AN ANALYTICAL FORMULATION FOR PREDICTION OF GEOMETRICAL DIMENSIONS OF A PHOTOCATHODE GUN FOR DESIRED RF PROPERTIES

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Abstract

Tuning of a photocathode RF gun for desired RF properties of the π mode, such as field balance (e_b) ~1, resonant frequency f_π = 2856 MHz, and waveguide to cavity coupling coefficient β_π ~1, requires precise tuning of independent cell frequencies and waveguide to full cell coupling coefficient (β_f). In this paper, we present an analytical formulation to predict geometrical dimensions of independent cells and the coupling slot on the full cell to obtain the desired π mode RF parameters during operation, taking into account the effect of brazing and vacuum. We also compare results obtained from low power RF measurements on a photocathode gun with those predicted by the above formulation.

INTRODUCTION

Laser driven photocathode RF guns are used to generate short pulse electron beams with low emittance. For high RF efficiency and good beam quality, the gun has to be tuned for the desired π mode frequency (f_π), with waveguide to cavity coupling coefficient (β_π) =1 and field balance e_b=1. Here, e_b is the ratio of the on-axis accelerating field in the full cell to that in the half cell. The quantity β_π determines the fraction of input RF power coupled to the cavity, and it is maximum at critical coupling for which β_π ~1 [1]. To minimize emittance growth, e_b should ideally be unity, and a deviation from unity gives a discontinuity in the accelerating field thereby causing emittance growth [2]. Similarly, the π mode frequency is also tuned to the design value for synchronization with laser and the linac.

RF parameters of a photocathode gun depend upon its geometry, which is designed by employing field solver computer codes [3]. However, machining errors and limitations of codes cause a deviation of predicted values of RF parameters from actual measured values, and the structure needs to be tuned by employing an iterative cut-and-measure technique which is tedious and time consuming. The interdependence of RF parameters further makes it difficult to tune the structures by the cut-and-measure technique. In addition to this, the physical condition of the gun during tuning is different from that during actual operation in the following ways: (1) tuning is done with the gun filled with dry air, while it is under vacuum during final operation, (2) tuning is done before brazing with a small brazing clearance between the full and half cells, while this clearance is filled up by the filler alloy during actual operation after brazing. These differences in the physical conditions also cause a variation in RF parameters, and should therefore be considered during tuning.

We have developed an analytical formulation for predicting the geometrical dimensions of the photocathode gun for desired RF parameters including the effects discussed above. In the next section, we discuss the methodology to predict the geometrical dimensions of photocathode gun, which is followed by a study of the effect of vacuum, brazing, and other geometrical dimensions on RF parameters of the gun in subsequent sections. This is followed by a discussion of experimental results from tuning of an S-band photocathode RF gun of the BNL/SLAC/UCLA design.

METHODOLOGY

A photocathode RF gun with RF coupling in the full cell can be represented by an LCR circuit as discussed in Ref. [4]. By applying Kirchoff's Law considering nearest neighbour coupling only, and by solving the resulting circuit equations, the coupled mode parameters for the gun in terms of the independent cell parameters are given by equations 5, 6 and 11 of Ref. [4]. As discussed in Ref. [4], the required values of the independent cell parameters are full cell frequency, f_f=2854.6 MHz, half cell frequency f_h=2854 MHz and β_f =1.62, to get the desired operating RF parameters: f_π=2856 MHz, e_b=1 and β_π=1.

This method simplifies the tuning of a photocathode RF gun by predicting the required independent cell RF parameters that need to be achieved by taking machining cuts on independent cells. To avoid or minimise these machining cuts, this LCR theory has been further extended to predict the relevant geometrical dimensions of the cells of the photocathode RF gun, viz. inner diameter (ID) of half and full cells and length of the RF coupling slot, to achieve the targeted independent cell RF parameters.

The variation of f_h and f_f with the radius of half and full cells has been studied using SUPERFISH [5] as discussed in the next section. The length of the RF coupling slot and its effect on f_f is predicted by employing Gao's scaling law [6]. A combination of these with the LCR circuit analysis discussed in Ref. [4] is employed to predict the ID of both cells and the length of RF coupling slot for the desired RF parameters of the π mode.

Since tuning of the gun is done before brazing with dry air in the cells, while actual operation is after brazing and under vacuum, this difference in the physical condition of the gun causes a variation in RF parameters

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during final operation. The effects of these physical differences have also been incorporated in the LCR circuit analysis to correctly predict the geometrical parameters of the independent cells of the gun to achieve the desired RF parameters during operation.

EFFECT OF VACUUM AND BRAZING ON π MODE PARAMETERS

The variation in π-mode frequency due to vacuum can be understood by employing an LCR circuit analogy.

The resonant frequency of a resonator is given by $f_0 = 1/\sqrt{LC}$ where L and C are the effective inductance and capacitance respectively of the resonator. For a cavity filled with air, $L_{air} = \mu_r L, C_{air} = \varepsilon_r C$, where μ_r and ε_r are the relative permeability and permittivity respectively of air. Hence, the change in resonant frequency due to vacuum, assuming $\mu_r = 1$, becomes

$$\Delta f = f_0 (1 - \sqrt{\varepsilon_r}) / \sqrt{\varepsilon_r} \ . \quad (1)$$

For dry air at normal temperature and pressure (NTP), ε_r =1.00059 [7] and for f_0 =2856 MHz, the corresponding Δf = -0.8421 MHz. This agrees very well with the frequency change of 0.9 MHz reported in literature [8]. Therefore, the gun has to be tuned for π mode at 2856 - 0.842 = 2855.158 MHz to finally achieve f_π = 2856 MHz under vacuum.

From brazing considerations, a clearance of ~ 50-100 μm is maintained between the full and half cells of a photocathode gun. This brazing clearance is present during tuning, but is filled up with filler alloy after brazing. This difference in physical condition causes variation in f_f and β_f after brazing from values during tuning. SUPERFISH simulations show that f_f , while β_f shows a step response and becomes 1.17 times its value before brazing. After this change, it remains constant showing no further dependence on brazing clearance. The variation of f_f with brazing clearance is shown in Fig. 1.

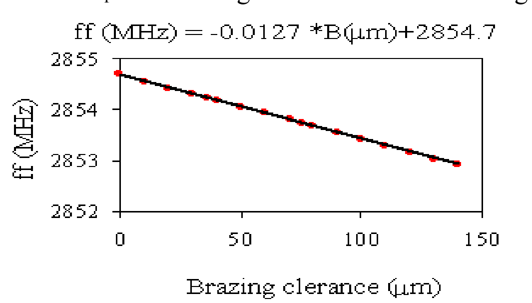

Figure 1: Variation in full cell frequency with brazing clearance.

VARIATION OF INDEPENDENT CELL FREQUENCY WITH RADIUS

To predict the ID of independent cells (half cell and full cell) of the photocathode RF gun, the variation of independent cell frequency with its radius has to be known. For an ideal pill box cavity, the resonant frequency of the TM_{010} mode varies with radius as $f = 2.405c/2\pi r$, where c is the velocity of light and r is the radius of the pill box cavity. Due to the presence of different ports for beam entry and exit, it is difficult to find an analytical expression for the dependence of resonant frequency of independent gun cells on the ID. The resonant frequency of an independent gun cell can be written as

$$f_i = a_i / r_i^{bi} \ , \quad (2)$$

where a_i and b_i are constants that depend upon the cell geometry and subscripts i = h, f refer to the half and full cell respectively. The constants for each cell can be determined by using SUPERFISH by detuning the other cell completely [9]. Figure 2a and 2b illustrate the procedure for detuning the half and full cells to obtain values of independent full and half cell frequencies respectively. The values of constants a_i and b_i in Eq. 5 for the half and full cells, as derived by this method are given in Table 1.

Table 1: Structural constants for full and half cell

Constants	a	b
Full cell	159202	1.0745
Half cell	68239	0.8521

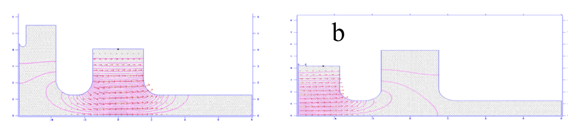

Figure 2: Field plots from SUPERFISH simulations to study the variation of independent cell frequency with its radius (a) For the full cell by detuning the half cell, and (b) for the half cell by detuning the full cell.

FULL-CELL ID AND RF COUPLING SLOT

In principle, the required ID of the full-cell for the required value of f_f predicted by the LCR circuit analysis can be determined using Eq. 2, after considering the contribution of vacuum and brazing clearance, However, Eq. 2 does not consider the RF coupling slot, the length of which has to be varied to get the desired β_f. The dependence of β_f on length of the slot is given by Gao's scaling law [10],

$$\beta_f = \frac{16 Z_0 \kappa_0 \Gamma_{10} e_0^4 l_1^6 \exp(-2\alpha_0 d)}{9ab[1 + \frac{3}{8} e_0^2 + \frac{15}{64} e_0^4 + \frac{315}{3072} e_0^6 + \ldots)^2} \frac{H_1^2}{P_c} \ . \quad (3)$$

Here d is the thickness of the RF coupling slot which depends upon the radius of full cell as

$$d = D - r_f \quad , \quad (4)$$

where D is the distance of outer wall of the RF coupling slot from the axis, and is a constant. To a first approximation, using Eqs. (2-3) and by considering the effect of brazing and vacuum, all geometrical dimensions of independent cells of a photocathode RF gun can be predicted under the assumption that the coupling slot does not affect the full cell frequency. Practically, however, this is not true and the full cell frequency varies with length of the coupling slot as

$$f_f^{\,2} = f_{(rf=const)}^2\{1-\frac{2\pi\mu_0 l_1^3 e_0^2}{12[K(e_0)-E(e_0)]}\frac{H_1^2}{U}(1-e^{-2\alpha d})\} \quad (5)$$

where $f_{\text{rf=const}}$ is the full cell frequency in the absence of RF coupling slot for a fixed ID of full cell [6]. From Eqs. 3, 4 and 5, when the length of the RF coupling slot is increased to increase β_f, the independent full-cell frequency is reduced, and the radius of the full-cell needs to be decreased to compensate this. A reduction in the full-cell radius increases the thickness of the RF coupling slot which reduces β_f. To correct the β_f, length of the RF coupling slot has to be increased further which again changes the full cell frequency and so on. This inter-dependence of f_f and β_f makes it difficult to predict the radius and RF coupling slot length for desired RF parameters. Hence, Eqs. 3-5 have to be solved simultaneously to predict the full-cell ID and RF coupling slot length. Since the radius of full cell and RF coupling slot length vary together, $f_{\text{rf=const}}$ in Eq. 5 is no longer a constant and should be replaced by Eq. 2, which gives

$$f_f^{\,2} = (\frac{a_f}{r_f^{bf}})\{1-\frac{2\pi\mu_0 l_1^3 e_0^2}{12[K(e_0)-E(e_0)]}\frac{H_1^2}{U}(1-e^{-2\alpha d})\}\,. \quad (6)$$

Equation 6 gives the variation in the full cell frequency with its radius as well as RF coupling slot length. Solving Eqs. 3, 4 and 6 together, it is possible to predict the ID of the full-cell and the length of the RF coupling slot for desired full cell parameters.

STEPS INVOLVED IN PREDICTION OF GEOMETRICAL DIMENSIONS OF GUN

To predict the geometrical dimensions of a photocathode RF gun, the following steps are involved:

Step1: Decide the π mode parameters (f_π, e_b, β_π) under vacuum.

Step2: Incorporate the effect of vacuum and get π mode parameters to be tuned in air.

Step 3: Use LCR circuit analysis to get the required independent cell RF parameters (f_h, f_f, β_f).

Step 4: Considering the effect of brazing on f_f and β_f, modify the required values of f_f and β_f.

Step 5: Determine the value of r_h using Eq. 2 and the values of r_f and length of RF coupling slot using equations 3, 4 and 6. With the independent cell dimensions obtained above, the photocathode RF gun will demonstrate the desired RF properties after brazing and under vacuum.

EXPERIMENTAL RESULTS

A photocathode RF gun made of OFE copper (OFEGUN) was tuned for f_π = 2856 MHz, e_b ~1, β_π~1 under vacuum. Initially, independent cells were fabricated with slightly smaller dimensions as compared to those predicted by the procedure discussed in the previous section. This gives flexibility in tuning the cells. For f_π = 2856 MHz with e_b = 1 and β_π = 1, the above analysis predicts the required dimensions of independent cells as r_h= 41.49 mm, r_f =41.9852 mm and RF coupling slot length L = 21 mm. Experimentally, f_π= 2855.98 MHz with e_b ~1 and β_π of 1.06 for r_h = 41.455 mm, r_f = 41.835 mm and L = 21.8 mm, showing reasonably good agreement with predictions of the analytical formulation. The deviations could be due to machining imperfections or due to measurement errors. All dimensions have been inspected on a Coordinate Measurement Machine (CMM).

CONCLUSIONS

An analytical formulation has been developed to predict geometrical dimensions of independent cells of a photocathode RF gun during tuning for desired π mode parameters under actual operating conditions. Using this procedure, a photocathode RF gun made of OFE copper has been tuned to the desired RF parameters. Good agreement has been observed between experimental results and predictions using the procedure developed.

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