STUDY ON PARTICLE LOSS DURING SLOW EXTRACTION FROM SIS-100

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Abstract

The heavy ion synchrotron SIS-100 will play a key role within the future FAIR project underway at GSI. Although this synchrotron is optimised for fast extraction, also slow extraction will be used. Slow extraction is based on beam excitation due to a third order resonance. The spread in the particle momenta generating a tune spread causes particle loss leading to an irradiation of the machine especially in a high-current operation. A major part of the losses is expected to occur at the electro-static (ES) septum. In the present study we apply a tracking method to model the extraction process to predict the losses, where, in a first step, high current effects are not taken into account.

INTRODUCTION

Slow extraction from SIS-100 is done by excitation of the third order resonance with the tune given by $n = 3\nu_x$ with the harmonic number of the resonance, $n = 52$. The resonance is excited by 11 sextupoles [1]. These sextupoles are distributed in pairs in the six SIS-100 periods. The only exception is in the period of the beam transfer from SIS-100 to SIS-300. There, it is only one sextupole found. The strength of each of the M_{rsext} sextupoles follows the formula

$$k_{2,m} = k_{2,a} \sin\left(h\frac{2\pi s_m}{C} + \phi\right),\ m = 1,\ \ldots,\ M_{rsext}, \tag{1}$$

where $k_{2,a}$ is the amplitude of the strength of the sextupoles for resonance excitation, h is the harmonic number, s_m is the location of the mth resonance sextupole, C is the ring's circumference, and ϕ is the sextupole phase which has to be set to adjust the orientation the separatrix so as to fit the tilt angle of the ES septum.

Furthermore, the SIS-100 lattice includes 48 sextupoles for chromaticity correction united in two families.

The main parameters of SIS-100 needed for this study are presented in Table 1.

STATUS

In its present status, the synchrotron SIS-100 is optimised in the way to partially fulfill the Hardt condition to reduce the spread of the separatrix going to the ES septum due to the momentum spread [1, 3]. In SIS-100, the Hardt condition [2] is basically fulfilled if the horizontal chromaticity defined by

$$\frac{\Delta\nu_x}{\nu_x} = -\xi_x\frac{\Delta p}{p} \tag{2}$$

Table 1: SIS-100 parameters as proposed in the Technical design report [1]. The horizontal position of the ES septum blade are negative although it is located at the outer side of the ring because the direction of beam motion in SIS-100 is counterclockwise so that the x axis points to the centre of the ring.

Circumference, C	1083.6 m
Working point, ν_x, ν_y	17.31, 17.8
Harmonic number of the resonance, n	52
Hor. Twiss functions at ES septum start:	
β_x	16.126 m
α_x	1.23
Number of sextupoles for	
resonance excitation, M_{rsext}	11
chromaticity correction, M_{csext}	48
Sextupole amplitude, $k_{2,a}L$, Equation (1)	$0.15\ \mathrm{m}^{-2}$
Chromatic sextupoles' strength, $k_{2,c}L$	$-0.41\ \mathrm{m}^{-2}$
Harmonic number h, Equation (1)	4
Hor. ES septum blade position, x_{sep}	−41 mm
Horizontal width of ES septum, d_{ES}	20 mm
Tilt angle of ES septum blade, x'_{sep}	1.3 mrad
RMS momentum spread, δ_{rms}	$5 \cdot 10^{-4}$

is reduced from the natural value, $\xi_{x,nat} = -1.17$, to $\xi_x = -0.06$. It was shown in [4] that such a strong reduction generated by the chromatic sextupoles leads to a strong decrease of the vertical dynamic aperture to a non-acceptable size. For this reason, the present status of optimisation includes a reduction of the horizontal chromaticity only to $\xi_x \approx -0.29$ at the working point $(\nu_x, \nu_y) = (17.31, 17.8)$.

In the present scheme, all chromaticity correction sextupoles have the same integrated strength, $k_{2,c}L \approx -0.4\ \mathrm{m}^{-2}$, to avoid a large maximum value in sextupole strength and so the further excitation of resonances. L is the sextupole length.

The amplitude of the strength of the resonance sextupoles has been chosen to $k_{2,a}L = 0.15\ \mathrm{m}^{-2}$. This value is small so that the triangle is so large that it includes the horizontal septum position leading to huge losses there if of the chromaticity correction system switched off.

If the sextupoles for chromaticity correction are switched on, three separatrices enclosing a triangular stable phase space area are formed, too. Due to octupole-like terms appearing in the second order on perturbation theory, the influence of the chromaticity correction sextupoles results in the formation of stable islands outside the stable triangular phase space area as shown in Figure 1. The

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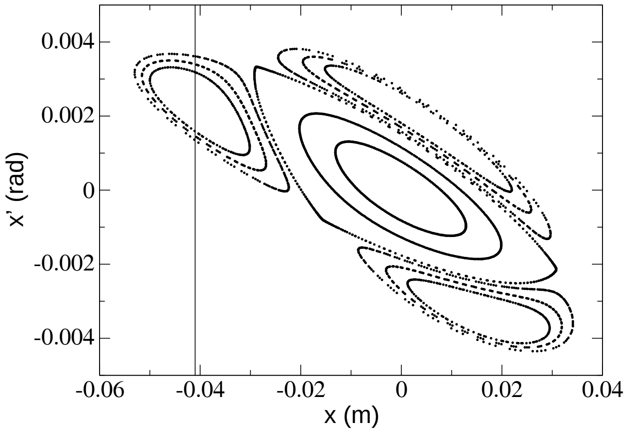

Figure 1: Stable particle trajectories in horizontal phase space for parameters according to table 1 with a momentum deviation $\delta = 0$.

islands cross the blade of the ES septum in a wide range in x'. That leads to an increase of the effective width in x' direction of the particle's path as Figure 2 shows. The resulting particle loss becomes larger than 10 %.

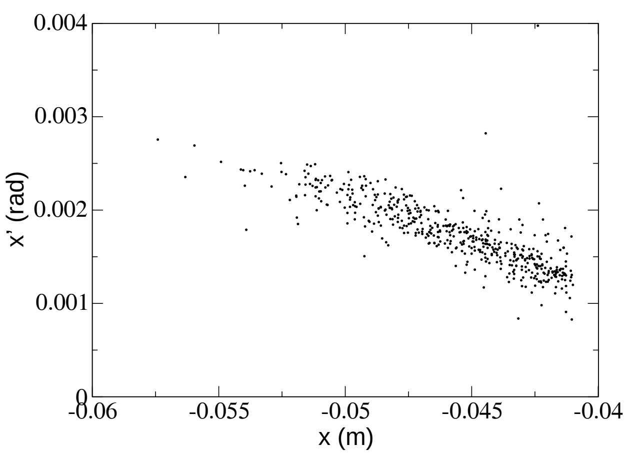

Figure 2: Position of the particles in phase space after they have passed the blade of the ES septum for parameters according to table 1. The vertical RMS emittance is $\epsilon_{y,rms} = 10$ mm mrad.

On the other hand, the attempt to cure this effect by increasing the resonance sextupole amplitude has to be done very carefully so as to avoid a larger jump length in phase space. In the case of too strong sextupoles, particles can jump over the whole septum and, so, become lost.

ANALYTIC ESTIMATE

We present here a first discussion on how to optimise the lattice. Here, we regard only difficulties arising from the tune spread due to the momentum spread in the presence of a finite chromaticity and from the chromaticity correction.

The transverse dynamics close to the third order resonance in first order perturbation theory can be described by Hamilton's equations of motion which can be written as [5]

$$\begin{aligned} \frac{\mathrm{d}r}{\mathrm{d}\phi} &= -gr^2 \sin(3\gamma + \psi) \\ \frac{\mathrm{d}\gamma}{\mathrm{d}\phi} &= \epsilon - gr\cos(3\gamma + \psi), \end{aligned} \tag{3}$$

where it is $r = \sqrt{J}$, with the Courant-Snyder invariant J, and $\gamma = n\phi/3 - \theta$ with the phase advance of the betatron oscillation during a particle's motion, $\phi(s) = \int_0^s \mathrm{d}t/[\beta(s)\nu]$, the initial phase θ, and the harmonic number of the resonance $n = 52$. $\epsilon = (n/3 - \nu)$ is the distance of the betatron tune ν from the resonance tune, and $g = A_n/8$ is the resonance width where A_n is the amplitude nth harmonic of the function $f(\phi) = [B''(\phi)/(2B\rho)]\nu\beta^{5/2}(\phi)$. Approximating the sextupoles as thin lenses, A_n as well as the according betatron phase at septum position, ϕ_{sep} can be found from

$$A_n \mathrm{e}^{3in\phi_{sep}} = \frac{1}{2\pi}\sum_{m=1}^{M_{rsext}} k_{2,m} L\, \beta_x^{3/2}(s_m)\mathrm{e}^{3in\phi(s_m)}. \tag{4}$$

Setting the expressions in equation (3) equal to zero yields the coordinates of the unstable fixed points (ufp).

$$r_{ufp} = \frac{\epsilon}{g} \text{ and } \gamma_{ufp} = -\frac{\psi}{3} + \left(0, -\frac{2\pi}{3}, -\frac{4\pi}{3}\right), \tag{5}$$

where the choice $\psi = \pi/2$ gives $\gamma_{ufp} = -\pi/6$ for the first fixed point. The spatial coordinate of this fixed points is

$$x_{ufp} = r_{ufp}\sqrt{\beta}\cos\frac{\pi}{6} = \frac{4\sqrt{3\beta}\epsilon}{A_n}. \tag{6}$$

From this expression we find the maximum value of the jump length [5],

$$\Delta x = \frac{3\pi A_n}{4\sqrt{\beta}}\left(x^2 - x_{ufp}^2\right). \tag{7}$$

It is obvious that, in the present scheme, $A_n = C_n k_{2,a}$ with the constant C_n. Furthermore, the maximum possible jump length is $\Delta x_{max} = d_{ES}$, where d_{ES} is the distance between the electrodes of the ES septum. Choosing $x = x_{sep}$, we can write

$$d_{ES} = \frac{3\pi}{4}\frac{C_n k_{2,a}}{\sqrt{\beta}}\left[x_{sep}^2 - \left(4\sqrt{3}\frac{\epsilon}{C_n k_{2,a}}\sqrt{\beta}\right)^2\right], \tag{8}$$

which can be transformed to a quadratic equation having for $\epsilon \neq 0$ always a positive and a negative solution, where the positive one is of interest. Therefore, it should always be possible to find a sextupole amplitude leading to a jump length Δx that is smaller than the distance between the electrodes of the ES septum, d_{ES}. Actually, $\Delta x = d_{ES}$ is reached with $k_{2,a}L = 1.15\ \mathrm{m}^{-2}$ and $r_{ufp} = 0.0065\ \mathrm{m}^{1/2}$ for parameters shown in Table 1.

If the chromaticity correction sextupoles are switched on the problem becomes more complicated because of the second order effects which lead to a decrease of the dynamic aperture but do do not affect A_n. For this reason, their influence is not visible in r_{ufp}, x_{ufp}, and Δx according to equations (5) – (7).

NUMERICAL MODELLING

The numerical modelling includes numerical calculations of separatrices as well as multi-particle simulations. Both are done using the thin lens tracking tool of MAD-X. This provides us a way to insert an exciter element necessary to simulate the whole knock-out extraction process.

The model introduced in the previous section is an approximation because the chromaticity correction does not affect the fixed points and the separatrices which remain straight lines. On the contrary, the tracking calculations show that the chromaticity correction reduces the stable phase space area by shifting the fixed points to the origin of the horizontal phase space, and the separatrices are, even without chromaticity correction, slightly bent. The latter could be a consequence of a too small number of resonance exciting sextupoles [2].

The usage of the sextupole amplitude $k_{2,a}L = 1.15\,\mathrm{m}^{-2}$ found from equation (8) for the conditions given in Table 1 in simulations yields unstable fixed points with $r_{ufp} = 0.0071\,\mathrm{m}^{1/2}$. This is slightly larger than that predicted by equation (5). The corresponding jump length Δx is significantly larger than that found from equation (7). To reach the desired jump length $|\Delta x| \approx d_{ES}$ the sextupole amplitude has to be reduced to $k_{2,a}L = 0.7\ \mathrm{m}^{-2}$ if all other parameters are kept constant.

Taking into account of the chromaticity correction sextupoles changes the results. To reduce the jump length we changed the sextupole amplitude in equation (1) from $\phi = \pi/2$ to $\phi = -0.9\,\pi$. This causes a clockwise rotation of the triangular stable phase space area shifting fixed point of interest closer to the septum blade. As in the initial case, the interesting fixed point is located to the left down as in figure 1. The choice of the horizontal tune $\nu_x = 17.3$ led to a small increase of the triangular stable phase space area. Here, the larger distance from the resonance tune ϵ causes a larger stable phase space area so that the unstable fixed point of interest is close to the septum blade. Additionally, the large stable phase space area is reached with resonance exciting sextupoles having strengths according to a sextupole amplitude $k_{2,a}L = 0.7\,\mathrm{m}^{-2}$ which is larger than that of the chromaticity correctors. So, island formation could be avoided. A multi-particle simulation showed that the resulting path of the particles in phase space behind the septum blade is narrower than with the initial parameters according to Table 1. This is shown in Figure 3. Hence, the particle loss due to particle collisions at the septum blade are reduced to about $5\ \%$.

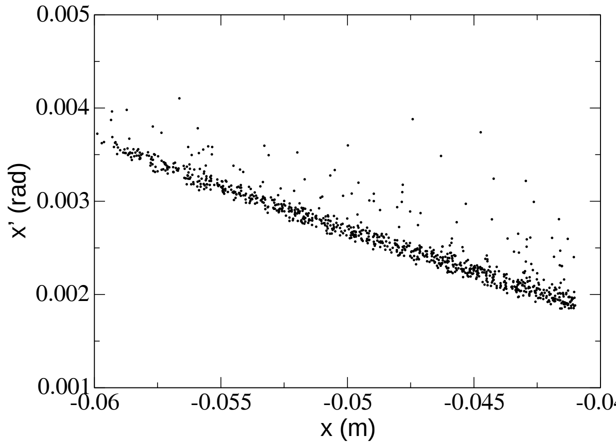

Figure 3: Position of the particles in phase space after they have passed the blade of the ES septum for parameters according to table 1 but $\nu_x = 17.3$ and a sextupole amplitude increased to $k_{2,a}L = 0.7\ \mathrm{m}^{-2}$. The vertical RMS emittance is $\epsilon_{y,rms} = 10\ \mathrm{mm\ mrad}$.

CONCLUSIONS

The present study is the first step to an optimisation of the sextupole settings of SIS-100 to minimise particle loss at the blade of the ES septum using the latest SIS-100 lattice. The goal was to find a settings which allows the motion of the particles in horizontal phase space to occur without islands formation and, so, to avoid the broadening of the path of the particles behind the ES septum blade leading to enhanced particle loss. In this first step, the basic scheme proposed in [1] is basically kept, only the amplitude of the strength of the resonance exciting sextupoles and the tune were used as varying parameters. In doing so, the acceptable vertical dynamic aperture was kept. Even this simple optimisation can provide a significant reduction of particle loss at the septum blade. A further simple step would be the adjustment of the sextupole phase ϕ to rotate the triangular stable phase space area and bring the unstable fixed point of interest closer to the ES septum. This extension to the present study is left to future work.

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