NON-LINEAR PARAMETRIC EFFECTS AND BEAM COLLAPSE AT MOTION OF ACCELERATED PARTICLES IN TRANSVERSAL FOCUSING FIELDS

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Abstract

The new type of oriental motion, so called parametric channeling of accelerated charged particles with internal energy structure in crystals or transversal focusing fields (TFF) is studied [1, 2]. Peculiarities of this motion are connected with parametric coupling of transversal oscillations of fast particle in TFF (e.g. averaged field of crystal plains) and oscillations caused by internal processes in particle. Parametric channeling is investigated for small charged mesomolecules, atomic ions and nuclei with internal resonances, relativistic electrons. It was shown that such parametric coupling leads to the possibility of beam cooling and "collapse": critical decrease of transversal oscillations of moving structured ion in TFF due to energy transfer from this ion to its own internal electron (for atomic ion) or its internal low energy nuclear state (for fast nuclei). Also it was shown that parametric beam cooling with the decrease of transversal energy can take place at axial relativistic electron beams channeling. This process is caused by the parametric coupling between quantized channeling states and electron spin states in effective magnetic field in moving system.

INTRODUCTION

A new type of self-organized orientational motion of charged particles with internal energy structure in crystals, i.e., parametric channeling, is proposed and examined in the article. The essence of this motion mode is associated with strong parametric coupling of characteristics of orientational oscillations caused by the interaction of the moving particle charge with averaged fields of crystal axes and planes and the transitions caused by intraparticle processes. The analysis of the parametric processes during channeling of molecules on the basis of the solution of classical equations of motion is given below. The results of the consideration of the parametric processes during channeling of atoms and electrons are presented in the second part of this paper.

PARAMETRIC CHANNELING AND COLLAPSE OF CHARGED PARTICLES BEAMS IN CRYSTALS

We consider the features of the effect of intramolecular vibrations on orientational motion of an ionized molecule in the planar channel. There is a wide variety of molecules and charge distributions in them. To illustrate the essence of the parametric channeling process, we restrict the analysis to the consideration of the motion of a diatomic molecule AB+ in which one atom is ionized and another is neutral. We analyze the problem on the basis of the classical consideration.

We independently consider two cases with different relative orientations (parallel or perpendicular) of the axes of the diatomic molecule and planar channel. The time of quantized spatial rotation of the moving molecule axis at a small orbital quantum number $l \leq 1$ in many cases much exceeds the time of flight of this molecule through a thin crystal in the channeling mode; therefore, the rotational motion of the molecule appears "frozen" in the channeling region. In this system, the total potential energy of the molecule under consideration consists of two components:

(i) The potential energy of the ion contained in the molecule, which is controlled by the interaction of the ion with channel walls;

(ii) The potential energy of the interaction of the neutral atom with the ion of the same molecule.

Using the Lagrange equation we can determine equations of motion for every particle, and after some simplifications, these expressions take the form of the standard set of equations, which defines oscillations of two coupled systems,

$$\ddot{x}_1 + f_1^2 x_1 - \alpha_1 x_2 = 0\,, \qquad \ddot{x}_2 + f_2^2 x_2 - \alpha_2 x_1 = 0\,. \quad (1)$$

The quantities f_1 and f_2 in (1) are partial frequencies of vibrations of the charged and neutral atoms in the molecule.

The maximum coupling corresponds to the case of equal partial frequencies of oscillators,

$$f_1 \equiv \sqrt{(k_1 + k_2)/m_1} = f_2 \equiv \sqrt{k_2/m_2} = f\,. \quad (2)$$

Let us consider the evolution of the entire system and the energy transfer from one oscillator to another under the following initial conditions

$$x_1(0) = x_0\,,\ x_2(0) = 0\,,\ \frac{dx_1(0)}{dt} = 0\,,\ \frac{dx_2(0)}{dt} = 0\,. \quad (3)$$

In such a case, the solutions to equations of motion take the final form

$$x_1 = x_0 \cos(\sqrt{m_2/4m_1}\,ft)\cos(ft)\,,$$

$$x_2 = x_0\sqrt{m_1/m_2}\,\sin(\sqrt{m_2/4m_1}\,ft)\sin(ft)\,. \quad (4)$$

We can see that the energy transfer from oscillatory motion of the molecule in the channel to internal

vibrations of the same molecule is a periodic process. In the associated and laboratory coordinate systems, the parametric energy exchange corresponds to variations in time and space, respectively. In the channel regions along the z axis, where the molecule vibration amplitude decreases to zero, $x_1 = 0$, local quasi-cooling of the beam occurs.

It is clear from solutions (4) that motion dynamics are independent of the initial vibration amplitude. For all molecules similarly oriented at the channel input, the time taken for maximum beam compression will be the same. Fig. 1 shows this result, which confirms the possibility of beam collapse.

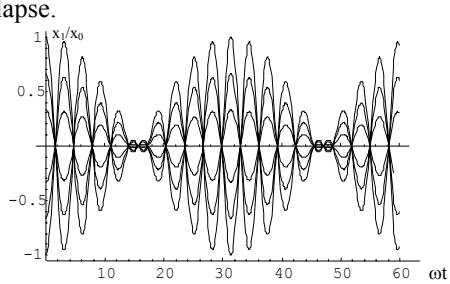

Figure 1: Periodic collapse of ion vibrations in various molecules in the channel (molecules arriving at the channel at various distances from its axis).

Now let us consider the features of orientational motion of a similar diatomic ionized molecule whose axis is oriented at a small angle to the channel axis. At such an orientation of the molecule axis, ionized molecules with longitudinal sizes greatly exceeding the channel width can be involved in channeling. Final solutions to the equation of motion of the particle are given by

$$x = X_0 \sec h(\sqrt{\frac{m_2}{(m_2+m_1)}}\frac{\omega_x X_0}{2l}t)\cos(\omega_x t), \quad (5)$$

$$z = Z_0 \,\mathrm{th}(\sqrt{\frac{m_2}{(m_2+m_1)}}\frac{\omega_x X_0}{2l}t)\,\sin(2\omega_x t), \quad (6)$$
$$Z_0 = X_0\sqrt{m_2/4(m_2+m_1)}, \omega_x = \sqrt{k_1/m_1}$$

If the molecule consists of two identical atoms, one of which is charged and another is neutral, the equations characterizing transverse and longitudinal vibrations of the molecule in this case are written as

$$x(t) = X_0 \sec h(t/\tau)\,\cos(\omega_x t), \quad (7)$$

$$z(t) = \{X_0/2\sqrt{2}\}\,\mathrm{th}(t/\tau)\,\sin(2\omega_x t),\ \tau = \sqrt{8m/k_1}/X_0 \quad (8)$$

An example of these solutions at the ratio $X_0/l = 0.2$ the initial transverse vibration amplitude and the length of the molecule is shown in Fig. 2.

We can see that in the case of a small-angle molecule orientation with respect to the channel axis the unidirectional energy transfer from the transverse oscillatory (channeled) motion of the ion in the channel to the internal longitudinal vibration of the molecule is observed, rather than a periodic energy exchange between two vibrations.

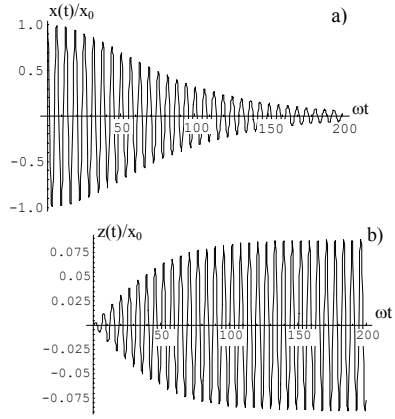

Figure 2: Variations of amplitudes of coupled (a) transverse and (b) longitudinal oscillations of the diatomic molecule moving in the channel and oriented at a small angle with respect to the channel axis.

FEATURES OF THE IMPLEMENTATION OF PARAMETRIC PROCESSES DURING CHANNELING OF ATOMIC IONS AND NUCLEI IN CRYSTALS.

Let us consider the features of orientational motion of the atomic ion with nucleus charge Ze and a single atomic electron in the planar channel. Let the nucleus coordinate with respect to the channel centre be $\vec{r}_n$ and the electron coordinate with respect to the nucleus be $\vec{r}_e$.

The Hamiltonian of the atomic ion in the channel can be written as

$$\hat{H}(x_n, \vec{r}_e) = \hat{H}_n(x_n) + \hat{H}_e(\vec{r}_e) + \hat{V}(x_n, \vec{r}_e). \quad (9)$$

$$\hat{H}_n(x_n) = \hat{T}_n(x_n) + U(x_n),\ U(x_n) = (Z-1)e\phi_{Cn}(x_n) \quad (10)$$

$$\hat{H}_e(\vec{r}_e) = \hat{T}_e(\vec{r}_e) - e\phi_e(\vec{r}_e) \quad (11)$$

are the Hamiltonians of the ionized atom and electron; the expression

$$\hat{V}(x_n, \vec{r}_e) \equiv -e\frac{\partial\phi_{Ch}(x_n)}{\partial x_n} r_e \cos\vartheta \quad (12)$$

defines the coupling (interaction) energy between the oscillatory systems of the atom and electron. The eigenfunctions $\psi_k(x_n), \psi_\beta(\vec{r}_e)$ of the operators $\hat{H}_n(x_n)$ and $\hat{H}_e(\vec{r}_e)$ define independent motion of the ion in the planar channel and atomic electron. The evolution of the electronic and nuclear subsystems is defined by the nonstationary Schrodinger equation

$$i\hbar\frac{\partial\Psi(x_n, \vec{r}_e, t)}{\partial t} = \hat{H}(x_n, \vec{r}_e)\Psi(x_n, \vec{r}_e, t) \quad (13)$$

After simple calculations the expressions for the probabilities of two alternative states can be found.

$$|c_{k\beta_0}(t)|^2 = 1 - \frac{\Omega_0^2}{\Omega^2}\sin^2\Omega t \qquad (14)$$

$$|c_{k_0\beta}(t)|^2 = \frac{\Omega_0^2}{\Omega^2}\sin^2\Omega t \qquad (15)$$

where $\Omega = [(\delta\omega/2)^2 + \Omega_0^2]^{1/2}$, $\Omega_0 = |V_{k\beta_0,\,k_0\beta}|/\hbar$ is the Rabi frequency, which defines the efficiency of the energy exchange between ion channeling states and the intra-atomic transition: $\delta\omega = \{(E_k + E_{\beta_0}) - (E_{k_0} + E_\beta)\}/\hbar$.

Equations (14)–(15) characterize the process of parametric coupling between channeled and intra-atomic states and allow us to consider the effect of parametric cooling of channeled particles with simultaneous excitation of intrinsic electronic states of the ion.

FEATURES OF THE PARAMETRIC INTERACTION AND COOLING OF THE ELECTRON BEAM DURING AXIAL CHANNELING

Detailed consideration shows that some parametric processes are completely inherent to channeling of nonrelativistic and relativistic elementary particles, despite the formal absence of their internal structure. The essence of such a paradoxical result is that these particles, apart from charge, are characterized by spin and, hence, intrinsic magnetic moment. The interaction of this moment with the effective magnetic field induced in the rest frame of the moving particle channeled in the field of crystal axes and planes controls an energy structure which is similar in a certain sense to the internal structure of atoms and molecules.

Let us consider the case of the evolution of the system forming two coupled states whose wave function is described by the superposition of two wave functions, the first one corresponds to the excited state of the channeled electron and the spin orientation projection $s_z = -\hbar/2$; the second wave function corresponds to the ground state of the channeled electron and the spin orientation projection $s_z = \hbar/2$.

Solving the Pauli we can obtain expressions for the probability of the parametric transition between channeling levels caused by synchronous transitions between spin states:

$$|c_{g,s_z=\hbar/2}(t)|^2 = \frac{\Omega_{eg(0)}^2}{\Omega_{eg}^2}\sin^2(\Omega_{eg}t) \qquad (16)$$

$$|c_{e,s_z=-\hbar/2}(t)|^2 = 1 - \frac{\Omega_{eg(0)}^2}{\Omega_{eg}^2}\sin^2(\Omega_{eg}t) \qquad (17)$$

where $\delta\omega_{eg} = (E_e - E_g)/\hbar$ is the frequency of the transition between channeling levels in the associated coordinate system and $\Omega_{eg} = \sqrt{\Omega_{eg(0)}^2 + (\delta\omega_{eg}/2)^2}$ is the effective Rabi frequency.

These solutions confirm the possibility of efficient energy exchange between channeling states of the charged particle in the crystal axis field and the states controlled by the interaction of the spin magnetic moment of the particle with the field of the same crystal axis.

CONCLUSIONS

The resulting motion of the molecular ion in the planar crystal channel is controlled by the evolution of two coupled oscillators and the possibility of their mutual transformation. The effect of such coupling depends strongly on the molecule orientation in the channel. In the case of the transverse with respect to the channel axis orientation, periodic vibrational energy exchange between these oscillators occurs. If the longitudinal crystal size corresponds to one of the values at which such beam self-compression occurs, a parametric collapse of the beam occurs with simultaneous excitation of the internal degrees of freedom of beam particles.

A completely different situation corresponds to the motion of diatomic ionized molecules with longitudinal axis oriented at a small angle to the channel axis. In this case, the parametric collapse of the molecular beam is irreversible and rapidly results in a similar parametric collapse of channeled ions.

The above consideration also shows that there exist prerequisites for providing the conditions of the parametric interaction and coupled oscillations between internal degrees of freedom of atomic ions, nuclei, and relativistic electrons on the one hand, and channeling states of the same particles in the crystal channel on the other hand. These interactions lead to the possibility of "parametric collapse" of the beam. The considered effect can also be used to cool particle beams due to energy transfer to internal nuclear states with low level energies.

Another application of this effect can be associated with the opportunity to form excited states of atoms and nuclei upon simultaneous cooling of beam particles in the channel.

REFERENCES

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