FIRST RESULTS OF SPACE CHARGE SIMULATIONS FOR THE NOVEL MULTI-TURN INJECTION

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Abstract

Recently, a novel multi-turn injection technique was proposed. It is based on beam merging via resonance crossing. The various beamlets are successively injected and merged back by crossing a stable resonance generated by non-linear magnetic fields. Space charge is usually a crucial effect at injection in a circular machine and it could have an adverse impact on the phase space topology required for merging the various beamlets. Numerical simulations were performed to assess the stability of the merging process as a function of injected beam charge. The results are presented and discussed in this paper.

INTRODUCTION

Recently it was proposed to use the same principle of transverse beam splitting [1] for a new multi-turn injection (MTI) that generates hollow beams in the horizontal plane [2]: this would be of great help in high-intensity low-energy rings, where the Coulomb repulsion among the particles limits the beam intensity. The underlying idea is the reverse process of the beam splitting: stable islands are created by injecting turn by turn several beamlets from an upstream injector around the fixed points of the horizontal phase space, generated by non-linear magnets. At the end of the injection, the islands are merged down towards the centre by crossing adiabatically the resonance. Most of the beam remains confined into an annulus around the centre, hence generating a hollow horizontal profile. This in turn leads to less severe space charge effects compared to a Gaussian profile, the peak beam intensity being lower. The aim of the present study is to get a deeper understanding of this space charge effect on the final beam distribution. In Fig. 1 the overall process is shown.

NUMERICAL STUDIES OF THE MTI

Simulations of the MTI have been ran, by injecting over four turns four beamlets with Gaussian distribution in the horizontal and vertical phase space

$$\rho_{Gaussian}(x, p_x) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+p_x^2}{2\sigma^2}} . \qquad (1)$$

As shown in Fig. 1, at the end of the crossing the islands will merge towards the center creating an annulus in phase space, and hence a hollow distribution. In Ref. [3] an analytic expansion of the final beam distribution is given in terms of Bessel functions. Such an expansion is characterised by two main parameters: μ representing the distance between the two peaks of the distribution and the center, and σ representing the width of the distribution *shoulders*.

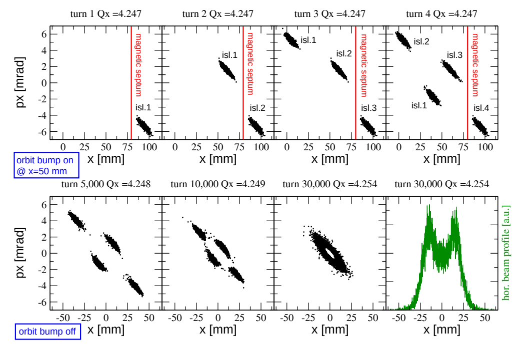

Figure 1: Top: Injection in the islands during four consecutive turns with the orbit bump on. Bottom: evolution of the horizontal phase space (after turning off the orbit bump) during the resonance crossing, when the islands are merged together. Bottom right: horizontal beam profile at the end of the process.

The accelerator model features a lattice similar to the one of the CERN PS Booster at injection energy of 160 MeV, i.e., a series of bending magnets, quadrupoles, and drift. The resonance is excited and the fixed points are created by including one octupole and one sextupole. The resonance is crossed by slowly changing the quadrupole gradient turn by turn.

Self-consistent simulations of the above accelerator structure have been carried out by means of the numerical libraries MIMAC [4]. These track an ensemble of macro-particles through a given array of magnets representing the accelerator lattice, computing at a defined time step the space charge force induced by the actual particle distribution, and applying it to the particles.

The space charge force is computed by solving the Poisson equation. The solution of the latter is found after deposing the charge distribution onto a grid, solving the equation on the grid nodes, and then computing the electric space charge field at any position via interpolation. Codes that solve equations on grids are usually referred as Particle-In-Cell (PIC) codes. Several boundary conditions may be imposed. The Poisson solver used in this study imposes Dirichlet boundary conditions (the potential goes to zero) on the 2D grid rectangle. Optionally, an elliptic contour may be chosen. Other codes implement open boundary conditions, where the potential goes to zero at infinity.

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Numerical Results

In order to understand the dependence of the final beam distribution on the process parameters (resonance crossing time, final horizontal tune and emittance of the injected beamlet), simulations have been carried out by scanning over their values without and with space charge effects. Simulations have been carried out with 10^5 particles.

Dependence on the Crossing Speed: As any adiabatic process, the islands merging needs to be slow in order to keep particles trapped in the islands and to move them towards the center without losses and emittance blow-up. Consequently, the dependence of the final beam distribution on the time needed to cross the resonance is of primary interest. As the process takes place at fixed energy (and hence revolution period), the crossing time is replaced by the number of turns set to cross the resonance.

Fig. 2 shows the dependence of μ and σ on the number of turns set to cross the resonance. It indicates that $\mu \propto \frac{1}{N^q}$ with $q \ll 1$ thus suggesting an asymptote to 0.

A number of turns that might be considered realistic for the MTI is at least of 5×10^4 as beyond this value the distribution parameters do not show any significant variation.

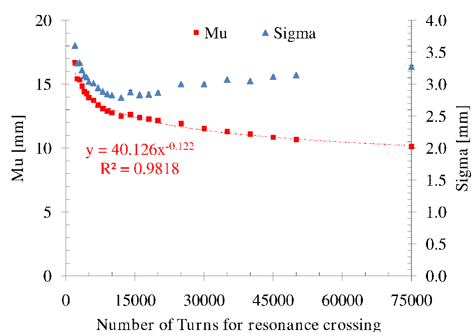

Figure 2: Dependence of μ and σ on the number of turns set to cross the resonance.

Dependence on the Initial Beamlet Emittance: Simulations were performed injecting beamlets into the four islands with different initial transverse RMS emittances (ϵ_x). Fig. 3 shows that when the initial emittance is too large the final beam distribution shows a satellite annulus (also called halo).

Both beam parameters increase linearly upon the initial beamlets emittance.

Dependence on the space charge: In order to evaluate the impact of space charge on the final beam distribution, a double scan has been performed, both on the total beam current (from 0 to 100 mA) and on the beamlet's horizontal emittance (from 1 to 4 mm mrad). The final tune was set to $Q_x = 4.258$, well outside the resonance stop-band, in order to account for the negative space-charge tune shift. The number of crossing turns has been increased accordingly in order to maintain the same tune change rate of the previous simulations, whose final tune was $Q_x = 4.254$.

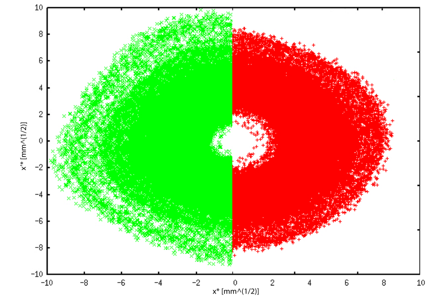

Figure 3: Final beam distribution in the Courant-Snyder coordinates for two cases of initial transverse emittance. On the right (red) $\epsilon_x = 0.49$ mm mrad, while on the left (green) $\epsilon_x = 1.49$ mm mrad.

The number of crossing turns has been increased to $91,660$ in order to keep the same tune derivative as in previous studies.

As far as μ is concerned (Fig. 4, upper), a power-law dependence $\mu = const + b \cdot I^\alpha$ was found, where $\alpha \simeq 0.5$ appears to be independent of the initial emittances. The impact of the latter is seen mainly in two extreme regimes: at low current (< 35 mA), μ increases linearly with the emittance. At higher current (> 35 mA), a slightly larger μ is found for beams with smaller emittances (hence greater brightness). It should be mentioned that the space charge changes the tune (it reduces it as it has a defocusing effect). Hence the final beamlets' position will be different with respect to the case with zero current.

Similar considerations apply for σ, Fig. 4 (lower). While high-brightness beam (red points in the plot) seems to induce a linear growth of σ with the beam current, low-brightness beams show a transient region at low current with little or no growth of σ. The extent of this region appears to be inversely proportional to the brightness.

For large $\epsilon_{x,i}$ the functional form of the final distribution is no more given by the sum of Bessel functions, even if it remains hollow. With space charge the core is no longer empty, as shown by the upper plots of Fig. 5. It can be noticed that the bigger the distribution emittance, the more the annulus has a squared shape.

It is interesting to note that the final horizontal beam distribution of a 100 mA beamlet with initial emittance $\epsilon_x = 1$ mm mrad appears to be squared and, more important, rather uniform (Fig. 6), with a parabolic profile.

It is worth mentioning that simulations with space charge eventually indicate that the final distribution preserves its hollow structure: during further 10,000 turns after the adiabatic crossing at 100 mA, both parameters μ and σ fluctuate of 5 % only.

Symmetry breaking due to the space charge: The resonance is crossed linearly from $Q_x = 4.247$ to $Q_x = 4.258$ with negative detuning with amplitude. The parameters of the final distribution, i.e. after merging the beamlets, are compared with the inverse crossing, namely with a tune variation from $Q_x = 4.258$ to $Q_x = 4.247$ and positive detuning.

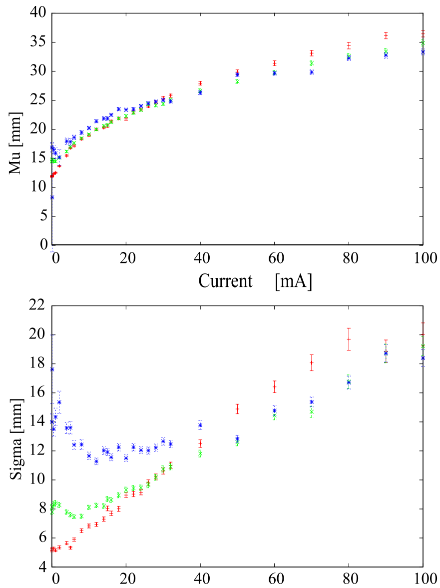

Figure 4: Dependence of μ (upper) and σ (lower) of the final distribution on beamlet current. Transverse initial emittance $\epsilon_x = 1$ mm mrad (Red), $\epsilon_x = 2$ mm mrad (Green), $\epsilon_x = 4$ mm mrad (Blue).

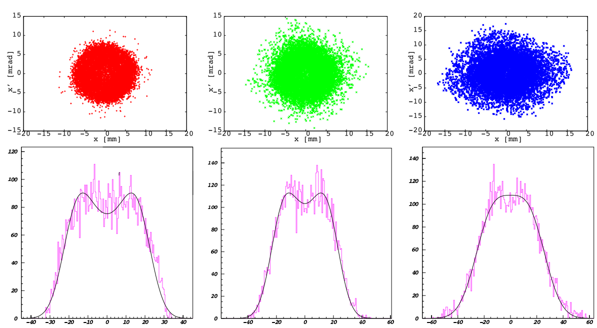

Figure 5: Phase space and corresponding profile of the distribution at the end of the MTI process with a current of 10 mA for transverse initial emittance $\epsilon_x = 1$ mm mrad, $\epsilon_x = 2$ mm mrad, $\epsilon_x = 4$ mm mrad (left to right).

In the absence of space charge, by simultaneously swapping the sign of the detuning term and the direction of the resonance crossing, the same phase space topology is retrieved. Hence, no major differences are expected to be found in the parameters of the final distribution. Indeed, by looking at the Hamiltonian of the system without space charge, it can be seen that the crossing of the resonance from above or below is symmetric. In other words, the distribution of the beam with a positive detuning term cannot be distinguished from the distribution with a negative one as it can be seen in Fig. (7) (upper). When space charge is included this is no longer the case, as the induced negative tune shift will create two "effective" crossings with different initial and final tunes. It can be observed that in the case of an intensity of 10 mA, the distribution changes depending on whether the resonance is crossed by the top or by the bottom. In fact, a large gap is found in the case of μ versus the tune between the two types of crossing.

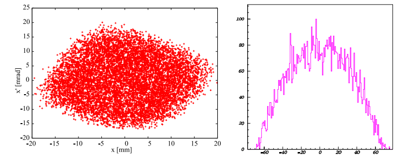

Figure 6: Left: Beam phase space for a current of 100 mA. The beam is almost uniformly populated. The squared form should also be noticed. Right: Corrresponding distribution beam profile. Important space charge effects lead to a parabolic distribution.

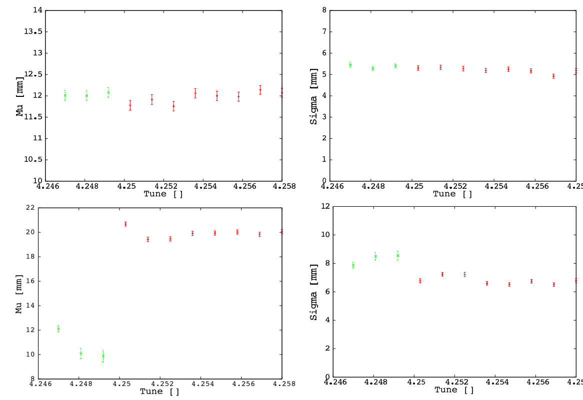

Figure 7: Top: Symmetry of resonance crossing from above (green) and below (red) without space charge. Bottom: Symmetry breaking of resonance crossing from above (green) and below (red) with space charge (10 mA).

CONCLUSIONS

Numerical simulations for the MTI have been presented without, but more importantly with space charge effects. The main result is the two regime behavior of the distribution with respect to the current. For small values of intensities, the impact of emittance is greater than the space charge effect. From a certain value of current, 35 mA, the impact of space charge is dominant and a bigger emittance lead to smaller distribution parameters μ and σ. An important point is that for very large values of current and small emittances, namely for high brightness, the distribution is losing the hollow beam property. The symmetry of the crossing process without space charge does not hold anymore with current.

ACKNOWLEDGMENTS

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