

# BEAM ENVELOPE CONTROL IN HEAVY ION SUPERCONDUCTING DRIFT TUBE LINAC\*

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## Abstract

At present a number of high energy heavy ion linear accelerator projects are discussed. FRIB accelerator is under R&D at Michigan University in USA, GANIL in France etc. The AEBF (RIA) project was designed in ANL, USA some years ago [1]. Using the independently phased short SC cavities with drift tubes is possible for beam acceleration and SC solenoids or quadruple can be used for focusing. The alternative phase focusing can be applied successfully too [2]. The beam envelope control is one of the main problems in these linacs. The method of analytical beam dynamics investigation is discussed. The conditions of beam envelope control are delved by using of especially averaging method, discussed in [3] initially.

## INTRODUCTION

During the past 40 years some different types of short superconducting low and medium energy cavities have been developed for ion and proton acceleration. Linacs are based on niobium SC interdigital cavities, which provide an accelerating gradient per cavity is about 1 MV. In the general case a geometrical velocity  $\beta_G$  of the RF wave is a constant for cavities and identical cavities operate at some initial driven phase  $\varphi$ . One can provide longitudinal beam dynamics stability in the system by controlling the accelerating structure driven phase and the distance between the cavities.

Beam focusing is provided by means of alternating-phase focusing (APF) and SC solenoid at the time. The layout of an accelerating structure period is sketched in Fig. 1. The low charge state beams and the low velocities require stronger transverse focusing than that is used in existing SC ion linac.

The technique of beam envelope control based on the frequencies analysis is discussed for low- $\beta$  ion with charge-to-mass ratio  $Z/A = 1/66$  in present paper.

## BASIC RELATIONS

The general equations of axisymmetric motion for ion beam moving in an accelerator can be written as

$$\left\{ \begin{aligned} \frac{d}{dt} \left( \gamma \frac{dz}{dt} \right) &= \frac{eZE_z}{Am} - \frac{1}{2\gamma} \frac{\partial}{\partial z} \left( \frac{eZA_0}{Am} \right)^2; \\ \frac{d}{dt} \left( \gamma \frac{dr}{dt} \right) &= (1 - \beta\beta_G) \frac{eZE_r}{Am} - \frac{1}{2\gamma} \frac{\partial}{\partial r} \left( \frac{eZA_0}{Am} \right)^2, \end{aligned} \right.$$

where  $\vec{R} = (z, r)$  is a beam radius-vector,  $\gamma$  is a reduced beam energy,  $A_0$  is the azimuthal vector-potential component of the solenoid magnetic field. We will suppose that  $A_0 = 0.5B(z)r$ ;  $B(z)$  is the modulus of magnetic induction vector.

RF field of periodic IH-cavity is represented as an expansion in spatial harmonics, namely

$$\left. \begin{aligned} E_z &= E_0 \cos(\omega t) \sum_n I_0(k_n r) \cos[k_n(z - z_i)]; \\ E_r &= E_0 \cos(\omega t) \sum_n I_1(k_n r) \sin[k_n(z - z_i)]. \end{aligned} \right\} \quad (1)$$

Here  $E_0$  is an amplitude of RF field at the axis ( $E_0 \neq 0$  if  $-L_r/2 < z - z_i < L_r/2$ ),  $k_n = \pi(1 + 2n)/D$ ,  $n = 0, 1, 2, \dots$ ,  $D = 0.5\beta_G\lambda$  is a cavity period length,  $L_r$  is a cavity length,  $z_i$  is a coordinate of the  $i$ th cavity center.  $I_0, I_1$  are modified Bessel functions. In our case the reference particle velocity  $\beta_c$  and the geometrical velocity  $\beta_G$  are close in each class of the identical cavities. Note that  $\omega t = k_0(z - z_i) + \varphi_i$ , where  $\varphi_i$  is the RF phase at the  $i$ th cavity center.

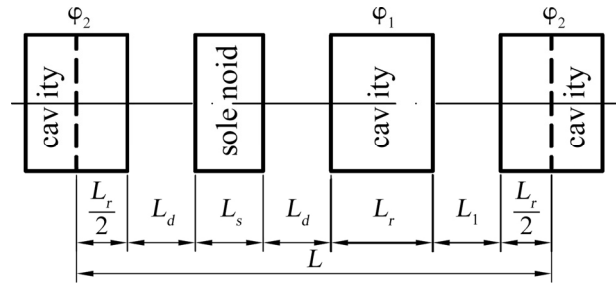


Figure 1: Layout of structure period.

## BEAM DYNAMICS ANALYSIS

It is well known that SC cavities provide high accelerating gradient but defocusing factor is much higher in comparison to the normal conducting ones at the same time. Conditions of longitudinal and transverse beam stability for the structure at hand were studied by using transfer matrix formalism previously [4,5]. There are severe disadvantages to this technique like as the impossibility of an investigation of radial-phase coupling effects, a bucket sizes calculation as well as correct bunch envelope control. In order to overcome the disadvantages pointed above we use the so called smooth approximation developed by professor E.S. Masunov with modification described in [3]. In terms of this approach we will take into account non-coherent particle oscillations in the

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beam being accelerated. To this end, one introduces a motion of a reference particle, i.e. a particle moving on the channel axis. This particle is at the point with coordinates  $(z_c, 0)$  at given moment of time. Then, one passes into the reference particle rest frame.

On averaging the beam motion equation we can write it in the form

$$\begin{cases} \frac{d^2\psi}{d\xi^2} + 3\kappa(\xi)\frac{d\psi}{d\xi} + \frac{\partial\Phi_{\text{ef}}}{\partial\psi} = 0; \\ \frac{d^2\rho}{d\xi^2} + \kappa(\xi)\frac{d\rho}{d\xi} + \frac{\partial\Phi_{\text{ef}}}{\partial\rho} = 0, \end{cases} \quad (2)$$

where  $\psi = \omega(t - t_c)$ ,  $t_c$  is the reference particle transit time,  $\rho = k_0 r$ ,  $\kappa(\xi) = d(\ln \beta_c \gamma_c)/d\xi$  and  $\Phi_{\text{ef}}$  has a view presented in [4], where straightforward waves were taken into consideration in Eq. 1 only.  $\Phi_{\text{ef}}$  plays role of the total mechanical energy in the dissipative system.

In the given case  $\Phi_{\text{ef}}$ -analysis allows one to formulate the conditions under which we can realize beam envelope control. Under beam envelope control we assume SC system parameters choice which ensures required beam sizes.  $\Phi_{\text{ef}}$  is expanded in Maclaurin's series as

$$2\Phi_{\text{ef}} = \Omega_z^2 \psi^2 + \Omega_r^2 \rho^2 + \dots, \quad (3)$$

where expansion coefficients depend on the interaction parameter  $\alpha = e\pi ZUL/2A\lambda mc^2 \beta_G^3 \gamma_G^3$ , the values of  $L_r/L$ ,  $L_s/L$  ( $L_s$  is the effective solenoid length) and the phase slip factor  $k_c = (2\pi/\lambda)(1/\beta_c - 1/\beta_G)$ . Therefore, the necessary conditions of the envelope control fulfillment are  $\Omega_z^2 > 0$ ,  $\Omega_r^2 > 0$ .

For the next study stated problem we will assume that cavity lattice was chosen in terms of condition  $\beta_c/\beta_G \cong 1$ . Now we can rewrite expansion coefficients as

$$\left. \begin{aligned} \Omega_z^2 &= -4\alpha \left[ \sin\varphi_1 + \sin\varphi_2 - 0.5\chi_1\alpha(\sin\varphi_1 + \sin\varphi_2)^2 - \right. \\ &\quad \left. - 0.5\chi_2\alpha(\sin\varphi_1 - \sin\varphi_2)^2 \right]; \\ \Omega_r^2 &= 0.25\Omega_z^2 + 3\alpha(\sin\varphi_1 + \sin\varphi_2) + BL_s/L. \end{aligned} \right\} \quad (4)$$

Here  $\varphi_1$  and  $\varphi_2$  are reference particle phase values in the inter-electrode gap centres;  $\chi_1$  and  $\chi_2$  are period filling factors,  $B$  is the zeroth Fourier coefficient of  $B(z)$ .

Table 1:  $\chi_{1,2}$ -values for different  $L_r/L$

$L_r/L$	$\chi_1$	$\chi_2$
0	1/6	1/2
1/4	1/24	1/3
1/2	0	1/6

$\Omega_z$  and  $\Omega_r$  as functions of  $\beta_G$  are shown in Fig. 2 for  $L_r/L = 0.25$ ,  $\varphi_1 = -30^\circ$ ,  $\varphi_2 = 20^\circ$  under different  $B$ -values. This figure shows that  $B$ -value is equal to 6 T does not ensure the transverse stability for all  $\beta_G$ -range. In turn, radial-phase coupling effects are seen to appear for  $B$  is equal to 9 T at the end of  $\beta_G$ -range. If  $B$ -value is equal to 12 T, it provides both the stable transverse and longitudinal beam motion. Note that magnetic field does not affect longitudinal motion.

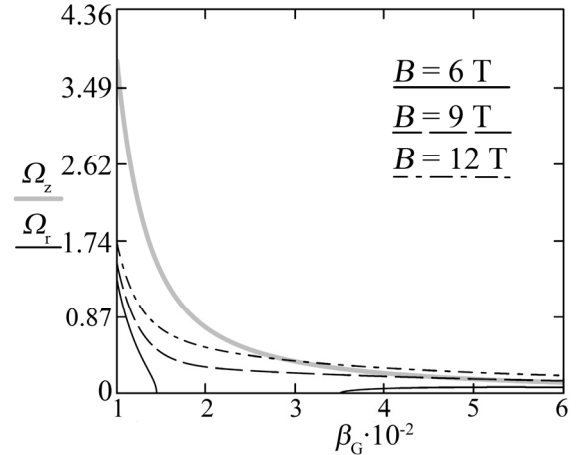


Figure 2: Free vibration frequencies.

## COMPUTER SIMULATION RESULTS

The analytical results obtained above were used to investigate the beam matching possibility at the linac output. For the analytical results to be verified computer simulations were carried out. The beam was the unbunched 50 keV/u tin ions  $\text{Sn}^{2+}$  with charge-to-mass ratio  $Z/A = 1/66$ . Initial radial deviation was 3 mm at most,  $L = 1$  m,  $L_r = 0.25$  m,  $L_s = 0.2$  m,  $B = 12$  T,  $\varphi_1 = -30^\circ$ ,  $\varphi_2 = 20^\circ$ ,  $f = 57.5$  MHz,  $\beta_G(0) = 0.01$ ,  $eU/E_0 = 1$ , amount of structure periods was equal to 25, beam current being not taken into account. The computer simulation results are presented in Fig. 3, Fig. 4 and Fig. 5.

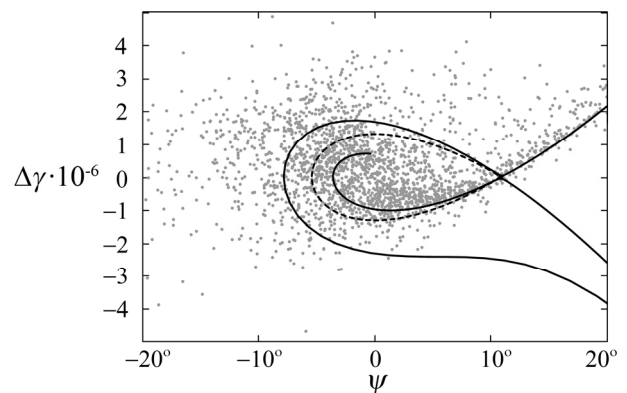


Figure 3: Longitudinal phase space.

Fig. 3 shows the longitudinal spread of beam particles at the end of accelerator, dotted curve defines longitudinal channel acceptance without dissipative effects and the solid line defines that taking into account decaying oscillations. One can see that longitudinal acceptance in the latter case is greater than in the former one.

Transversal spread of beam particle at the end of accelerator is represented in Fig. 4. Spread ellipse which confines 95% of particles within radius value is equal to 3 mm is also plotted there.

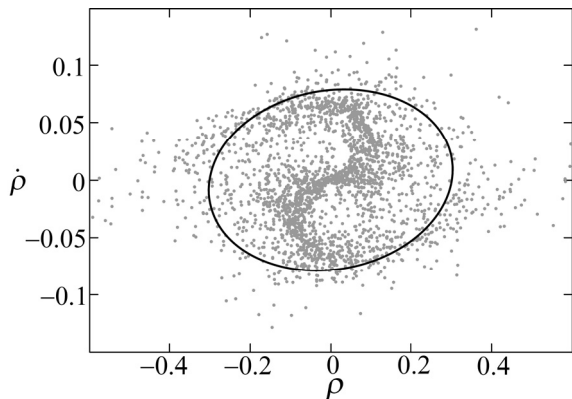


Figure 4: Transverse phase space.

Beam envelope control realized under  $\Omega_z^2 > 0$ ,  $\Omega_r^2 > 0$  conditions is demonstrated in Fig. 5 for a few particle trajectories.

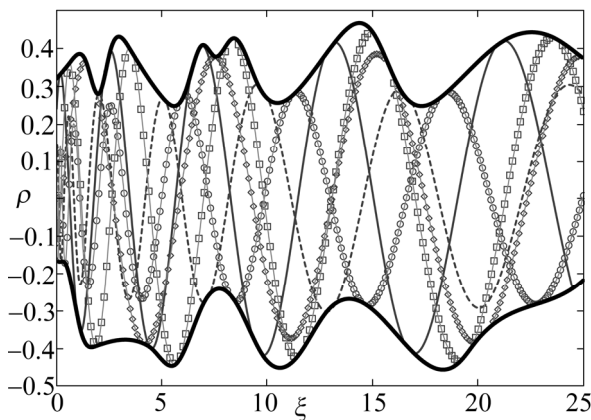


Figure 5: Particle trajectories.

### CONCLUSION

The method of the beam envelope control is presented. It is shown that the smooth approximation allows one to find necessary restrictions on the beam dynamics. Analytical results are derived and verified by means of computer simulations. All predicted results turn out to be correct.

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