

# APPLICATION OF FREQUENCY MAP ANALYSIS TO BEAM-BEAM EFFECTS STUDY IN CRAB WAIST COLLISION SCHEME

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## Abstract

We applied Frequency Map Analysis (FMA) – a method that is widely used to explore dynamics of Hamiltonian systems – to beam-beam effects study. The method turned out to be rather informative and illustrative in the case of a novel Crab Waist collision approach, when “crab” focusing of colliding beams results in significant suppression of betatron coupling resonances. Application of FMA provides visible information about all working resonances, their widths and locations in the planes of betatron tunes and betatron amplitudes, so the process of resonances suppression due to the beams crabbing is clearly seen.

## INTRODUCTION

Crab Waist (CW) collision scheme was proposed in [1] to enhance the luminosity of electron-positron colliders. Such a scheme provides suppression of betatron and synchro-betatron coupling resonances, thus increasing the  $\xi_y$  limit by a factor of about three [2, 3]. Firstly it was predicted by simulations and then observed experimentally at DAΦNE Φ-factory [4]. Frequency Map Analysis [5, 6] turned out to be very useful for CW investigations, as it provides visible information about all working resonances, their widths and locations in the planes of betatron tunes and betatron amplitudes, so the process of resonances suppression due to the beams crabbing is clearly seen. We applied this technique to see how CW works at DAΦNE Φ-factory, where a very good agreement between simulations and experimental data has been obtained [7]. Then we performed similar studies for SuperB project [8]. Many interesting observations were made which considerably enriched our understanding of beam dynamics in CW scheme. See also [9] for more information.

## BEAM-BEAM INTERACTION IN DAΦNE WITH CRAB WAIST

The DAΦNE Φ-factory was upgraded in the second half of 2007 in order to increase luminosity and test the Crab Waist idea [10, 11]. As a result the peak luminosity was boosted by a factor of about three. The gain could be even larger, but it was limited by the crab sextupoles strength and the effects disturbing positron beam at large currents (e.g. electron cloud instability) [4, 12]. In order to investigate the innovative CW collision scheme and find its abilities and limitations due to the beam-beam effects, a “weak-strong” experiment was carried out in May 2009; the results will be published soon by the DAΦNE Team. In our beam-beam simulations we used the machine parameters corresponding

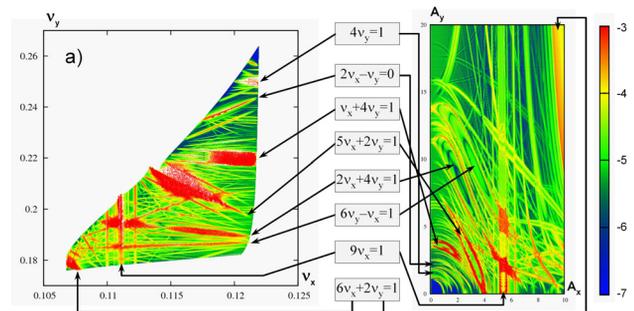


Figure 1: Beam-beam resonances in the tune and amplitude planes for DAΦNE, Crab=0.4. Correspondence between color and diffusion index is shown in the color palette.

to the best luminosity achieved in this experiment, and we got a very good agreement with the experimental data. This gave us more confidence in the tracking code which then was used in our FMA studies.

An example of the output data is presented in Fig. 1, where the FMA plots for DAΦNE with Crab=0.4 are shown. Apparently, many beam-beam resonances including the high-order ones can be clearly seen and identified on both tune and amplitude planes. Among them one can distinguish synchro-betatron resonances of different orders – in the plane of tunes they are parallel to the generative betatron ones. Note how the strong resonances look like. In the amplitude plane their widths can be recognized by the red contours corresponding to the separatrix, while in the center of resonance the motion is more regular (blue and green colors). In the plane of tunes strong resonances (e.g.  $4\nu_y = 1$ ) can be surrounded by specific white areas.

In order to study how the resonances are suppressed by the CW transformation we performed a scan of Crab value in the range of 0 to 1.0 with a step of 0.2, see Fig. 2. One of important features facilitating the comparison is that the location of resonances in the tune and amplitude planes is almost not affected. Yet note how the area occupied by the footprint shrinks when the resonances are suppressed.

Using the FMA technique we found a good illustration on the well-known Chirikov’s criterion of stochasticity. Pay attention to the resonances  $2\nu_x + 4\nu_y = 1$  and  $6\nu_y - \nu_x = 1$ . When the beam crabbing decreases from 0.4 to 0.2, their widths increase and start to overlap, thus creating a stochastic layer in the overlapping region, see the areas indicated by arrows in Fig. 2 (left column). The effect is clearly seen on both the tune and the amplitude planes. One more interesting observation is that the optimum Crab can be different for different resonances. For example, see the resonances  $\nu_x + 4\nu_y = 1$  and  $5\nu_x + 2\nu_y = 1$  indi-

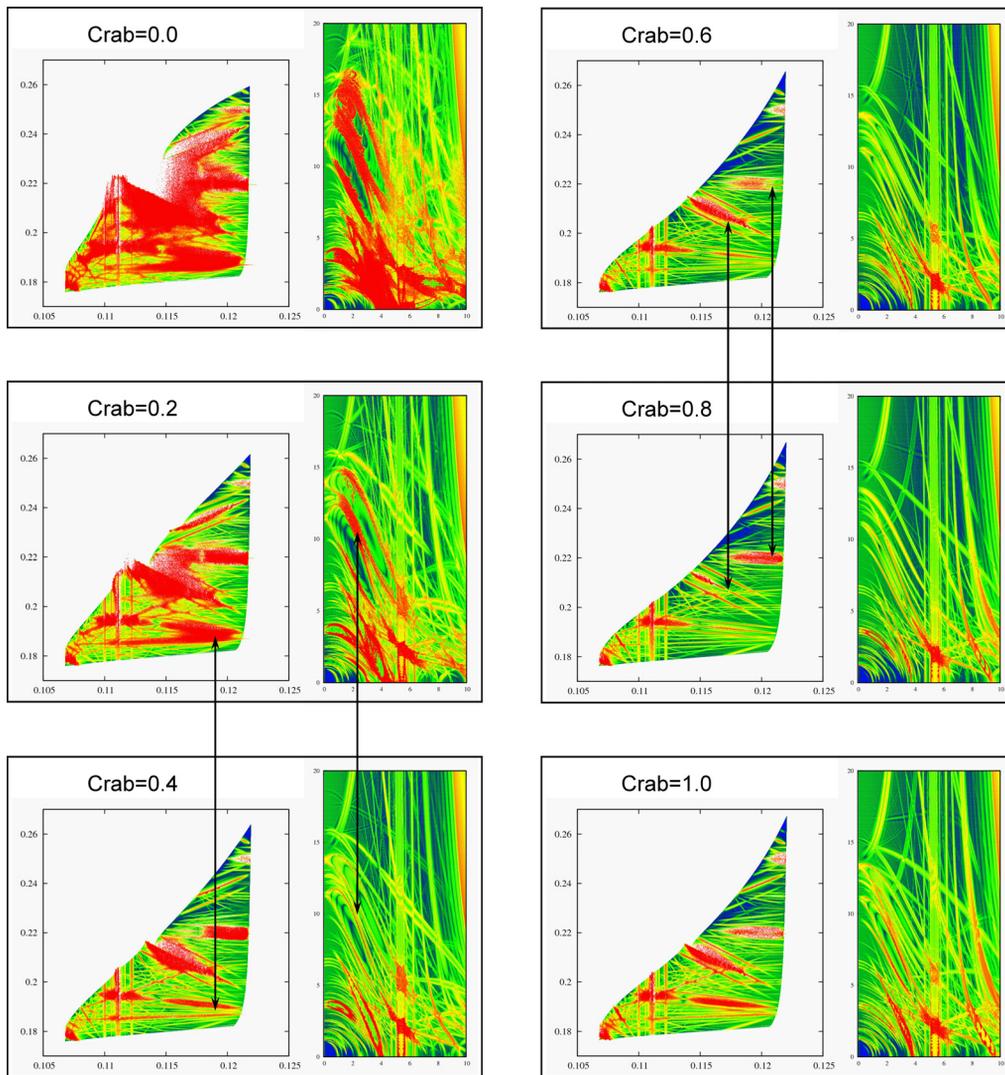


Figure 2: Beam-beam resonances in the tune and amplitude planes versus the Crab value for DAΦNE.

cated by arrows in Fig. 2 (right column). Evidently, the optimum Crab value is 0.6 for the first one and 0.8 for the second. As for the whole picture, the optimum value lies somewhere between 0.7 and 0.8.

### BEAM-BEAM SIMULATIONS FOR SUPERB

In the current version of SuperB design [13], there is an asymmetry between HER and LER lattices: emittances and beta functions are different, though the vertical beam sizes at IP are the same. Such asymmetry noticeably affects the beam dynamics. In particular, the beam-beam effects become much stronger for LER regardless of the fact that the “nominal” tune shifts are the same for both rings [14]. On the other hand, the designed beam-beam tune shift  $\xi_y \sim 0.1$  is far below the limit, which is about 0.2 for the SuperB parameters with CW.

It is worth mentioning that the effect of CW becomes

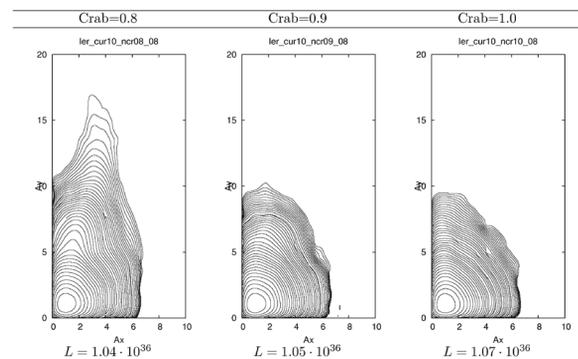


Figure 3: Contour plots of the equilibrium density distribution for SuperB LER and luminosity (numbers at the bottom) for Crab values of 0.8, 0.9 and 1.0.

valuable only for large  $\xi_y$  (high bunch currents) and it decreases when  $\xi_y$  is getting relatively small. The latter was confirmed once again by our simulations, see Fig. 3.

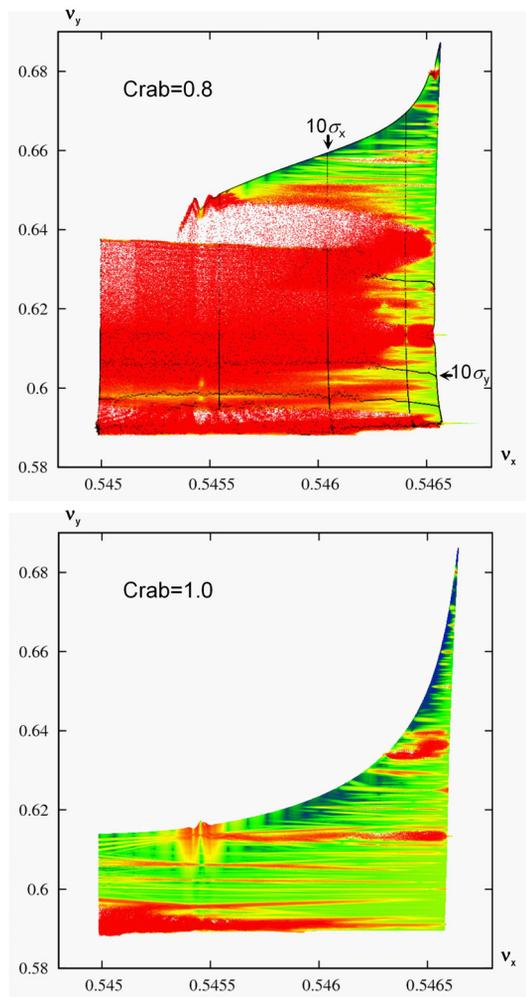


Figure 4: Beam-beam resonances in the plane of betatron tunes for SuperB LER versus the Crab value.

Though the optimum there can be determined, both the luminosity and the beam tails remain almost the same for Crab value in the range of 0.8 to 1.0. It is interesting that the FMA technique turned out to be very sensitive to the fine tuning of parameters. So, the differences look much more pronounced in Fig 4.

Besides, a number of interesting observations can be made from the footprints even without FMA. Note how the footprint shape differs from the “classical” one and how strongly it depends on the crabbing. One more surprise is connected with the actual horizontal tune spread in the beams. As it is seen in Fig. 3, the equilibrium beam distribution is located well within  $6\sigma_x$  horizontally. Looking at the footprint for Crab=0.8, where the grid of betatron amplitudes is shown (5, 10, 15 and 20 sigma), we may conclude that the actual spread of  $\nu_x$  is not greater than 0.0003. That is about 1/15 of the horizontal tune shift  $\xi_x$  which is also very small itself.

In order to make the FMA plots more informative and allow more resonances to be identified, we enlarged the plotting area to 20 sigma in both directions, while the actual

beam density occupies only a small part of it. As we see, the main differences are located at larger amplitudes, that is why they are not seen in Fig. 3.

## CONCLUSION

We found the FMA technique to be very useful for beam-beam interaction studies. Demonstration of how the Crab Waist works is clear and impressive. The capabilities of investigating every particular resonance, provided by FMA, can be very helpful for a better understanding of various mechanisms affecting the nonlinear beam dynamics.

The *old-style* tracking (e.g. see Fig. 3) produces the luminosity and equilibrium density distribution, for that damping and noises must be taken into account; but identification of resonances in the density contour plots is rather difficult. On the contrary, tracking for FMA must be without noise and damping; this results in a very high resolution of resonances, but cannot give the numbers for luminosity and beam density. Thereby we should use both techniques, as they answer different questions.

## ACKNOWLEDGMENTS

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