INTRABEAM SCATTERING AT LOW TEMPERATURE RANGE*

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Abstract

During the process of ion beam crystallization, the main heating source is Intra-beam scattering (IBS), in which the Coulomb collisions among particles lead to a growth in the 6D phase space volume of the beam. The results of molecular dynamics (MD) simulation have shown an increase of heating rate as the temperature is increased from absolute zero, but then a peak in the heating rate, and subsequent decrease with ever increasing temperature. [J. Wei, H. Okamoto, and A. M. Sessler, Phys. Rev. Lett. 80, 2606 (1994)]. This phenomenon has been carefully studied by Y. Yuri, H. Okamoto, and H. Sugimoto [Y. Yuri, H. Okamoto, and H. Sugimoto, J. Phys. Soc. Jpn. 78, 124501 (2009)]. On the other hand, in the traditional IBS theory valid at high temperatures, heating rate is an ever increasing as the temperature becomes lower and lower. [A. Piwinski, Lect. Notes Phys. 296, 297 (1988)]. In this paper we attempt to extend the traditional IBS theory valid at high temperatures to relatively low temperature range, by including some many body effects in the traditional IBS theory. In particular we take into account the static and dynamic effect of the self-electromagnetic field of the beam. We shall show how these effects modify the traditional IBS theory, and present the evaluation of IBS heating rate of an ion beam in the low temperature range.

INTRODUCTION

During the past few decades, there have been theoretical efforts to study the crystalline ion beams, and one of the topics of interest is the melting process of a low temperature ion beam [1–3].While the beam propagates through the AG-focusing lattice, the Coulomb interactions among the particles leads to beam emittance growth. At high temperature range, this phenomenon is studied by the intra-beam scattering (IBS) theory [4–7]; near the ground state of the beam crystal, the heating process is simulated by the Molecular Dynamics (MD) algorithm [1].

The MD algorithm, and IBS theory are developed for beam at ultra low, and high temperature respectfully. It is interesting to extend the MD simulation to higher temperature, and see whether the result recovers that of the IBS. Likewise, we could extend the IBS theory to lower temperature, and attempt to match the MD simulation at relatively low temperature range. In this paper, we extend the IBS theory to relatively low temperature by including the static and dynamic effect of the beam self-electromagnetic field. In addition, according to MD simulation data, we discuss the lower temperature limit below which the IBS theory can no longer be applied.

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IBS IN LOW TEMPERATURE BEAM

Static effect

At a low temperature range, the static effect of the beam self-field becomes significant, which is equivalent to a defocusing effect in the AG-focusing lattice and would modulate the betatron tune and machine function of the lattice for the space-charge-dominated beam [8]. To obtain the self-consistent envelope and dispersion function, one has to include the self-potential of the particle beam [9]:

$$\begin{aligned} a'' + K_x a - \frac{K_{sc}}{a+b} - \frac{\epsilon_x^2}{a^3} &= 0 \\ b'' + K_y b - \frac{K_{sc}}{a+b} - \frac{\epsilon_y^2}{b^3} &= 0 \\ D_x'' + K_x D_x - \frac{K_{sc}}{a(a+b)} D_x - \frac{1}{\rho} &= 0 \end{aligned} \quad (1)$$

where ρ is the dipole radius; K_x and K_y are the strength of the AG-focusing elements, $K_{sc} = 2Nr_0/\beta^2\gamma^3$ is the beam perveance; r_0 is the classical radius of the beam; a and b are the horizontal and vertical envelope when the dispersion term is neglected; β and γ are the Lorentz factors. The beta and dispersion function averaged over the ring roughly satisfy the condition $\beta_{x,y} = \beta_{x,y0}/\eta$, $\alpha_{x,y} = \alpha_{x,y0}/\eta$, $D_x = D_{x0}/\eta^2$, and $D'_x = D'_{x0}/\eta^2$, where $\eta = \nu/\nu_0$ is the tune depression determined by [10]

$$\Gamma = \frac{4}{a_{ws}\lambda} \frac{(1-\eta^2)^{4/3}}{\eta^2} \quad (2)$$

where Γ is the Coulomb coupling factor, λ is the line density in the beam rest frame, and a_{ws} is the Wigner-Seitz Radius. As the beam temperature gets lower, the decreasing tune depression leads to changes in the machine functions. Although the phase space density of the beam would remain unchanged, the form factor in the IBS formulas will change. Hence, we take into account the changes of machine functions when evaluating the IBS heating rate.

Dynamic effect

In the IBS literature, it is presumed that the binary collision among particles is the dominant factor for beam temperature growth [4–7]. It is further presumed that small angle scattering dominates in the scattering process. Owing to the reason that the collision integral diverges logarithmatically at large impact parameter , an 'ad hoc' cut-off of the impact parameter at $b = b_{\max}$ is introduced to make the integral converge. In the IBS literature, the smallest beam size $\sigma_{\min}$ is often chosen to be $b_{\max}$. However, when the beam temperature is sufficiently low and beam density is high, we can consider the ion beam as a non-neutral plasma subject to the AG-focusing lattice [11]. The non-neutral plasma responses to perturbation in the time scale of the plasma frequency ω_p; hence, we can treat the beam as in quasi-equilibrium state when typical time scale of the emittance growth due to Coulomb collision is much larger than ω_p^{-1} [11].

As a result, it is natural to consider the Debye shielding effect in the Coulomb collision of ion beam, since it is possible that the Debye length λ_D is smaller than $\sigma_{\min}$ at low temperature. If we simplify this screening effect such that there is no Coulomb force between the two particles whose distance is larger than Debye length, the shielding provides a natural cut-off of the impact parameter at the Debye length $\lambda_D = \sqrt{\epsilon_0 T/nZ^2e^2}$, where ϵ_0 is the vacuum permittivity, n is the beam density, Ze is the particle charge, and T is the beam temperature.

Moreover, besides providing a well-defined $b_{\max}$, Debye shielding also changes the impact parameter probability density function $g(b)$ for a typical test particle. Unlike the case in an infinitely large plasma that has a linear dependency between b and its probability, the impact parameter probability density function is determined by the particles within the Debye sphere of the test particle.

IBS formulas modified by dynamic effect

We assume that particles move freely in the beam rest frame, and the majority exchange of their momentum take places when they achieve minimal distance. As presented in Fig. 1, if we assume that no Coulomb force exerts on particle P_1 when outside the Debye sphere of the test particle P_2, the relation between the impact parameter b and scattering angle ψ can be expressed as:

$$b^2 = \frac{\cot^2(\psi/2)\lambda_D^2}{\cot^2(\psi/2) + A^2} \tag{3}$$

where $A = 1 + 2\overline{\beta}^2\lambda_D/r_0$, and $\overline{\beta}$ is the relative velocity of the two particles. Moreover, unlike the case in plasma with infinitely large size, the impact parameter probability density function within the Debye sphere is:

$$g(b)db = \frac{3b}{\lambda_D^3}\sqrt{\lambda_D^2 - b^2}db \tag{4}$$

The corresponding scattering angle distribution can be expressed as:

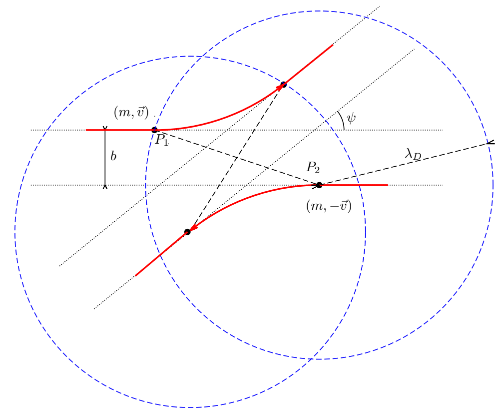

Figure 1: Binary collision of two particles with Debye shielding. The Debye spheres of P_2 at two critical positions are presented by the blue circles. The motions of the two particles are illustrated by the red trajectories.

$$f(\psi)d\psi = \frac{3}{2}\frac{A^3\cot(\psi/2)\csc^2(\psi/2)}{[\cot^2(\psi/2) + A^2]^{5/2}}d\psi \tag{5}$$

As evident from Eq. (3) and (5), the scattering angle ψ can be integrated from $[0, \pi]$ with $\psi = 0$ corresponding to $b = \lambda_D$. Note that collision integral no longer diverges at small scattering angle; hence, we omit the assumption that small angle collisions dominate in our derivation.

Following the approaches in Ref. [4, 5], we could calculate the IBS heating rate of an ion beam with Gaussian distribution. We find that the results follow the same form as Eq. (28) in Ref. [5], with the Coulomb logarithm term $\ln(1 + C^2)$ substituted by

$$\frac{C^2D^2}{2(D^2-1)^{7/2}}\Big[-\sqrt{D^2-1}(4 + 11D^2) + 3D(3 + 2D^2)\text{arccosh}(D)\Big] \tag{6}$$

where $C = q^2(\xi^2 + \zeta^2 + \theta^2)/4$, $D = 1 + C$, $q = 2\beta\gamma\sqrt{2b_{\max}/r_0}$ with $b_{\max} = \lambda_D$, and we follow the definition of ξ, ζ, θ in Ref. [5]. It can be confirmed that in the limit of $\lambda_D \to \infty$, Eq. (5) recovers the traditional result of Rutherford scattering cross section, and Eq. (6) recovers the tradition Coulomb logarithm.

NUMERICAL RESULTS OF THE MODIFIED IBS FORMULAS

We numerically integrate the modified IBS formulas and average over the ring lattice. The resulting heating curve at relatively low temperature is presented in Fig. 2. In the calculation, we take into account the change in machine functions owing to the static effect as the beam temperature varies. To better illustrate the two effects, we numerically added the situations when the two effects are included separately in the heating rate evaluation. The calculation parameters used in the integration are presented in Tab. 1. As evident in Fig. 2, the modified IBS recovers the traditional IBS models at high temperature range. At around $\epsilon_n = 1 \times 10^{-10}$ [12], the static effect has become quite significant, and the corresponding changes in machine functions cause the heating curve go up from the traditional slope line (blue dotted line). As the temperature gets lower, at around $\epsilon_n = 1 \times 10^{-11}$ the modified IBS curve starts to bend over owing to the dynamic effect which eliminates binary collisions at distance greater than Debye length.

LOW TEMPERATURE LIMIT FOR IBS

A presumption in the IBS formulas derivation is that particles move freely in the beam rest frame. However, when the Coulomb ordering is formed, the particles no longer move freely in space, and one should alternatively adopt the physical picture of particles oscillating in the Coulomb potential. MD results indicate that on the left side of the MD heating curve peak, the particles are confined longitudinally and seldom pass one another. On the right side of the peak, particles start to move freely longitudinally. Typical particle motion on the left side is described similar to the ground state motion as described in Ref. [2]; while the typical particle motion on the right side consists of free betatron oscillations. To some extent, the MD heating curve peak is the watershed for the two different types of particle motion. Hence, it is evident that the IBS theory cannot be applied in the temperature lower than the MD heating curve peak. The temperature at which the MD heating curve peak stands using the parameters in Tab. 1 is illustrated by black dotted line in Fig. 2. Below this temperature, the IBS presumptions that particles move freely breaks down, and the IBS theory can no longer be adopted.

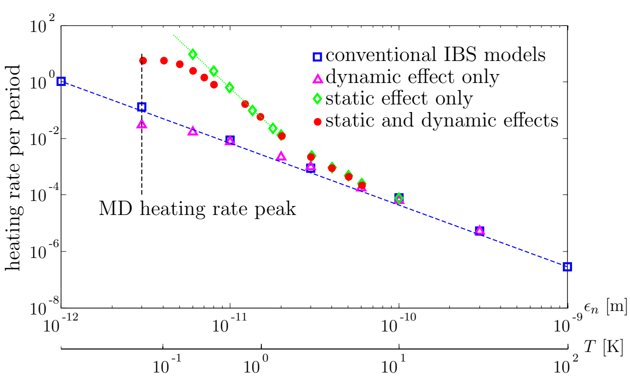

Figure 2: The numerical results of the modified IBS formulas and conventional IBS models. We cross check the result at high temperature with the conventional IBS models [4–6] using the BETACOOL code [13].

SUMMARY AND DISCUSSIONS

We extend the IBS formulas to low temperature by including the dynamic and static effect of the self-fields of the beam into the collision process, and compare its result

Table 1: Parameters used in the numerical evaluation of the modified IBS formulas.

Parameters	Values
Ion species	$^{24}Mg^{+}$
Beam energy [keV]	35
Lattice	S-LSR lattice [14]
Dipole radius [m]	1.05
Quadrupole strength	-1.5238
Tune (ν_x, ν_y)	1.44, 1.44
Super period	6
Line density [m^{-1}]	3×10^5

with the tradition IBS formulas. The dynamic effect eliminates binary collisions of which the impact parameter is greater than Debye length, hence reducing the IBS heating rate; while the static effect decreases the tune depression, changes the lattice functions, and increases the IBS heating rate. It is shown that as the temperature is lowered the heating curve of the modified IBS formulas first goes up from the traditional slope line owing to the static effect, then bends over owing to the dynamic effect.

According to MD simulation results, we find that at temperature lower than the MD heating curve peak, the particles motion are confined in the longitudinal direction, and the presumption that particles move freely in the beam rest frame breaks down. This sets a lower temperature limit below which the IBS theory can no longer be applied.

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[12] $T_{x,y} = \beta^2\gamma^2 M_0 c^2 \epsilon_{x,y}/2k_B\langle\beta_{x,y}\rangle$. Note that $\langle\beta_{x,y}\rangle$ varies as the emittance is lowered. Furthermore, at temperature lower than the heating rate peak, the conventional definition of beam temperature needs to be revised to include the coherent motion of the particles, as indicated in Ref. [10].

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