

SIMULATION STUDY ON COHERENT RESONANT INSTABILITY OF NON-NEUTRAL PLASMAS CONFINED IN A LINEAR PAUL TRAP*

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Abstract

A numerical simulation for “S-POD” (Simulator for Particle Orbit Dynamics) is developed using the Particle-In-Cell code “WARP”, which allows multi-dimensional Vlasov simulations. S-POD is a linear Paul trap system constructed at Hiroshima University and is dedicated to studying various collective effects in space-charge-dominated beams. In this paper, a transverse two-dimensional model is employed to save computing time and carry out systematic comparisons with S-POD experiments on linear and non-linear coherent resonances. A large number of simulations verified that stop band distributions produced by the simulation runs are consistent with experimental results. It is confirmed that coherent resonances are excited when one of the coherent tunes is close to a half integer. Simulations also suggest that the stop band in a FODO quadrupole lattice slightly depends on the filling factor of quadrupoles in the lattice and is almost identical to that of a sinusoidal focusing when the filling factor is 1/2.

INTRODUCTION

One of the critical issues in recent high-power accelerators is collective instabilities induced by space-charge. A novel experimental method using a non-neutral plasma trap was proposed to study such instabilities [1]. The method is based on the physical equivalence between the collective motion of a charged particle travelling thorough a beam focusing channel and that of a non-neutral plasma confined in a trap. At Hiroshima University, radio-frequency quadrupole traps and a solenoid trap system were developed, and these systems are being used to investigate a wide variety of space-charge-induced phenomena [2-3]. A similar linear Paul

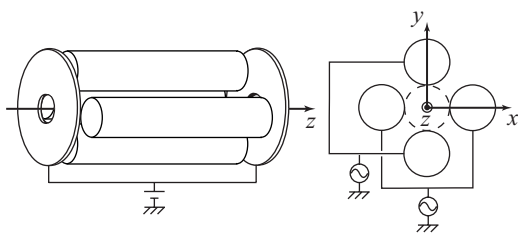


Figure 1: A schematic of a linear Paul trap.

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trap system was constructed at Princeton University and is being employed to study beam dynamics issues [4].

A schematic of a typical linear Paul trap is illustrated in Fig. 1. The plasma confinement in transverse (x - and y -) direction is achieved by an applied RF electric field with quadrupole symmetry. For the confinement in axial (z -) direction, static voltages are applied to end plates. Consider an infinitely long plasma column along the z direction since the plasma extent in the z -direction is much longer than that in the transverse direction in typical S-POD experiments [2]. Thus, the particle motion in the transverse plane is approximately governed by the following Hamiltonian,

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}K_x(\tau)x^2 + \frac{1}{2}K_y(\tau)y^2 + \frac{q}{mc^2}\phi(x, y; \tau), \quad (1)$$

where q and m are charge and mass of particles, c is the speed of light in vacuum, $K_{x(y)}(\tau)$ is a “lattice function” which is proportional to the rf quadrupole potential, and the independent variable is $\tau = ct$. The scalar potential ϕ and particle distribution f satisfy the Vlasov-Poisson equations. The Hamiltonian Eq. (1) is mathematically equivalent to that of a beam propagating through a linear transport channel. This fact is an evidence for that a plasma trap can be used for the study of beam dynamics.

A theoretical study using the linearized Vlasov analysis predicted that coherent resonances are excited under the condition [5]

$$\nu_0 - C_m \Delta\nu = N_{sp} \frac{n}{2m} \quad (2)$$

where m and n are both integers, N_{sp} is the lattice super-periodicity, ν_0 and $\Delta\nu$ are the bare betatron tune and the incoherent tune shift from space-charge, and C_m is a constant smaller than unity which depends on mode number m . The validity of the resonance condition Eq. (2) has been experimentally confirmed using S-POD [3].

S-POD system has been established as a useful experimental tool for the study of beam physics, but it should become even more useful when combined with reliable numerical simulations. For this purpose, we developed S-POD simulations using WARP code [6]. In this paper, we present simulations of S-POD experiments on linear and non-linear coherent resonances.

SIMULATION SETUP

Although WARP is primarily a full 3D simulation code, computational resources required for systematic 3D simulations tend to be large, therefore we apply a transverse 2D model where the plasma is axially uniform ($\partial/\partial z = 0$). Since the computational time of 2D

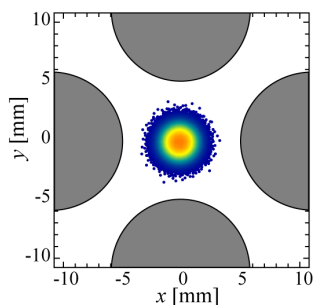


Figure 2: Simulation area.

simulations are small relative to 3D simulations, systematic 2D simulations with high grid resolution and particle statistics can be carried out. The 2D simulation also allows us to check how well S-POD reproduces a transverse coasting beam. Furthermore, the 2D simulation provides a helpful tool to setup future 3D runs since many of the code tools are the same in 2D and 3D and needed resolutions can be more easily established in 2D.

Figure 2 shows an example of geometry of simulation area. Both applied and self-field forces are calculated with quadrupole symmetry biased circular electrodes using a multi-grid field solver. Neumann boundary conditions are applied on the square simulation box to realize small applied field model improvement in region between rods. The aperture diameter $2R$ is chosen to be 10 mm, where R is equal to the minimum distance from the origin to the surface of the electrodes. The radius of each rod is $1.15R$ to minimize the strength of low-order non-linear field components of the applied field. However, non-linear components are also amplified by electrode misalignments. To model such situation, any arbitrary multi-pole of rf component can be introduced. Simulation setup including choices in the field-solver, particle-loader and diagnostics can be changed without recompiling by simply modifying an interactive Python interface which controls the interpreter based WARP code.

SIMULATION RESULTS

We here show an example of WARP simulations of an S-POD experiment on coherent resonances of charged-particle beams. In the simulations, we assume a storage ring composed of 12 FODO cells, thus 12 sinusoidal rf periods correspond to one turn of the ring. The frequency of the sinusoidal wave is chosen to be 996 kHz. A perturbation wave whose wavelength is 4 times longer than that of the primary focusing wave is superimposed to change N_{sp} from 12 to 3. The amplitude of the $N_{sp}=3$ wave is set to 0.5 % of the main rf wave. The third and fourth order multi-pole rf components are artificially introduced to enhance non-linear resonances, and the amplitudes of these multi-poles are chosen to be, respectively 6% and 1 % of the fundamental quadrupole field strength at the pole. These values were chosen to be much larger than expected values of S-POD in order to reduce computing time required to identify low order non-linear resonances. The typical simulation mesh size is

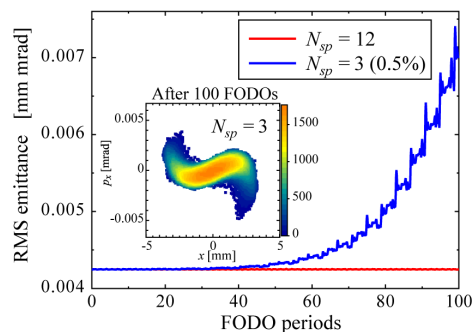


Figure 3: Evolution of transverse emittance.

less than 0.1mm, and 10^4 macro particles are initially loaded. The equations of motion are numerically integrated using 200 leap-frog steps per one main rf wave.

A typical simulation result is shown in Fig. 3 where evolution of average transverse rms emittance $(\epsilon_x + \epsilon_y)/2$ and the phase space distribution of $^{40}\text{Ar}^+$ ions are plotted. In the simulations, the bare tune is set to 1.54 and waterbag distributions are assumed initially with rms envelope conditions matched to the main rf focusing field. The initial waterbag distribution is canonically transformed from equivalent continuous focusing to alternating gradient focusing for a good initial beam match. The plasma temperature and rms equivalent beam tune depression are set to 0.1 eV and 0.95 respectively. It is clear from Fig. 3 and Eq. (2) that a linear resonance is excited due to the lattice symmetry breaking, and this collective instability results in large emittance growth.

Comparison with Experimental Data

Figure 4 summarizes results of a large number of simulations assuming 12- and 3- fold symmetric rings. The ordinate represents the ratio of rms emittance after 0.1 msec to its initial value when the initial distribution is an rms-matched waterbag. The temperature and rms tune depression are chosen to be 0.1 eV and 0.95 respectively. Similar results are obtained when initial KV and thermal distributions are launched. The 12-fold symmetric ring has no strong stop band in the region below $\nu_0 \sim 3.0$. On the other hand, a strong stop band appears around $\nu_0 \sim 1.7$ in the 3-fold symmetric ring. Some weak stop bands are also observed around $\nu_0 \sim 1.2, 2.25, 2.6$ and 2.75 . The phase space distributions at 100 periods shown in Fig. 4 indicate that the instabilities near $\nu_0 \sim 2.25$ and 2.75 are caused by third and fifth order resonances respectively.

Corresponding experimental results are shown in Fig. 5 where the ordinate is the number of $^{40}\text{Ar}^+$ ions surviving after 1 msec as measured in a Faraday cup. The simulations presented in Fig. 4 reproduce the resonances at $\nu_0 \sim 1.2, 1.7, 2.25$ and 2.75 in Fig. 5. We also confirmed the existence of the resonant instability near $\nu_0 \sim 0.8$ in separate simulations with increased $N_{sp}=3$ wave amplitude. However, one notices that the shape of the strong stop band near $\nu_0 \sim 1.7$ in Fig. 4 and 5 are different. In Fig. 5, the instability region splits into two separate stop bands. The stop band on left and right side

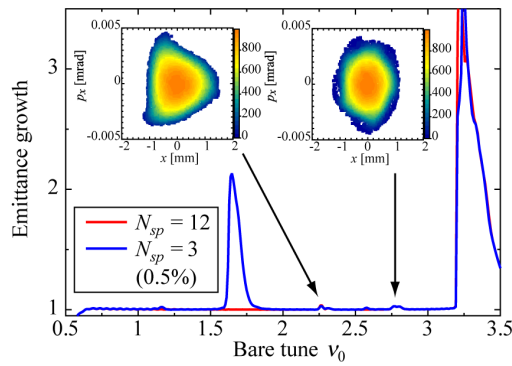


Figure 4: Simulation results.

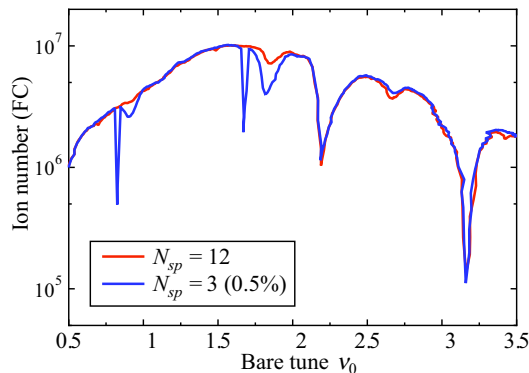


Figure 5: Experimental results.

should have its origins in 2nd and fourth order resonances respectively since the coherent tune shifts are different for the modes as predicted in Eq. (2). However, the corresponding stop band in Fig. 4 does not appear to have double peaks, and the instability is caused mainly by a linear resonance as shown in Fig. 3. Such discrepancies may be explained by differences in the initial plasma distributions, external forces, non-linear space-charge forces, etc., but further studies are needed to clarify the reason for the discrepancy.

Pulsed Excitation

In past S-POD experiments as well as ordinary applications of a Paul trap, the waveform of voltages applied to quadrupole electrode is sinusoidally varying in time. It is also possible to employ pulsed voltages in our trap system to represent commonly modeled piecewise constant quadrupole focusing functions. We here present results of a preliminary simulation for an S-POD experiment where pulsed wave is used in place of a sinusoidal wave. Analogous simulation results to Fig. 4 are given in Fig. 6 where the initial distribution is waterbag, and the rms tune depression is 0.8. Artificial non-linear rf components and the lattice-induced wave are not included for simplicity. Sinusoidal and pulsed waves with three different fractional period filling factors α are simulated. Figure 6 suggests that the stop band of a pulsed wave case is different from that of a sinusoidal case when the occupancy α is small. Note that the sinusoidal and pulsed stop bands are almost identical when $\alpha = 0.5$.

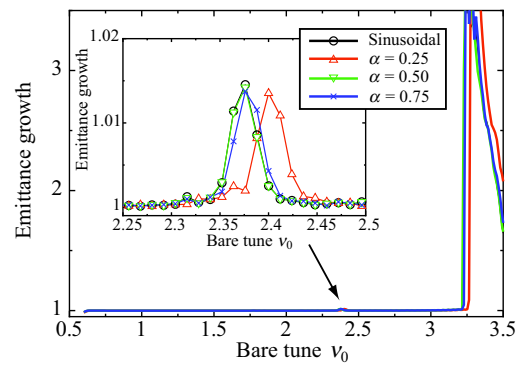


Figure 6: Simulations comparing results of a piecewise constant rf wave to a sinusoidal wave.

SUMMARY

We have carried out numerical simulations of the Hiroshima University Paul trap (S-POD) for accelerator beam modeling using the PIC code WARP. In the present simulations, a transverse 2D model is employed to save computational resources and enable large numbers of parametric runs to be carried out. Numerous options for field modeling, field solver, diagnostics and initial particle loading are available. The developed WARP simulation capability will be employed to guide future S-POD experiments.

As an example, we presented a simulation study on coherent resonant instabilities observed in S-POD. The simulation results are in good agreement with the experimental results. This agreement supports that S-POD reproduces an axially unbunched beam well. It is also confirmed from these results that coherent resonances occur when one of the coherent tunes is close to a half integer. Finally, we performed simulations when pulsed rf voltages are applied to the trap electrodes instead of sinusoidally varying voltages. Simulation results suggest that stop bands shift for pulsed voltages relative to sinusoidal voltages when the filling factor is small.

As the future simulation work, full 3D geometry of S-POD will be modeled. The 2D simulations will be valuable to guide this work. We are convinced that the combined use of plasma trap experiments and numerical simulation provides an efficient, low-cost approach to understanding collective phenomena of space-charge-dominated beams.

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