm ext}$, the magnetic field has almost no effect on the propagation because electron velocities are parallel to the field. When $\vec{k}$ is parallel to $\vec{B}_{\rm ext}$, there is Faraday rotation of the polarization direction, assuming both polarizations are allowed by the waveguide. In more general cases, the typical solutions will involve some reflected component or other dissipation.

Large Dipole Field

Simulations were performed using WARP with similar parameters as before. A uniform magnetic field of 1.1 T is included, either horizontal, vertical, or longitudinal. The cyclotron frequency is 30.79 GHz. In addition to a rectangular waveguide, a square waveguide with both diameters 0.09 m was used as a simplified model of a cylindrical waveguide. For a cylindrical waveguide, the modes mix components and are more complicated. The cutoff frequency for a circular pipe with diameter 0.09 m would be 1.95 GHz, which is even closer to the input frequency.

When the cyclotron frequency is much higher than the input EM wave frequency, the phase shift from the plasma can be strongly suppressed. When the electric field is aligned with the external magnetic field, there is no effect. When the external field is orthogonal to both the electric fields and the waveguide axis, the phase shift should be reduced by ω^2/Ω_c^2 compared to zero external field. On the other hand, a second polarization component is generated that is out of phase and a factor of $\omega_p^2/\omega\Omega_c$ smaller in amplitude. The fields are shown in Figure 2; note that the wave has close to 8 periods in the simulation volume, very similar to the case with no electron cloud. The horizontal fields might be easier to detect, but will not propagate outside the electron cloud due to their higher-order structure. This effect is only moderately sensitive to the geometry of the waveguide.

For a longitudinal magnetic field, Faraday rotation will generate an increasing component of the opposite polarization, so long as the other polarization is able to propagate. The rotation rate is proportional to the electron density and magnetic field. An example for a square waveguide is shown in Figure 3. For a circular beam pipe, the variation of the electric field vector with position further complicates the interaction. By comparing different orientations of the TE wave, one could obtain more detailed information about the location of the electrons. Any decrease in transmission is very weak.

The kinetic energy of the electrons at first increases linearly as the EM wave fills the volume, then saturates, suggesting that any electron heating effects are either higher order in EM power or occur on a much longer time scale.

Cyclotron Resonance

Close to the cyclotron resonance, there is typically strong damping or reflection due to the mismatch between the normal waveguide modes and the waves allowed by the dispersion equation. There can also be significant heating of the electrons. The frequency for maximal electron heating rather than reflection is the "upper hybrid" frequency, $\omega_{uh}^2 = \Omega_c^2 + \omega_p^2$. For solenoid fields, where the left-hand

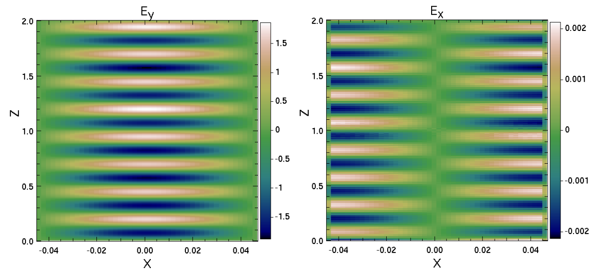

Figure 2: Transmission of a vertically-polarized TE wave in a square waveguide with a strong horizontal magnetic field and electron density of 10^{15} m^{-3}.

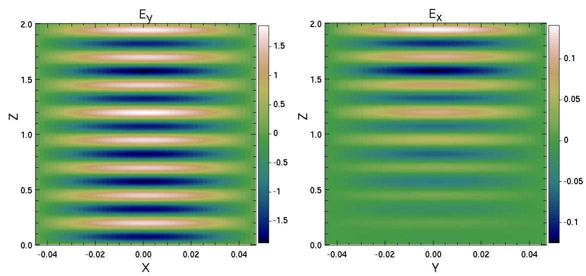

Figure 3: Transmission of a vertically-polarized TE wave in a square waveguide with a strong axial magnetic field and electron density of 10^{15} m^{-3}.

circularly polarized mode is insensitive to the resonance, half of the power will still be transmitted if this mode is allowed by the geometry of the waveguide. An example is shown in Figure 4 for a magnetic field of 0.07 T, where the cyclotron frequency is 1.96 GHz. The mode is left-hand circularly polarized, and has roughly half the power of the original mode. There is a transition region in the first few wavelengths as the right-hand polarized fields decay.

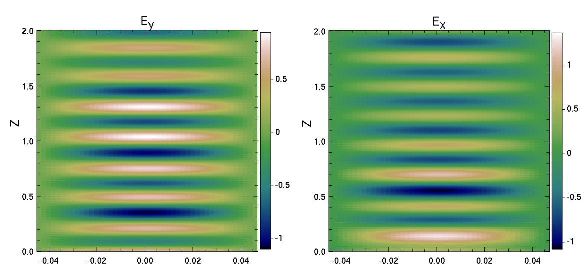

Figure 4: Transmission of a vertically-polarized TE wave in a square waveguide with vertical polarization and an axial magnetic field near resonance. The electron density is 10^{15} m^{-3}. A left-hand circularly polarized wave is transmitted at roughly half the original power.

CONCLUSIONS

Simulations have been performed in WARP to model electron cloud measurements through TE waves using the beam pipe as the waveguide. Waveguide modes in the presence of a cold plasma can be simply related to a weighted average of the electron density. However, thermal effects alter this behavior and external magnetic fields introduce significant complexity to the guided mode properties. Furthermore, while propagation near the cyclotron resonance significantly enhances the phase shift and reduce transmissions, large magnetic fields can suppress the phase shift effect for all but "O" waves. Other signals of electron cloud, such as reflection, changes in polarization, or asymmetries between polarizations may then become more useful as a signal. Here it was assumed that the electron cloud density is slowly varying compared to the wavelength of the guided mode. Close to cutoff, the wavelength increases and sharp electron density gradients must be considered.

REFERENCES

[1] J. Izawa, Y. Sato, and T. Toyomasu, "The vertical instability in a positron bunched beam", Phys. Rev. Lett. **74** (1995) 5044–5047.

[2] M.A. Furman, "Formation and dissipation of the electron cloud", *Proceedings of PAC03, Portland, Oregon 2003*, p. 297 (paper TOPC001).

[3] S. De Santis, J.M. Byrd, F. Caspers, A. Krasnykh, T. Kroyer, M.T.F. Pivi, and K.G. Sonnad, "Measurement of Electron Clouds in Large Accelerators by Microwave Dispersion", Phys. Rev. Lett. **100** (2008) 094801.

[4] S. De Santis, M.G. Billing, M. Palmer, J. Sikora, B. Carlson, "TE Wave Measurements of the Electron Cloud in the Cesr-TA Synchrotron Ring", *Proceedings of IPAC10, Kyoto, Japan 2010*, MOPE088.

[5] K.G. Sonnad, M. Furman, S. Veitzer, P. Stoltz, and J. Cary, "Simulation and analysis of microwave transmission through an electron cloud, a comparison of results", *Proceedings of PAC07, Albuquerque, New Mexico 2007*, THPAS008.

[6] J.C. Slater, "Microwave Electronics", Reviews of Modern Physics **18** (1946) pp. 441–512; in particular, Eqn. III.89.

[7] J.-L. Vay, A. Friedman and D.P. Grote, "Self-consistent simulations of high-intensity beams and e-clouds with WARP/POSINST", *Proceedings of ICAP06, Chamonix, France 2006*, Vol. 1, p. 262 (paper WEA3MP02).

[8] M.A. Palmer, J. Alexander, M. Billing, J. Calvey, et al., "The conversion and operation of the Cornell electron storage ring as a test accelerator (CesrTA) for damping rings research and development", *Proceedings of PAC09, Vancouver, Canada 2009*, paper FR1RAI02.

[9] S.T. Ivanov and E.G. Alexov, "Electromagnetic waves in a plasma waveguide", J. Plasma Physics **43** (1990) 51–67.

[10] B.J. Hu and C.L. Ruan, "Propagation properties of a plasma waveguide in an external magnetic field", J. Phys. D: Appl. Phys. **31** (1998) 2151–2154.

[11] A. Ganguli, M.K.Akhtar, R.D. Tarey, "Theory of high-frequency guided waves in a plasma-loaded waveguide", Phys. Plasmas **5** (1998) 1178–1189.

[12] T.H. Stix, *The Theory of Plasma Waves*, McGraw-Hill, 1962.