BUNCH LENGTH EFFECTS IN THE BEAM-BEAM COMPENSATION WITH AN ELECTRON LENS*

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Abstract

Electron lenses for the head-on beam-beam compensation are under construction at the Relativistic Heavy Ion Collider. The bunch length is of the same order as the β-function at the interaction point, and a proton passing through proton bunch experiences a substantial phase shift which modifies the beam-beam interaction. We review the effect of the bunch length in the single pass beam-beam interaction, apply the same analysis to a proton passing through a long electron lens, and study the single pass beam-beam compensation with long bunches.

INTRODUCTION

In proton-proton collisions exact beam-beam compensation can be achieved with short bunches and a short electron lens if three conditions are met [1]: First, there are no nonlinearities between the 2 collisions. Second, the phase advance between the p-p and p-e collision is a multiple of 180 deg. Third, the proton and the electron beams produce the same amplitude dependent forces by having the same effective charge and transverse profile.

In practice these conditions can only be met approximately. We investigate how long bunches affect the beam-beam compensation. We study a test proton that interacts with a long bunch, is transported through a linear channel of variable phase advance, and interacts with a long electron lens. We consider only transversely round beams.

Our study is motivated by the head-on beam-beam compensation effort for the Brookhaven Relativistic Heavy Ion Collider (RHIC) [1], following the use of electron lenses in the Fermilab Tevatron [2–4]. The main parameters for our case are given in Tab. 1. We will first calculate the change in the transverse coordinates (r, r') of a test particle, including a time delay δt, treat the case of a long electron lens, and finally study the beam-beam compensation for proton-proton collisions with an electron lens.

SINGLE PASS PROTON INTERACTION WITH LONG BUNCH

We consider round Gaussian bunches of rms size σ_p and rms length $\sigma_s \gg \sigma_p$. We choose the longitudinal coordinate $s = 0$ at the IP, and the time $t = 0$ when the opposing proton bunch center passes through $s = 0$. The test particle shall arrive at $s = 0$ at time δt, so that its time dependent s-position is $s = \beta_{p_1} c(t - \delta t)$, where c is the speed of light, β_{p_1} the relativistic factor of the test particle, and β_{p_2} for the other beam. Following Ref. [5], radial force for a proton in the field of the other bunch is

$$F_r(r) = +\frac{n_p e^2(1+\beta_{p_1}\beta_{p_2})}{2\pi\epsilon_0 r}\left[1-\exp\left(-\frac{r^2}{2\sigma_p(s)^2}\right)\right] \quad (1)$$

where r is the radius, n_p is the proton line density, e the elementary charge, and ϵ_0 the permittivity of vacuum. The rms beam size depends on the s-position as $\sigma_p(s) = \sigma_p(0)\sqrt{1+s^2/\beta^{*2}}$, where β_p^* is the lattice function at the IP. The line density n_p, centered at $s = -\beta_{p_2} ct$, is given by

$$n_p(s,t) = \frac{N_p}{\sqrt{2\pi}\sigma_s}\exp\left[-\frac{(s+\beta_{p_2} ct)^2}{2\sigma_s^2}\right]. \quad (2)$$

N_p is the number of protons in the opposing bunch. The time evolution of (r, r') is then given by

$$\begin{aligned} \frac{dr}{dt} &= cr' \\ \frac{dr'}{dt} &= \frac{2N_p r_p c(1+\beta_{p_1}\beta_{p_2})}{\sqrt{2\pi}\beta_{p_1}\gamma_{p_1}\sigma_s}\times \\ &\times\exp\left[-\frac{c^2((\beta_{p_1}+\beta_{p_2})t-\beta_{p_1}\delta t)^2}{2\sigma_s^2}\right]\times \\ &\times\frac{1}{r}\left[1-\exp\left(-\frac{r^2}{2\sigma_p^2(\beta_{p_1}c(t-\delta t))}\right)\right]. \end{aligned} \quad (3)$$

r_p is the classical proton radius. Eqs. (3) can be integrated numerically, for example with MATHEMATICA [6].

Table 1: Reference case for RHIC beam-beam and beam-lens interactions with parameters from Ref. [1].

quantity	unit	value
proton beam parameters		
total energy E_p	GeV	250
bunch intensity N_p	10^{11}	2.0
rms beam size at IP6, IP8 σ_p^*	μm	70
rms beam size at IP10 σ_p^*	μm	310
rms bunch length σ_s	m	0.25
hourglass factor F, initial	...	0.88
beam-beam parameter ξ/IP	...	0.010
number of beam-beam IPs	...	2+1*
electron lens parameters		
distance of center from IP	m	2.0
effective length L_e	m	2.1
kinetic energy E_e	kV	6.4
relativistic factor β_e	...	0.16
electron line density n_e	10^{11}m^{-1}	0.82
electrons in lens N_{e1}	10^{11}	1.7
electrons encountered N_{e2}	10^{11}	2.0
current I_e	A	0.62

*One head-on collision in IP6 and IP8 each, and a compensating head-on collision in IP10.

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We consider a test particle with initial coordinates $(r_i(\delta t), r_i'(\delta t))$ at the IP. We transform the coordinates to a location $s < 0$ so that the test particle is outside the opposing bunch and integrate Eqs. (3) over time t until the test particle is again outside the opposing bunch. We transform the coordinates back to the IP, resulting in (r_f, r_f'). We use the vector $R = \sqrt{(r_f - r_i)^2 + (r_f' - r_i')^2}$ to describe the change in the coordinates. We compare the result of the integration with the case of an infinitely short bunch, denoted by the subscript "sb", ($\sigma_s \to 0$) and $\delta t = 0$ [5], for which we use the vector $\Delta R = \sqrt{(r_f - r_{f,sb})^2 + (r_f' - r_{i,sb}')^2}$.

The numerical integration of Eqs. (3) does generally not guarantee symplecticity, nor does it account for the energy change of the test particle due to the electric field of the opposing bunch. A full 6D symplectic treatment of the beam-beam interaction can be done with synchro-beam mapping (SBM) [7]. In Ref. [8] the SBM technique was applied to the a more general case with coupling and crossing angle.

Figure 1 (a) shows the vector (r_f, r_f') as a function of (r_i, r_i') for an infinitely short bunch. (b) shows the deviation from case (a) for a long bunch with parameters in Tab. 1, and (c) for a long bunch and a test particle with time delay $\delta t = 3\sigma_t$. Take note of the R and ΔR values. For test particles with a large time delay, a large r' value and an initial r value of about an rms beam size, the beam-beam kick is almost reversed.

SINGLE PASS PROTON INTERACTION WITH LONG ELECTRON LENSES

In RHIC the electrons in the lens are non-relativistic, and both the rms electron beam size σ_e and line density n_e are constant. To accommodate two lenses in a common beam pipe section, the lenses are placed close to but not at the IP (see Tab. 1). With this the equivalent of Eqs. (3) is

$$\frac{dr}{dt} = cr'$$
$$\frac{dr'}{dt} = -\frac{2n_e r_p c(1+\beta_{p1}\beta_e)}{\beta_{p1}\gamma_{p1}} \times \frac{1}{r}\left[1 - \exp\left(-\frac{r^2}{2\sigma_e^2}\right)\right]. \tag{4}$$

The integration of Eqs. (4) extends over the length of the electron lens L_e, i.e. the time $L_e/(\beta_{p1}c)$. The electron lens is characterized by the beam size σ_e, and its integrated strength $N_{e1} = (n_e L_e)$ (or the current $I_e = e n_e \beta_e c$).

SINGLE PASS HEAD-ON BEAM-BEAM COMPENSATION

We now consider a beam-beam interaction with a long bunch at IP8 followed by a linear transport channel and an interaction with a long electron lens near IP10 (with parameters in Tab. 1). Our expectation for a phase deviation from a multiple of 180 deg is $\Delta\psi \approx 10$ deg. The electron beam size is given by $\sigma_e = \sigma_{e0} + \Delta\sigma_e$ where σ_{e0} matches the proton beam size. The electron current is given by $I_e = I_{e0} + \Delta I_e$ with $I_{e0} = \left(\frac{N_p}{L_e}\right)\frac{e\beta_e c}{1+\beta_e/\beta_{p1}}$. I_{e0} matches the proton bunch intensity N_P of the opposing beam. Note that for a short bunch and electron lens, and $\Delta\psi = 0$, $\Delta\sigma_e = 0$, $\Delta I_e = 0$ the head-on beam-beam compensation is exact.

Figure 2 (a) shows (r_f, r_f') of a proton at IP10 after interaction with a long bunch, a linear transport channel of varying phase advance, and a long electron lens. The initial (r_i, r_i') are taken at IP8. Part (b) is with HOBBC with a phase error of $\Delta\psi = -10$ deg, and a current error of $\Delta I_e/I_{e0} = -5$ deg. Part (c) also includes a time delay of the test particle of $\delta t = -3\sigma_t$.

SUMMARY

The deviation of the beam-beam interaction from the short bunch case is most pronounced for particles with initially large r' and $r \approx 1$ rms beam size. For these particles the beam-beam effect is reduced; for the parameters in Tab. 1, by about 10%. The interaction is further modified for test particles with a time delay δt. For the parameter set under study, particles with initially large r' and $r \approx 1$ rms beam size the changes in (r, r') change sign compared to the short bunch case. The beam-beam effect in a long electron lens is also modified compared to a short electron lens. In the RHIC case the effect is small because of the size of the β-function at the electron lens. Consistent with these results we find single-pass head-on beam-beam compensation ineffective for particles with initially large r' and $r \approx 1$ rms beam size, and large δt. Compensation should still be effective when averaged over many turns.

The single pass head-on compensation calculations can be used to establish goals for the betatron phase error between between the p-p and p-e interactions ($\Delta\psi \leq 10$ deg), the electron beam size error ($\Delta\sigma_e/\sigma_{e0} \leq 20\%$) and the electron current error ($\Delta I_e/I_{e0} \leq 5\%$) [9]. A full investigation of the effects of long bunches requires long-term tracking of many particles. The modification of resonance driving terms due to the bunch length was investigated in Ref. [10].

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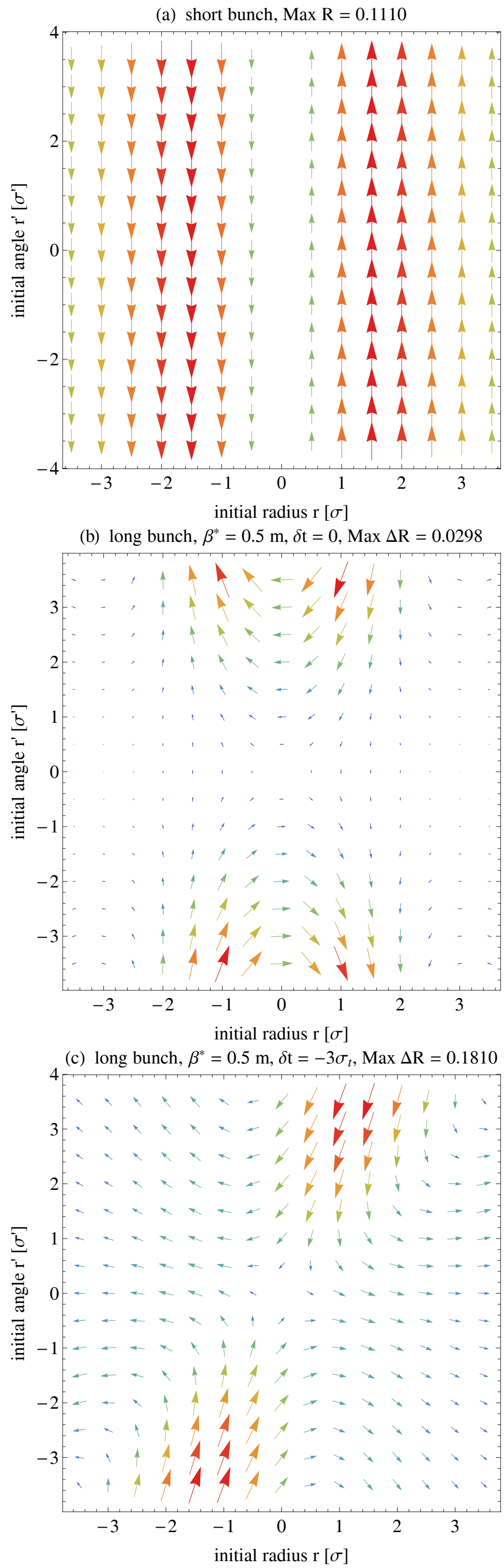

Figure 1: Change of (r, r') as a function of (r_i, r_i') for a test particle interacting with a long proton bunch. (a) shows the the change of (r, r') for an infinitely short bunch. (b) shows the deviation from case (a) for a long bunch with parameters in Tab. 1, and (c) for a long bunch and a test particle with time delay. Take note of the R and ΔR values.

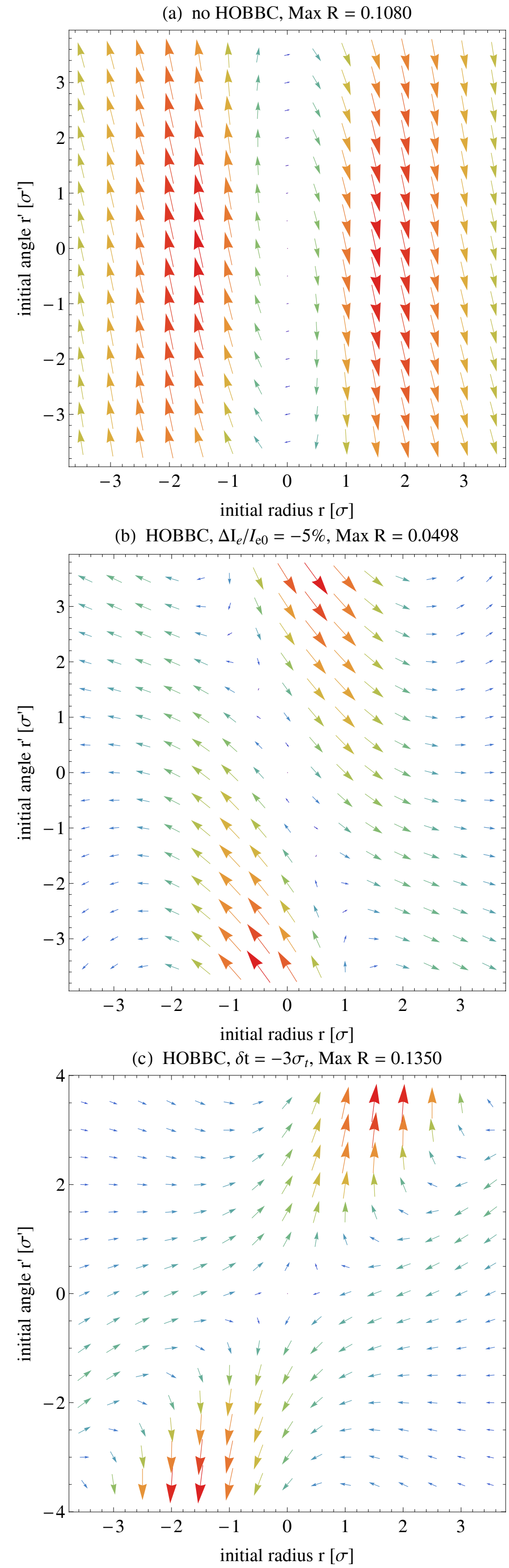

Figure 2: (r_f, r_f') at IP10 as a function of (r_i, r_i') at IP8 for a test particle. (a) shows the situation without head-on beam-beam compensation (HOBBC), and (b) with HOBBC with a phase error of $\Delta\psi = -10$ deg, and a current error of $\Delta I_e/I_{e0} = -5$ deg. Part (c) also includes a time delay of the test particle of $\delta t = -3\sigma_t$.