LINEAR COUPLING WITH SPACE CHARGE IN SIS18

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Abstract

For high current synchrotrons and for the SIS18 operation as booster of the projected SIS100 it is important to improve the multi-turn injection efficiency, which can be achieved by coupling the transverse motion. Linear betatron coupling due to skew quadrupole components in SIS18 with space charge was studied by simulation and measurement using different diagnostic methods. Finally, a preliminary test of skew injection is briefly discussed.

INTRODUCTION

The importance of studing linear betatron coupling and how to compensate it is due to the fact that the SIS18 working point at injection is usually chosen to be close to the difference resonance $Q_x - Q_y = 1$. Linear betatron coupling resonance is driven by skew quadrupoles due to either random rotation around the z-axis in the normal quadrupole magnets or off-centered orbits in sextupoles. Correcting linear betatron coupling prevents beam losses as a result of emittance exchange where part of the horizontal energy will be transformed to the smaller vertical acceptance. At SIS18, a controlled linear coupling can be used to improve multi-turn injection [1] by using the installed set of skew quadrupoles.

THE COUPLED TUNES

The fractional parts of tunes of the two coupled eigenmodes in the transverse plane can only approach each other up to a distance $|C|$ [2, 3], where q_x, q_y are the fractional parts of tunes of the uncoupled system,

$$q_u - q_v = \sqrt{(q_x - q_y)^2 + |C|^2} \quad (1)$$

In general, transverse Schottky spectrum analysis [4] is used to measure the fractional part of the tune q which can be estimated from the upper f^+ and the lower f^- side bands, where f_0 is the revolution frequency and m is the harmonic number, from the relations,

$$\begin{aligned} f^+ + f^- &= 2\, f_0\, m \\ q &= m \frac{f^+ - f^-}{f^+ + f^-} \end{aligned} \quad (2)$$

This method was used to measure the coupled tunes $q_{u,v}$ in SIS18 at injection. A static crossing of the coupling resonance was performed by changing the vertical set tunes and keeping the horizontal set tune fixed on $Q_x = 4.26$ with

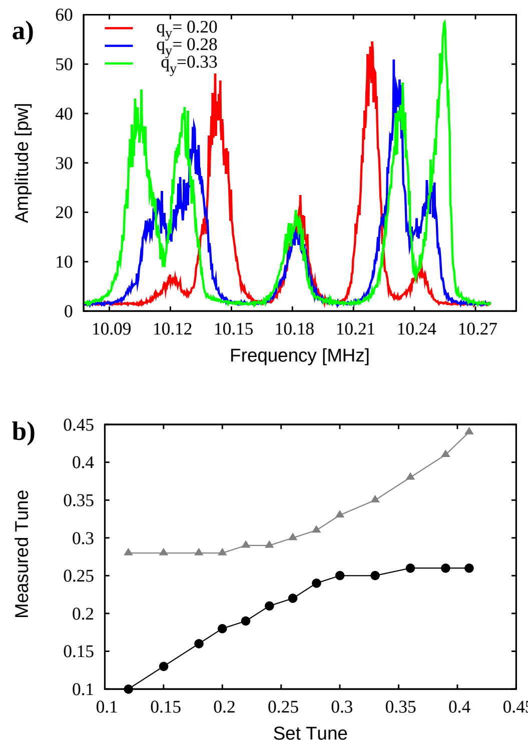

Figure 1: a) Schottky vertical spectrum measurement at SIS18 during static scan of the vertical set tune. b) the coupled tunes against the fractional vertical set tune.

an external skew quadrupole switched on with strength $k_{sq} = 50 \times 10^{-3}\,\mathrm{m}^{-1}$. The vertical spectrum for different vertical set tunes is shown in Fig. 1a. Applying Gaussian fitting to Fig.1a, we obtained the side band frequencies. Then the coupled tunes were estimated from Eq. (2) and plotted in Fig. 1b against the vertical set tunes. The minimum tune separation $|q_u - q_v|$ gives the strength of betatron coupling $|C|$, which was measured to be 0.07.

TRANSVERSE EMITTANCES EXCHANGE

In the absence of space charge and close to the difference linear betatron coupling resonance, the turn by turn evolution of the transverse emittances is given by [1],

$$\begin{aligned} \epsilon_x(N) &= \epsilon_{x0} + \epsilon_1 + \epsilon_2 + \epsilon_3 \quad (3) \\ \epsilon_1 &= \frac{|C|^2 \sin^2\Theta}{\delta^2 + |C|^2} \frac{(\epsilon_{y0} - \epsilon_{x0})}{2} \\ \epsilon_2 &= \frac{|C| \sin^2\Theta}{\delta^2 + |C|^2} \delta \sqrt{\epsilon_{x0}\,\epsilon_{y0}} \cos\phi \end{aligned}$$

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$$\begin{aligned} \epsilon_3 &= \frac{|C| \sin\Theta \cos\Theta}{\delta^2 + |C|^2} \sqrt{\epsilon_{x0}\epsilon_{y0}(\delta^2+|C|^2)} \sin\phi \\ \epsilon_x + \epsilon_y &= \text{constant} \end{aligned} \tag{4}$$

where

$$\begin{aligned} |C| &= C = \frac{1}{2\pi} k_{sq} \sqrt{\beta_x \beta_y} \\ \Theta &= \sqrt{\delta^2 + |C|^2}\, 2\pi N \\ N_s &= \frac{1}{\sqrt{\delta^2 + |C|^2}}, \qquad (\text{for } \Theta = 2\pi) \end{aligned}$$

$\epsilon_{x0,y0}$: initial uncoupled system emittances
δ : distance from the resonance center
N : turn number
N_s : exchange turn number
$|C|$: stop band width of coupling resonance
$\beta_{x,y}$: betatron functions at the skew quadrupoles.

The maximum number of turns at $\delta = 0$, which can be considered as a measure of the skew quadrupole strength, is $N_{s,max} = 1/|C|$. On the resonance center and for fixed tunes, the transverse emittances will oscillate with a period $1/|C|$. These oscillation cannot be observed if the integration time of the measurement device is too long which is the case for the residual gas monitor (RGM) installed in SIS18. Therefore, the observable quantity is an average over $N \gg 1/|C|$ of Eq. (3) and the averaged RMS emittances are given by

$$\epsilon_{x,y} = \epsilon_{x0,y0} \pm \frac{|C|^2}{\delta^2 + |C|^2} \frac{\epsilon_{y0} - \epsilon_{x0}}{2} \tag{5}$$

In the 4-D phase space, according to Liouville theorem, the coupled system invariant is proportional to the determinant of the beam matrix. Also, from simulations [6], the sum of the transverse emittances is constant up to 10^{-3}.

The transverse emittance exchange in a constant focusing lattice for zero space charge was simulated using PARMTRA, which is a multi-particle code developed at GSI [5], see Fig. 2. The skew quadrupole effect was assumed as a thin lens kick with strength $k_{sq} = 8 \times 10^{-3}\,\mathrm{m}^{-1}$ and the horizontal tune was fixed on $Q_x = 4.26$. We found, under the previous assumptions, that the emittances exchange is periodic, Fig. 2a and Fig. 2b. For a working point far from the resonance center, the exchange is fast, but the emittances are only partially shared, as in Fig. 2a. The contrary is true for a working point on the resonance, where the exchange is maximum, Fig. 2b. The width of the coupling resonance is proportional to the skew quadrupole strength, see Fig. 2c. Also, we simulated with PARMTRA the RMS emittances exchange for high intensity beams. At injection energy, a Gaussian particle distribution was generated and tracked in the SIS18 triplet linear lattice. A xy Poisson solver was used to calculate space charge forces. The linear coupling was introduced by adding a 45 degree rotated quadrupole to the first period, which applied coupling with strength $|C| = 0.0067$. The RMS transverse emittances

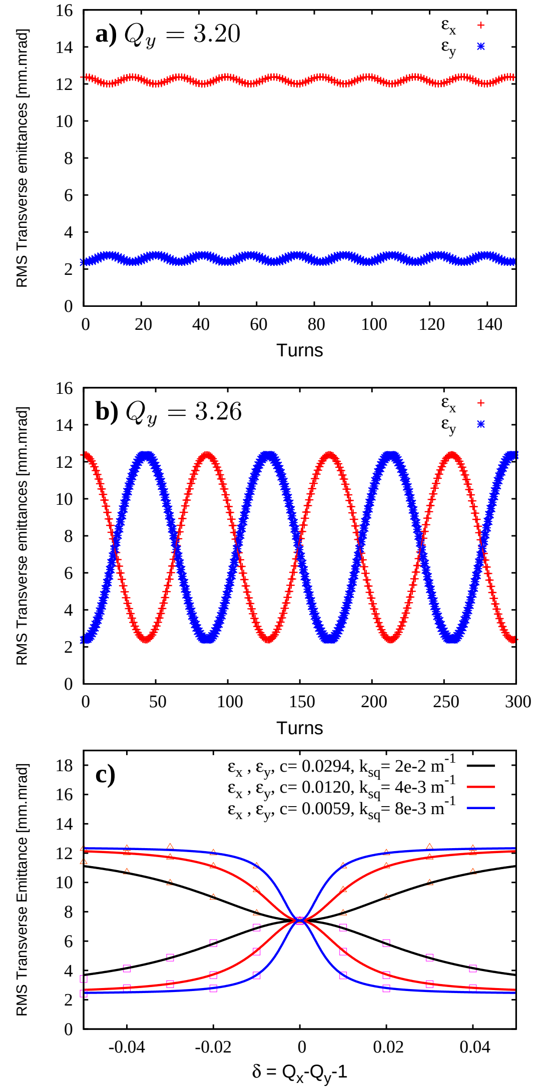

Figure 2: Transverse RMS emittance exchange simulation for zero space charge in a constant focusing lattice.

were computed turn by turn. Then we considered the averaged value to obtain Fig. 3, where we compare the results with the zero space charge case (black curves). According to [6], space charge effects are: broadening the stop band width, reduction of the maximum emittance transfer (depending on the skew quadrupole strength), shifting of the resonance center above the single particle resonance as $\epsilon_x/\epsilon_y > 1$ and modification of the exchange curve so it becomes asymmetric. The results of our simulation agree with it and with the invariance condition.

Using a residual gas monitor (RGM) [7] the averaged RMS transverse emittance exchange in SIS18 was measured for low and high intensity beams. We concluded an overestimation of the transverse beam sizes, which could be a result of the abrasion of the MCPs in the RGM. The actual beam sizes are supposed to fulfill the invariance condition $\epsilon_x + \epsilon_y = \text{constant}$ [1, 6]. In Fig. 4, we show the

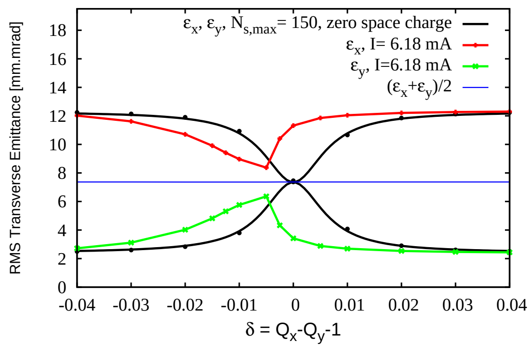

Figure 3: Emittance exchange simulation using PARMTRA. The incoherent tunes shift are $\Delta Q_x = -0.02, \Delta Q_y = -0.11$ with applied skew quadrupole strength, $k_{sq} = 5 \times 10^{-3}$ m^{-1}.

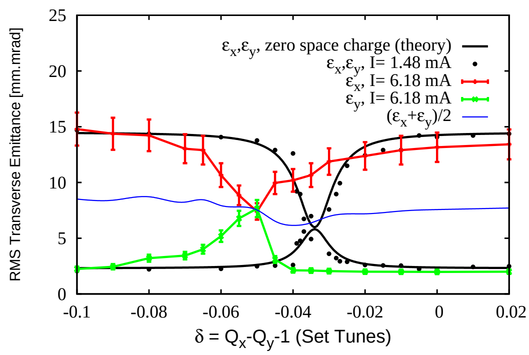

Figure 4: Emittance exchange measurement (06Aug2009) using RGM due to random skew quadrupole components. The incoherent tune shifts for the high intensity beams are $\Delta Q_x = -0.01, \Delta Q_y = -0.10$.

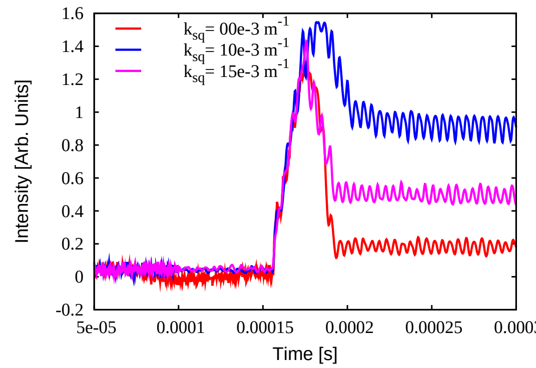

Figure 5: Current intensity in SIS18 versus time measured during multi turn injection with fast current transformer.

case with the coupling driven by random skew quadrupole components for high and low intensity beams. The random coupling strength $|C|$ in SIS18 was estimated from the low intensity measurement and from previous measurement [3] to be 0.008 ± 0.003. The large tune shift of the resonance center is due to space charge effect in addition to the known systematic deviations that is attributed to the software used in the control system at GSI [3].

APPLICATION: EFFECT ON INJECTION EFFICIENCY

Skew quadrupoles can be used to improve the multi-turn injection [1]. For a preliminary test we checked first single turn injection setting the working point to be $Q_x = 4.26$ and $Q_y = 3.35$. The injection was optimized so the bump amplitude 100 mm, the bump flank 500 μs and the delay 245 μs. For this slow bump we observed in case of applied skew quadrupoles off, 30% loss. The injected beamlet had practically no loss for switching on an applied skew quadrupole with $k_{sq} = 10 \times 10^{-3}$ m^{-1}. In the second part of the measurement the chopper window was increased to 25 μs. The bump setting and the working point was chosen such that the loss were 84% (red curve), see Fig. 5. When switching on an applied skew quadrupole with $k_{sq} = 15 \times 10^{-3}$ m^{-1} the loss decreased to 66% (pink curve), and with $k_{sq} = 10 \times 10^{-3}$ m^{-1} the loss decreased to 31% (blue curve). Further examination of skew quadrupoles effects together with bump setting is needed for multi-turn injection optimization.

CONCLUSION

Linear coupling with space charge was studied by simulation and measurement. We measured the coupled tune in SIS18 using Schottky noise for low intensity beam and from the closest tune approach we estimated the coupling strength. The transverse emittance exchange for zero space charge was compared with finite space charge case and we found an agreement between theory and simulation, but the measurement for high intensity beam showed slight deviations. As a first test and with careful injection setting, we concluded that the skew quadrupole can be used to improve the injection efficiency.

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